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Internal Flow Management in a Multi-Zone Climate Control Unit

C. De Persis, J.J. Jessen, R. Izadi-Zamanabadi, H. Schiøler

Abstract— **In this contribution, we examine a dynamic model describing the evolution of internal climate conditions in a closed environment partitioned into zones for which different climate conditions must be guaranteed. The zones are not separated, large air masses are exchanged among them, and the behavior of each zone is strongly affected by those in the neighbor zones. We discuss a control strategy which, by acting on the heating and ventilation devices of the overall system, is able to achieve the control task while efficiently managing the internal flow. It is pointed out that the controller is hybrid and decentralized. An additional feature of the controller is that it takes on values in a finite set. The possible implementation in a networked environment is briefly discussed.**

Nomenclature

T_i	Indoor air temperature of Zone i $[^\circ C]$
T^{amb}	Temperature of the supplied air $[^\circ C]$
x_i	Normalized indoor temperature of Zone i $[^\circ C]$
$Q_{in,i}$	Airflow through inlet of zone i$[m^3/s]$
$Q_{out,i}$	Airflow through outlet of zone i $[m^3/s]$
$[Q_{ij}]^+$	Internal airflow from zone i to j $[m^3/s]$
$[Q_{ij}]^-$	Internal airflow from zone j to i $[m^3/s]$
u_T	Controlled heat production $[J/s]$
w_T	Indoor heat production (disturbance) $[J/s]$
V_i	Volume of zone i $[m^3]$
c_{air}	Air Heat Capacity $[J/kg/^\circ C]$
ρ_{air}	Air Density $[kg/m^3]$

I. INTRODUCTION

We discuss the problem of guaranteeing prescribed indoor climate conditions in a building partitioned into communicating zones which exchange air flows. The prescribed climate conditions may differ very much from zone to zone. The ultimate goal is to act on the heating and ventilation devices in such a way that the climate requirement for each zone is reached even when large air masses are being exchanged and time-varying disturbances are present. These prescribed climate conditions typically mean that temperature and humidity should evolve within an interval of values (the “thermal region”). The focus of the paper will be on the temperature behavior only, but extensions to include the humidity dynamics are possible, although more involved. We refer the reader to [8], [6], [1] and references therein for recent contributions on the problem of climate control, with special emphasis on agricultural and livestock buildings, which was the initial motivation for the present investigation. Although we do not have space here to thoroughly compare our results with others, it is important to stress the main features of our contribution. We devise a controller which is suitable for implementation in a networked environment, in which sensors, controllers and actuators may be physically separated. Our controller is event-based, thus requiring sporadic measurements only. The actuators are required to provide control laws which take values in a finite and discrete set of values. Each controller governs the behavior of a single zone using information from neighbor zones, and cooperate with neighbor controllers to achieve different compatible control objectives and avoid conflicts. As a result of our approach, the overall controller turns out to be *hybrid* ([9], [2]) and *decentralized*.

We concentrate on actively cause such air masses exchange so as to make the heating and ventilation mechanism more efficient. This means that we aim to achieve an automatic mechanism to redirect warm air from hot zones (which need to be cooled down) to cold zones (which need to be heated up), and to draw as much fresh air as possible to hot zones, relying on the ventilation capacity of neighboring “collaborative” zones. In order to achieve a controller capable of maintaining the climate conditions within the various thermal regions and at the same time capable of managing the internal flow, we introduce a set of coordinating logic variables ([3]) which express the willingness of each zone to cooperate in the flow exchange, depending on the climate conditions of that zone and the neighboring ones. Then, the controller is designed to solve at each time a game theoretic problem ([3], [5]) aimed to keep the state within the thermal zone despite the action of competitive players, namely thermal disturbances, given the constraints imposed by the neighboring zones and due to their willingness to cooperate or not in the air exchange. In additions, other constraints must be fulfilled at any time, namely flow balance for each zone.

