University of Groningen

Instantons and cosmologies in string theory

Collinucci, Giulio

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Publisher's PDF, also known as Version of record

Publication date:
2005

Link to publication in University of Groningen/UMCG research database

Citation for published version (APA):
Collinucci, G. (2005). *Instantons and cosmologies in string theory*. s.n.

Copyright
Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the “Taverne” license. More information can be found on the University of Groningen website: https://www.rug.nl/library/open-access/self-archiving-pure/taverne-amendment.

Take-down policy
If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): http://www.rug.nl/research/portal. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.

Download date: 04-06-2023

Instantons and Cosmologies in String Theory

Andrés Collinucci

Para mamá
Per papà

The work presented in this thesis was performed at the Centre for Theoretical Physics of the Rijks*universiteit* Groningen, in The Netherlands.

Printed by Universal Press - Science Publishers / Veenendaal, The Netherlands.

Rijks*universiteit* Groningen

Instantons and Cosmologies in String Theory

Proefschrift

ter verkrijging van het doctoraat in de
Wiskunde en Natuurwetenschappen
aan de Rijks*universiteit* Groningen
op gezag van de
Rector Magnificus, dr. F. Zwarts,
in het openbaar te verdedigen op
vrijdag 7 oktober 2005
om 13.15 uur

door

Giulio Andrea Collinucci

geboren op 10 juli 1977
te Genève

Promotor: Prof. dr. E. A. Bergshoeff

Beoordelingscommissie: Prof. dr. T. Ortín
Prof. dr. A. Van Proeyen
Prof. dr. J. G. Russo

ISBN: 90-367-2360-4

Contents

Introduction

> *If we knew what we were doing, it wouldn't be called research, would it?*
>
> A. Einstein

Unification is one of the driving principles of modern theoretical physics. Maxwell showed us around the 1860's that electricity and magnetism were not to be thought of as separate forces, but as two different manifestations of the same entity, *electromagnetism*. Later on, in the beginning of the twentieth century, Einstein asked the famous question: "What do you see if you chase a ray of light? Can you see it in its rest frame?" Although the question was not motivated by any unexplained experimental results, the answer led to a great revolution in physics. To answer his question, Einstein had to find a way to combine the framework Galilean mechanics with Maxwell's electromagnetism. This led to *special relativity*, and eventually *general relativity*, which completely modified the way we view space, time, and gravity.

In the early twentieth century, quantum mechanics was well-established as *the* theory to describe the electron in the atom. As experiments at the subatomic level became more sophisticated, being able to collide subatomic particles at higher and higher energies, it was noticed that the number of particles is not conserved in a collision process. A new theory was needed that could at the same time deal with the fact that particles are small, and hence quantum mechanical, and highly relativistic, traveling at speeds comparable to the speed of light. This led to the development of *relativistic quantum mechanics* and eventually *quantum field theory*, between the late 1920's and the 1950's. The latter is built to incorporate special relativity and quantum mechanics in one framework. Because it is relativistic, quantum field theory treats mass and energy as a single entity. Consequently, it no longer requires the conservation of the particle number in a process, as long as the total energy/mass at the end of a process is the same as that at the beginning. Because it is quantum mechanical, it also allows for a temporary violation of energy/mass conservation, leading to the off-shell intermediate states that one sees in Feynman diagrams.

The ultimate success of quantum field theory came from its concrete application to the *standard model* of particles. This model, which was developed in the 1970's, describes three of the four fundamental forces of nature:

- **Electromagnetism**: this force is responsible for most of the phenomena we observe in our lives besides gravity, such as the electric repulsion that keeps solid objects from simply merging into each other, and the fact that we can chat on cellular phones.

- **The strong nuclear force**: this is what keeps nuclei from flying apart due to their electric repulsion.
- **The weak nuclear force**: this force is responsible for radioactivity.

The standard model describes these three forces and the particles that are *charged* under them in a single gauge theory with $SU(3) \times SU(2) \times U(1)$ as its symmetry group. This theory has been experimentally confirmed within its regime of validity beyond a shadow of doubt.