In the next section, the dynamic model is introduced. The controller is chosen within the set of safety controllers, i.e. controllers which guarantee the temperature to lie within the thermal zone for all the times. The set of these controllers is described in Section III, and then refined to characterize the safety controller with the additional capability to manage the internal flow. That the proposed controller guarantees the achievement of the control objective while fulfilling all the constraints (including the flow balance) is explicitly proven in Section IV. In Section V, we illustrate the functioning of the controller for a three-zone climate control unit and

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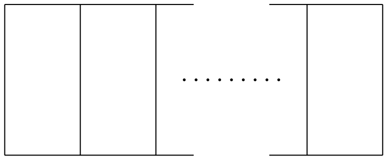

Fig. 1. Cascade connection of N zones.

conclusions are drawn in Section VI.
Due to space limitations, most of the proofs are omitted.

II. System Description and Model

In this paper, we consider a cascade connection of N rectangular zones, as illustrated in Figure 1. This corresponds to the arrangement of zones in many real-life situations, such as livestock buildings.

Each zone i, with $i \neq 1, N$, can exchange air with zones $i-1$ and $i+1$, while zone 1 and N can only exchange air with zone 2 and, respectively, $N-1$. For each $i = 1, \dots, N-1$, we denote by $[Q_{i,i+1}]^{\pm}$ the amount of air flow exchanged between zone i and zone $i+1$. More specifically, we have

$$[Q_{01}]^{\pm} = 0 \,, \quad [Q_{N,N+1}]^{\pm} = 0 \,, \tag{1}$$

and, for each $i = 1, \dots, N-1$,

$$[Q_{i,i+1}]^{+} = \begin{cases} |Q_{i,i+1}| & \text{if air flows from } i \text{ to } i+1 \\ 0 & \text{otherwise} \end{cases}$$

$$[Q_{i,i+1}]^{-} = \begin{cases} |Q_{i,i+1}| & \text{if air flows from } i+1 \text{ to } i \\ 0 & \text{otherwise ,} \end{cases}$$

where the symbol $|Q_{i,i+1}|$ denotes a nonzero and positive constant value. We naturally assume that, it is not possible to have simultaneously air exchange from zone i to zone $i+1$ and in the opposite direction. In other words, we assume that

$$[Q_{i,i+1}]^{+}[Q_{i,i+1}]^{-} = 0 \tag{2}$$

for each $i = 1, \dots, N$. Each zone is equipped with an inlet, an outlet, and a ventilation fan, which allow the zone to exchange air with the outside environment and with the neighboring zones. Indeed, by turning on the fan, air is forced out of each zone through the outlet. The amount of air outflow is denoted by the symbol $Q_{out,i}$. An amount $Q_{in,i}$ of inflow enters the zone through the inlet, and the following flow balance must hold: For each $i = 1, 2, \dots, N$,

$$\begin{aligned} &Q_{in,i} + [Q_{i-1,i}]^{+} + [Q_{i,i+1}]^{-} = \\ &Q_{out,i} + [Q_{i-1,i}]^{-} + [Q_{i,i+1}]^{+} \,. \end{aligned} \tag{3}$$

We explicitly remark that the amount of inflow depends on the outflow caused by the ventilation fan at the outlet. We now turn our attention to the equations describing the climate condition for each zone. Relevant quantities are the internal temperature $T_i \in \mathbb{R}$ and humidity $h_i \in \mathbb{R}_{\geq 0}$. For the sake of simplicity, in this paper we focus on temperature behavior only, which is therefore taken as state variable. In addition to the ventilation rates $Q_{out,i}$ provided by the fans, and the inflows $Q_{in,i}$ flowing through the inlets, another degree of control is given by the heating system, which provides a controlled amount u_i of heat. Moreover, we shall model the effect of internal disturbances which provide an additional amount of heat w_{Ti} power. Associated to the air masses which are flowing through the zones is an amount of power proportional to their temperature and the air heat capacity, which gives rise to changes in the temperature inside each zone. By balancing such power in each zone, the following equations are easily obtained (cf. e.g. [4],[1]) for $i = 1, 2, \dots, N$:

$$\begin{aligned} &\rho_{air} c_{air} V_i \frac{d}{dt} T_i = \rho_{air} c_{air} \big([Q_{i-1,i}]^{+} T_{i-1} + Q_{in,i} \times \\ &\times T_{amb} - [Q_{i,i+1}]^{+} T_i - Q_{out,i} T_i - [Q_{i-1,i}]^{-} T_i + \\ &[Q_{i,i+1}]^{-} T_{i+1} \big) + u_i + w_{Ti} \,. \end{aligned} \tag{4}$$

Setting, by a slight abuse of notation,

$$u_i = u_i / (\rho_{air} c_{air}) \,, \quad w_{Ti} = w_{Ti} / (\rho_{air} c_{air}) \,,$$

assuming that outside temperature T_{amb} is constant, and introducing the change of coordinates

$$x_i = T_i - T_{amb} \,, \quad i = 1, \dots, N \,,$$

we obtain, bearing in mind (3), and after easy calculations, the equations, for $i = 1, 2, \dots, N$,

$$\begin{aligned} &V_i \frac{d}{dt} x_i = [Q_{i-1,i}]^{+} x_{i-1} - [Q_{i,i+1}]^{+} x_i - Q_{out,i} x_i - \\ &[Q_{i-1,i}]^{-} x_i + [Q_{i,i+1}]^{-} x_{i+1} + u_i + w_{Ti} \,. \end{aligned} \tag{5}$$

In what follows, we shall refer to the x_i's simply as the temperature variables, although they differ from the actual temperature variables by a constant offset.
There are limitations on the control effort which can be delivered. In particular, the outflow $Q_{out,i}$ and the controlled heat must respectively fulfill

$$Q_{out,i} \in [0, Q_{out,i}^{M}] \,, \quad u_i \in [0, u_i^{M}] \,, \tag{6}$$

for some known constants $Q_{out,i}^{M}$ and u_i^{M}. The only way to regulate the amount of inflow is acting on the opening angle of a moving screen at the inlet, which can take only a finite number of positions. As a consequence, we assume that the inflow through the inlets can take only a finite number of values, i.e.

$$Q_{in,i} \in \Delta_i \,, \tag{7}$$

with Δ_i a finite set of nonnegative values which will become clear later (see (17) and the remark following it). We stack in the vector U all the control signals $[Q_{i-1,i}]^{\pm}$, $i = 2, \dots, N$, $[Q_{i,i+1}]^{\pm}$, $i = 1, \dots, N-1$, $Q_{in,i}$, $Q_{out,i}$, u_i, $i = 1, 2, \dots, N$ and denote by $\mathcal{U}$ the set of admissible (piecewise continuous) control signals which satisfy (1), (2), (3), (6), (7). Note that not all the components of the vector U are independent, as they are related through the constraints (2), (3). Additional constraints will be added by the introduction of the coordinating logic variables in the next subsection. Finally, we denote by $\mathbf{U}$ the set of values taken by the vector U, letting, for $i = 1, \dots, N$, $Q_{out,i}$ and u_i range in the intervals given in (6), $Q_{in,i}$ take values in the set (7), and $[Q_{i-1,i}]^{\pm}$, $[Q_{i,i+1}]^{\pm}$ be such that (2), (3) are fulfilled.

The disturbance signals are not measured, but they are bounded $w_{Ti} \in [w_{Ti}^m, w_{Ti}^M]$, and the upper and lower bounds are assumed to be known. The vector $W = (w_{T1}, \ldots, w_{TN})^T$ of disturbance signals taking values on the above intervals is said to belong to the class $\mathcal{W}$ of admissible disturbances. The set $\mathbf{W} := [w_{T1}^m, w_{T1}^M] \times \ldots \times [w_{TN}^m, w_{TN}^M]$ denotes the range of values taken by the vector W.

A. Coordinating logic variables

To systematically resolve conflicts, which may arise when the control objectives of neighbor zones are contrasting, we introduce coordinating logic variables ([3]). Without loss of generality, we regard such variables as state variables which takes values in the binary set and whose derivatives are constantly equal to zero. Their values are reset from time to time by the hybrid controller to be specified below. For Zone 1, the logic variables are $[Q_{12}]_1^+$, $[Q_{12}]_1^-$, for Zone N, $[Q_{N-1,N}]_N^+$, $[Q_{N-1,N}]_N^-$, and for each zone $i \neq 1, N$, $[Q_{i-1,i}]_i^+$, $[Q_{i-1,i}]_i^-$, $[Q_{i,i+1}]_i^+$, $[Q_{i,i+1}]_i^-$. Each one of the logic variables takes values in the set $\{0,1\}$. If Q is the vector in which all the logic variable are stacked, we have $Q \in \{0,1\}^{2N}$. The logic variables $[Q_{i-1,i}]_i^{\pm}$ are set by zone i. Loosely speaking, if $[Q_{i-1,i}]_i^+ = 0$, this means that zone i does not want to accept air flow from zone i. On the contrary, if $[Q_{i-1,i}]_i^+ = 1$, the zone is willing to accept air flow from zone i. Note that $[Q_{i-1,i}]_i^+ = 1$ does not necessarily imply that flow will occur from zone $i-1$ to i, i.e. $[Q_{i-1,i}]^+ \neq 0$, as this depends on whether or not zone $i-1$ is willing to provide air to zone i. Similarly for the other logic variables. The rules followed to set the logic variables to a new value and when this should take place is discussed in the next section. Furthermore, for each zone, we introduce "cumulative" variables, which describe the amount of internal flow that the neighboring zones are willing to exchange in either one of the two directions. Such variables are recursively defined as follows:

$$\begin{array}{rcl} Q_N^+ & = & 0 \\ Q_i^+ & = & (Q_{i+1}^+ + 1)[Q_{i,i+1}]_i^+ \cdot \\ & & [Q_{i,i+1}]_{i+1}^+ \; i = N-1, \ldots, 1 \; , \end{array} \quad (8)$$

and

$$\begin{array}{rcl} Q_1^- & = & 0 \\ Q_i^- & = & (Q_{i-1}^- + 1)[Q_{i-1,i}]_{i-1}^- \cdot \\ & & [Q_{i-1,i}]_i^- \; i = 2, \ldots, N \; . \end{array} \quad (9)$$

III. Safety Controllers

In this section we characterize the set of all safety controllers, and the maximal controlled invariant set [5]. Later, we shall single out in such a set the controller which additionally allows to manage the internal flows among the zones efficiently. By safety controller, we mean that controller which is able to maintain the state of the system within the so-called *thermal region*:

$$\begin{array}{rcl} F & := & \{x_i : x_i \in [x_{im}, x_{iM}] \; , \quad i = 1, \ldots, N\} \\ & = & \Pi_{i=1}^N F_i := \Pi_{i=1}^N \{x_i : x_i \in [x_{im}, x_{iM}]\} \; , \end{array}$$

where

$$x_{im} = T_i^m - T^{amb} \geq 0 \; , \quad x_{iM} = T_i^M - T^{amb} > x_{im} \; ,$$

for all the times, for any initial vector state, and under the action of any admissible disturbance $W \in \mathcal{W}$. The controller is designed following the indications of [5]. In the next subsection, we briefly recall the design procedure tackled there and refer the interested reader to the original source for more details. The procedure is applied to the design of the controller for each single zone. The interaction with the neighboring zones is tackled by a wise use of the coordination variables.

A. Design Procedure

The problem is that of designing a controller which guarantees the state x_i which describes the evolution of the temperature of zone i to belong to F_i, the projection on the x_i-axis of the thermal region F, for all the times. Following [5], the problem is addressed by formulating the two game problems:

$$\begin{array}{l} J_i^{1*}(x,t) = \max\limits_{U(\cdot)\in\mathcal{U}} \min\limits_{W(\cdot)\in\mathcal{W}} J_i^1(x, U(\cdot), W(\cdot), t) \; , \\ J_i^{2*}(x,t) = \max\limits_{U(\cdot)\in\mathcal{U}} \min\limits_{W(\cdot)\in\mathcal{W}} J_i^2(x, U(\cdot), W(\cdot), t) \; , \end{array} \quad (10)$$

where the value functions

$$J_i^1(x, U(\cdot), W(\cdot), t) = \ell_i^1(x(0)) := x_i(0) - x_{im} \; ,$$
$$J_i^2(x, U(\cdot), W(\cdot), t) = \ell_i^2(x(0)) := -x_i(0) + x_{iM} \; ,$$

represent the cost of a trajectory $x(\cdot)$ which starts from x at time $t \leq 0$, evolves according to the equations (5) under the action of the control $U(\cdot)$ and the disturbance $W(\cdot)$. Clearly, $F_i = \{x : \ell_i^j(x) \geq 0 \text{ for } j = 1, 2\}$. In [5], the set of safe sets is defined as $\{x : J_i^{j*}(x) := \lim_{t\to-\infty} J_i^{j*}(x,t) \geq 0\}$, where the function $J_i^{j*}(x,t)$, $j = 1, 2$, is found by solving the Hamilton-Jacobi equation

$$\begin{array}{rcl} -\dfrac{\partial J_i^{j*}(x,t)}{\partial t} & = & \min\left\{0, H_i^{j*}\left(x, \dfrac{\partial J_i^{j*}(x,t)}{\partial x}\right)\right\} \\ J_i^{j*}(x,0) & = & \ell_i^j(x) \; , \end{array} \quad (11)$$