We could now ask a question that is not *yet* motivated by unexplained observational data, but is in the spirit of unification: “What takes place inside a black hole?" The first notion of a black hole was discovered by Schwarzschild, as the first solution to the Einstein equations ever written down. A lot of efforts have been made, and are still being made, in order to understand the real physical meaning of this mathematical solution. What is interesting about black holes, is that they provide us with a Gedankenexperiment that forces general relativity and quantum mechanics together. The former is necessary because it is the framework for strong gravitational fields, whereas the latter is necessary because black holes are made of matter that is compressed to a very small space. This is where we notice the shortcomings of quantum field theory, and general relativity. They are seemingly incompatible. Although a theory of quantum gravity does not yet exist, there are two candidate theories: string theory, and loop quantum gravity. In this thesis, we will work with string theory.

String theory is an attempt to describe very high energy densities such as the inside of a black hole. However, it is more ambitious than that. It also has the potential to unify gravity with the other aforementioned forces of nature into one single framework, which would be valid in all possible regimes of energy and size. String theorists hope to formulate a *theory of everything*.

The theory is derived from the very simple idea that fundamental particles, which were always thought of as points (i.e. objects of zero size), are actually tiny vibrating strings of Planck length size (i.e. $\sim 10^{-35}$ m). The strings do not have fixed length, but a fixed tension, or energy density. This means that the mass of any given string is determined by its vibrational state. For instance, if it spins really fast, it will tend to stretch by centrifugal force, and will have a higher mass than a string that does not spin. Whereas a particle cannot have angular momentum, but only intrinsic spin, a string does have angular momentum. So string theory regards all different kinds of particles as being made out of the same ‘fabric’, and properties such as spin and mass are no longer intrinsic[1], but simply labels of the states in which the strings are. Trying to formulate a quantum theory of a relativistic strings (special or general relativistic), creates a world of mathematical structure that is both beautiful and complicated.

Needless to say, the path toward such an ambitious goal as formulating the theory of everything is plagued with obstacles. Although the theory has been around for several decades, as of this writing, it is still in its infancy. One might even say that string theory has so far made bigger contributions to mathematics than to physics. A major drawback of string theory is that it is only defined *perturbatively*. This means one has to assume that strings interact *weakly* with each other, in order to even define the theory. In order to be able to perform calculations, however, one often has to make one more approximation: the low energy approximation. This approximation requires that one only consider the massless states of the string. It also requires

---

[1] However, the difference between fermionic and bosonic strings is in some sense still intrinsic.

that spacetime curvature be weak. Once those criteria are met, one can treat string theory as a field theory. To be specific, the field theories used to approximate string theory are called *supergravities*. Throughout this thesis, we will be working with this approximation.

In chapter 3, we will discuss D-instantons. These are objects that arise in the supergravity approximation of string theory, yet they can actually provide us with *non-perturbative* information about string theory, i.e. they show effects that cannot be found by means of naïve perturbation theory. They are analogous to instantons in ordinary field theory in that they can only be found in the Euclidean formulation of the path integral. The D-instanton can be interpreted as a quantum field theoretic tunneling amplitude between two states of the spacetime metric, and the *axion-dilaton* scalar of type IIB supergravity. It yields a non-perturbative contribution to the calculation of the path integral. We will be studying a non-supersymmetric kind of D-instanton. We will show its relation to the better known supersymmetric D-instanton in terms of the $SL(2,\mathbb{R})$ duality symmetry of type IIB supergravity. We will also show how the general D-instanton can be viewed as a spatial section of a charged black hole, one dimension higher.