$H_i^{j*}(x,p)$, the optimal Hamiltonian, is computed through the point-wise optimization problem

$$H_i^{j*}(x,p) = \max_{U\in\mathbf{U}} \min_{W\in\mathbf{W}} H_i^{j*}(x,p,U,W) \; , \quad (12)$$

and

$$H_i^{j*}(x,p,U,W) = p^T f(x,U,W) \; .$$

Notice that, by (11), at each time $J_i^{j*}(x,t)$ is non decreasing. Hence, if $J_i^{j*}(x) \geq 0$, then $J_i^{j*}(x,0) \geq 0$ as well, i.e. $\ell_i^j(x) \geq 0$. In other words, as expected, the set of safe states $\{x : J_i^{j*}(x) \geq 0\}$ is included in the set $\{x : \ell_i^j(x) \geq 0\}$.

B. Maximal Controlled Invariant Set

The maximal controlled invariant contained in F is the largest set of initial conditions for the state variables for which there exists a control action which maintains the state within F no matter what is the admissible disturbance acting on the system. In what follows, we show that in the present case, such a set coincides with F itself. This should not come as a surprise, the system being *bilinear* with the disturbance appearing linearly.

Lemma 1: For any $i = 1, 2, \ldots, N$, if

$$u_i^M \geq -w_{Ti}^m \ , \tag{13}$$

we have

$$\{x \, : \, J_i^{1*}(x) \geq 0\} = \{x \, : \, \ell_i^1(x) \geq 0\} \ . \tag{14}$$

Lemma 2: For any $i = 1, 2, \ldots, N$, if

$$Q_{out,i}^M x_{iM} - w_{Ti}^M \geq 0 \ , \tag{15}$$

then

$$\{x \, : \, J_i^{2*}(x) \geq 0\} = \{x \, : \, -x_i + x_{iM} \geq 0\} \ . \tag{16}$$

The two lemma above lead us to trivially conclude the following:

Proposition 1: If for any $i = 1, 2, \ldots, N$ (13) and (15) hold, then the maximal controlled invariant set coincides with F.

Proof: It is enough to notice that the entire set of safe sets, namely

$$\bigcap_{i=1}^{N} \bigcap_{j=1}^{2} \{x \, : \, J_i^{j*}(x) \geq 0\} \ ,$$

coincides with the set F, and hence any other controlled invariant set must be contained in the one given above. ∎

Remark. Conditions (13) and (15) are very frequently encountered in practice and, loosely speaking, they are necessary for the control objective to be achieved. Notice that, as for condition (13), even for condition (15) a smaller safety set does not help to relax these requirements. Indeed, (13) is independent of the state, while (15) is such that if it holds for any x_i which is inside the thermal region, then it is *a fortiori* true for $x_i = x_{iM}$.

C. Safety Controller with Internal Flow Management

In this section we propose a safety controller which enjoys additional important features. First, for each zone, it takes into account the constraints imposed by the neighbor zones. In doing so, it is able to guarantee flow exchange among zones when all the zones are willing to carry out this action, while it avoids the raise of conflicts when the actions carried out by neighbor zones are not compatible with each other. We shall operate under the following:

Assumption 1: At each time, each zone is either cooling down or heating up.

Remark. To have this fulfilled, it suffices to have conditions (13) and (15) fulfilled with strict inequalities.

Given that, for each $i = 1, \ldots, N$, the local controller has access at each time to the temperatures x_{i-1}, x_i, x_{i+1} (to x_i, x_{i+1} if $i = 1$, and to x_{i-1}, x_i if $i = N$), and to the coordinating variables, and that it also trivially knows whether the zone is in the "cooling down" or "heating up" mode, the values for the coordinating logic variables and controls are chosen so as to enforce the maximizing controller $U(\cdot)$ for the game $J_i^{1*}(x,t)$, if the zone is heating up, or for the game $J_i^{2*}(x,t)$, if the zone is cooling down, and taking into account the additional constraints imposed by the logic variables of the neighboring zones. Notice that we use the notation $Q_{in,i}^M$ to denote the value[1]

$$Q_{in,i}^M := Q_{out,i}^M + \sum_{j=1}^{Q_i^-} Q_{out,i-j}^M + \sum_{j=1}^{Q_i^+} Q_{out,j+i}^M \ . \tag{17}$$

Remark. Depending on the values of Q_i^-, Q_i^+, which in turn depend on the values taken by the coordinating logic variables, the variable $Q_{in,i}^M$ can represent different values. All the possible values for $Q_{in,i}^M$ obtained from (17) define the set Δ_i introduced in Section II.

We now introduce, for each Zone i, the controller which is able to handle the conflicting scenarios. To this purpose we need to explicitly take into account the conditions at the neighbor zones, namely temperatures and logic variables. As a result, for each Zone i, we precisely characterize the optimal controller which satisfies the game problems (10). Furthermore, by construction, whenever the neighbor zones agree on the actions to carry out (and this can be seen on the values taken by the coordinating logic variables), warm air is redirected from zones which are cooling down to zones which are heating up and are at lower temperatures. At the same time, the zones which are heating up collaborate with the neighbor zones which are cooling down to increase the amount of outflow. The controller is summarized in Table 2. For the special cases $i = 1, N$, the controller simplifies, and it becomes easy to represent the behavior of the switched controller by a graph – see Figure 3 for $i = 1$.

Proposition 2: Let Assumptions 1 and, for each $i = 1, 2, \ldots, N$, (13), (15) hold. Suppose additionally that, for each $i = 1, 2, \ldots, N-1$ (respectively, $i = 2, \ldots, N-2, N-1$),

$$[Q_{i,i+1}]^+ = \sum_{j=1}^{Q_i^+} Q_{out,j+i}^M \quad \left([Q_{i-1,i}]^- = \sum_{j=1}^{Q_i^-} Q_{out,i-j}^M \right) \ . \tag{18}$$

Then, for each $i = 1, 2, \ldots, N$, the controller described in Table 2, renders F_i invariant and is the maximizing controller of the game problems (10).

Remark. As already mentioned, if both (13) and (15) hold with strict inequalities, then Assumption 1 can be removed from the statement. Moreover, we shall prove in the next

[1] In the sums below, if $Q_i^+ = 0$ ($Q_i^- = 0$), then $[Q_{i,i+1}]^+ = 0$ ($[Q_{i-1,i}]^- = 0$).

Mode	Zone i Cooling Down							
		$Q_{out,i}$	$Q_{in,i}$	u_i	$[Q_{i-1,i}]_i^+$	$[Q_{i-1,i}]_i^-$	$[Q_{i,i+1}]_i^+$	$[Q_{i,i+1}]_i^-$
1	Under any condition	$Q_{out,i}^M$	$Q_{in,i}^M$	0	0	1	1	0
	Zone i Heating Up							
2	$x_{i-1} \leq x_i \wedge x_i \geq x_{i+1}$	0	0	u_{iM}	0	0	0	0
3	$x_{i-1} > x_i \wedge x_i \geq x_{i+1}$ $[Q_{i-1,i}]_{i-1}^+ = 0$	0	0	u_{iM}	1	0	0	0
4	$x_{i-1} > x_i \wedge x_i \geq x_{i+1}$ $[Q_{i-1,i}]_{i-1}^+ = 1$	$Q_{out,i}^M$	0	u_{iM}	1	0	1	0
5	$x_{i-1} \leq x_i \wedge x_i < x_{i+1}$ $[Q_{i,i+1}]_{i+1}^- = 0$	0	0	u_{iM}	0	0	0	1
6	$x_{i-1} \leq x_i \wedge x_i < x_{i+1}$ $[Q_{i,i+1}]_{i+1}^- = 1$	$Q_{out,i}^M$	0	u_{iM}	0	1	0	1
7	$x_{i-1} > x_i \wedge x_i < x_{i+1}$ $[Q_{i-1,i}]_{i-1}^+ = 0 \wedge [Q_{i,i+1}]_{i+1}^- = 0$	0	0	u_{iM}	1	0	0	1
8	$x_{i-1} > x_i \wedge x_i < x_{i+1}$ $[Q_{i-1,i}]_{i-1}^+ = 1 \wedge [Q_{i,i+1}]_{i+1}^- = 0$	$Q_{out,i}^M$	0	u_{iM}	1	0	1	0
9	$x_{i-1} > x_i \wedge x_i < x_{i+1}$ $[Q_{i-1,i}]_{i-1}^+ = 0 \wedge [Q_{i,i+1}]_{i+1}^- = 1$	$Q_{out,i}^M$	0	u_{iM}	0	1	0	1
10	$x_{i-1} > x_i \wedge x_i < x_{i+1} \wedge x_{i-1} \geq x_{i+1}$ $[Q_{i-1,i}]_{i-1}^+ = 1 \wedge [Q_{i,i+1}]_{i+1}^- = 1$	$Q_{out,i}^M$	0	u_{iM}	1	0	1	0
11	$x_{i-1} > x_i \wedge x_i < x_{i+1} \wedge x_{i-1} < x_{i+1}$ $[Q_{i-1,i}]_{i-1}^+ = 1 \wedge [Q_{i,i+1}]_{i+1}^- = 1$	$Q_{out,i}^M$	0	u_{iM}	0	1	0	1