Another challenge of string theory is that it manifests itself in different forms. Until the mid 1990's, there were actually five different consistent formulations of string theory, which was very unsettling for those who believed it to be a theory of everything. However, in the mid 1990's, Edward Witten and other physicists showed that these five theories, together with eleven-dimensional supergravity (a bonus theory, so to speak), were actually different limits of one unique theory now known as *M-theory*. Unfortunately, not much is known about M-theory itself. Even the origin of its name is a mystery. One often illustrates this novel understanding of string theory by drawing a hexagon, where the corners represent all six limiting cases of M-theory, the latter being represented by the content of the polygon. The six theories are related to each other via so-called *dualities*. A duality can be thought of as the abstract generalization of the Fourier transform. Fourier transforming a differential equation means writing down the same problem in different variables, according to a certain map. A problem that seems impossible in one set of variables, can be a one-line calculation in the new variables. String theory dualities relate different theories in their opposite regimes, or sometimes they relate a theory to itself. For instance, type IIB string theory is *S-dual* to itself. This S-duality manifests itself via the action of the group $SL(2,\mathbb{Z})$ on the degrees of freedom of the theory, and it sometimes maps weakly coupled string theory to its strongly coupled counterpart.

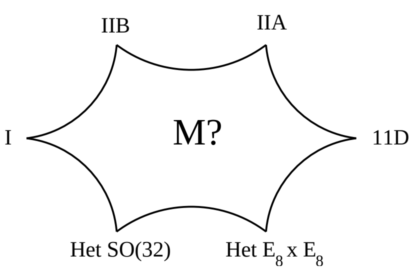

Figure 1: *The five string theories, 11-D supergravity, and M-theory.*

Although this picture shows us what we know about string and M-theory, it mainly shows us what we do not yet know. We know only the six corners of the hexagon, i.e. the extreme regimes of these theories. Everything between the corners is uncharted territory.

Another interesting discovery of the 1990's is Maldacena's AdS/CFT conjecture [1]. The latter asserts that type IIB string theory in a certain background geometry, namely $AdS_5 \times S^5$, is fully equivalent to supersymmetric Yang-Mills theory in four dimensions ($\mathcal{N} = 4, d = 4$ SYM). This is another example of a strong/weak duality. It relates the two theories in their opposite regimes of coupling strength. Therefore, this is useful for exploring the weakly and strongly coupled phases of both theories, but not the intermediate phases.

The lack of a viable framework for M-theory prevents us from deriving the laws of physics that govern strings. String theory, as it is currently formulated, does not tell us in what spacetime manifold we actually live. It treats the spacetime metric as a non-dynamical background, and imposes the Einstein equation on it as a consistency condition that restricts the kinds of allowed manifolds. In order to be able to pinpoint a unique background for string theory, one would need a theory with a *vacuum selection principle*. This is analogous to having a system with degenerate vacua and no potential that can lift the degeneracy. The best one can do is to look for backgrounds through trial and error, and see which ones are most consistent with the physical world we live in. A new emergent philosophy among string theorists, the so-called *landscape* scenario, suggests that there is no vacuum selection principle, but that all possible universes actually coexist as *bubbles* in a *megaverse*. According to this picture, we happen to live in one of the few universes where the constants of nature are such that life is possible, but other universes where it is not possible also exist. However, these are causally disconnected from ours.

So far, no verifiable or falsifiable prediction has been made by string theory. This is due to two reasons: first, technological limitations make it impossible to measure any string effects in a particle accelerator. Second, even if particle accelerators were capable of making measurements at an arbitrary energy level, string theory has not told us yet what we would see, due to its complicated nature.

Recently, however, hopes of getting string theory to make contact with reality have been revived by cosmology. First of all, cosmological processes such as supernovae are the ultimate particle accelerators, reaching energies far higher than CERN could ever dream of. Secondly, recent measurements have confirmed that our universe is undergoing a period of *accelerated expansion*. This provides string theorists with the challenge/opportunity to derive a scenario from string theory that produces accelerated expansion that is consistent with observations.

In chapters 5 and 6, we will be studying a certain class of cosmological models containing Einstein gravity and scalar fields, some of which are derivable from string theory, and some with yet unknown fundamental origins. We will specifically see when these models lead to accelerating universes, be it *eternal* or *transient* acceleration.