Fig. 2. Summary of the control law.

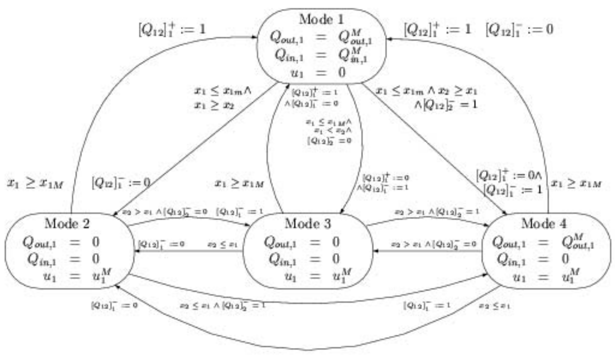

Fig. 3. Hybrid controller for Zone 1. Following standard convention, when the edge has two labels the first one represents the guard, the second one (denoted by :=) the reset. When no reset is present, then all the variables remain unchanged upon the transition. All the guards at the edges linking the three states at the bottom should include the clause $\neg x_1 \geq x_{1M}$, which is omitted for the sake of simplicity. For each mode (discrete state) only three representative values are indicated. The remaining values can be derived from Table 2 specialized to the case $i = 1$.

section that the controller to be designed below guarantees (18) to be actually satisfied.

Before ending this section, we explicitly mention that the controller introduced above clearly renders F an invariant set. It is enough to verify that, on each edge at the boundary of F, the controller makes the velocity vector to point inwards F or to be tangent at the boundary of F. This is a straightforward exercise and is left as an exercise to the reader. In the next section, we show that the safety controller characterized here is actually a feasible controller, meaning that the flow balance (3) is fulfilled for each zone. As a consequence, it will be clear that (18) is actually guaranteed by our design of the controller.

IV. Feasibility of the safety controller

The main obstacle to prove the feasibility of the safety controller investigated in the previous section comes from the fact that the dynamics of each zone are closely intertwined with those of the neighbor zones and that the number of zones are arbitrarily large. Nevertheless, we can exploit the topology of the system, namely the configuration according to which the zones are positioned, to approach the problem by an inductive argument. In particular, we shall characterize conditions under which the flow balance is fulfilled for the first 2 zones. Then we shall proceed by showing the conditions under which, assuming that the flow balance is fulfilled up to Zone i, the flow balance is fulfilled even for Zone $i+1$ and, finally, concluding the argument considering the zones $N-1$ and N.

Lemma 3: Consider the multi-zone climate control unit depicted in Fig. 1. The flow balance (3) is fulfilled for $i = 1, 2$, i.e. for Zone 1 and 2, provided that:

- If Zone 2 is in Mode 1, 4 or 8,

$$[Q_{23}]^+ = \sum_{j=1}^{Q_2^+} Q_{out,\,j+2}^M \;, \tag{19}$$

with $Q_2^+ = (Q_3^+ + 1)[Q_{23}]_3^+$, and $[Q_{23}]^- = 0$.