The Big Bang scenario, which is a widely accepted account for the early history of our universe, states that the latter was once very dense and hot, emitting perfect *blackbody radiation*. The microwave spectrum of this radiation, the famous *Cosmic Microwave Background Radiation*, has been observed and thoroughly studied, and is consistent with the Big Bang theory. Earlier in this introduction, I mentioned that black holes provided us with a 'theoretical laboratory' in which to study quantum gravity. The Big Bang is actually a real life laboratory for quantum gravity, as it describes a very dense, and hence highly curved spacetime, where short-distance physics is dominant. If string theory is a theory of everything, it must ultimately explain and ‘smooth out’ the Big Bang singularity.

This thesis is organized as follows: chapter 1 is a basic introduction to the bosonic string, and its quantization. There, I will also briefly explain the conformal field theory approach to string theory, and the physical interpretation of spacetime backgrounds. Finally, a brief summary of superstring and supergravity theories will be provided.

In chapter 2, I will introduce instantons in quantum mechanics and field theory, thereby explaining the semiclassical approximation in a Euclidean signature. This will be illustrated with examples, including the Yang-Mills instanton. Then, I will present a brief introduction to solitons. Finally, I will explain the correspondence between solitons and instantons.

Chapter 3, which is based on a publication, concerns type IIB non-extremal D-instantons. First, I will review the $SL(2,\mathbb{R})$ symmetry of type IIB supergravity and generalize to arbitrary dimensions and dilaton coupling. Later, this theory will also be generalized to theories with multiple scalars. Then, the solutions will be presented, as well as their $SL(2,\mathbb{R})$ properties. After a brief introduction to Euclidean wormholes, we will see that one class of solutions gives rise to such geometries. In analogy with the soliton-instanton correspondence explained in chapter 2, a correspondence between D-instantons and charged black holes; and D-instantons and p-branes will be established. The calculation of the action for these instanton solutions will be presented, alongside with a discussion about the potential quantum mechanical role of non-extremal D-instantons in string theory. Finally, I will comment on some work in progress, where these D-instantons are put to work in the AdS/CFT context.

In chapter 4, I will give a basic introduction to modern cosmology and its issues. I will begin by introducing the *Friedmann-Lemaître-Robertson-Walker* metric and the standard terminology for the matter and energy content of the universe. Then, I will review three main problems in cosmology: the *horizon*, *flatness*, and *relics* problems, and we will see how these are solved by inflation. I will then discuss present day acceleration, mentioning some of the current methods being used by string theorists to *derive* it.

The goal of chapter 5, which is based on a publication, will be to describe gravity-scalar models for cosmology with single-exponential potentials. We will see that these systems can be formulated as autonomous systems, and that power-law and de Sitter solutions can be regarded as critical points. We will then analyze the solutions that interpolate between critical points, paying attention to trajectories that have periods of acceleration.

In chapter 6, we will generalize on the previous chapter by analyzing multiple-exponential potentials. This chapter is based on a publication, in which the critical points are given for the most general case for the first time. This analysis is novel in that it includes cases that are even more general than what is known as ‘generalized assisted inflation’. The analysis will be illustrated by some examples of potentials with higher-dimensional origins via compactifications of gravity over three-dimensional *group manifolds*.

Just as instantons and solitons have similar mathematical structures, D-instantons and FLRW cosmologies are also mathematically similar. They are both gravity-scalar configurations that depend on only one coordinate (be it time-like or space-like). They both asymptote to ‘trivial’ configurations, but have non-trivial interpolating behavior, much like kink solutions. They can probably be viewed as sections of non-trivial bundles over the circle. In chapter 7, this parallelism will be pursued in two ways: first, we will see that some D-instantons can be related to cosmologies via Wick rotation. Then, we will see that D-instantons and scalar cosmologies can be viewed as the trajectories of particles in a fictitious *scalar manifold* or *target space*. This interpretation not only puts these solutions on equal footing, it even patches them as two portions of the same trajectory. We will see how this suggests a possible resolution of the Big Bang singularity, by means of smooth Big Crunch to Big Bang transitions that have an intermediate Euclidean period.