- If Zone 2 is in Mode 2, 3, 5, 7 or 10,

$$[Q_{23}]^+ = 0 \text{ and } [Q_{23}]^- = 0 \;. \tag{20}$$

- If Zone 2 is in Mode 6, 9 or 11, $[Q_{23}]^+ = 0$ and

$$[Q_{23}]^- = \sum_{j=1}^{Q_3^-} Q_{out,\,3-j}^M \;, \tag{21}$$

with $Q_3^- = Q_2^- + 1$.

To make the statements below more concise we introduce the following definition:

Definition. Let i be an integer such that $2 \leq i \leq N-1$. Zones $1, 2, \ldots, i$ are said to *conditionally* satisfy the flow balance (3), if (3) is satisfied for $j = 1, 2, \ldots, i$ provided that (20), (19), (21) are satisfied with index 2 replaced by i and 3 by $i+1$.

Then along the lines of the previous lemma, the following statement can be proven:

Lemma 4: For some integer $2 \leq i \leq N-2$, if Zones $1, 2, \ldots, i$ conditionally satisfy the flow balance (3), then also Zones $1, 2, \ldots, i, i+1$ conditionally satisfy the flow balance (3).

Zone N is different from the preceding zones in that air can be exchanged only through one side. Nevertheless, a similar statement holds:

Lemma 5: Suppose Zones $1, 2, \ldots, N-1$ conditionally satisfy the flow balance (3), then Zones $1, 2, \ldots, N-1, N$ satisfy the flow balance (3).

The previous statements allow us to conclude immediately with the following:

Proposition 3: For each $i = 1, 2, \ldots, N$, let (13), (15) hold with strict inequalities. Then, the controller described in Table 2 renders F_i invariant and satisfies (18).

V. Numerical results for a 3-zone climate control unit

In this section we present the outcome of a numerical simulation for the three zone case. In order to demonstrate the applicability of the presented controller we define the

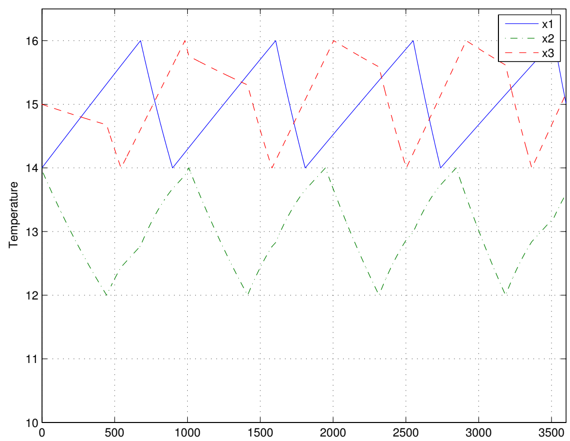

Fig. 4. Trajectory of the system. Vertical axis represents absolute temperature.

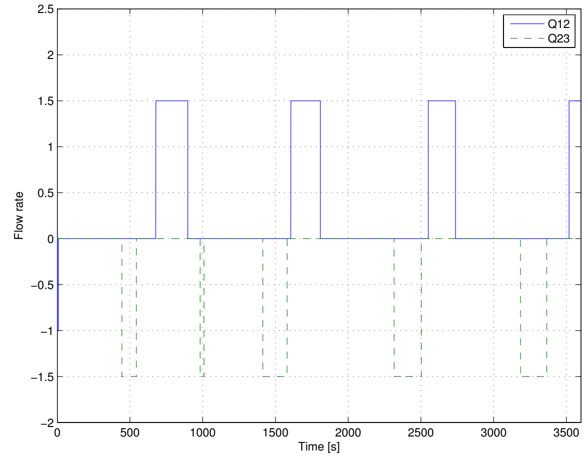

Fig. 5. Time profile for the internal flows.

zones to be of different sizes, internal disturbances, ventilation rates, heating capacities and finally different thermal regions. The numerical values are set to resemble the values for a real livestock building.

We set the zone volumes to: $V_1 = 2000,\ V_2 = 1600,\ V_3 = 1800$ and the fan capacities: $Q^M_{out,1} = 1,\ Q^M_{out,2} = 1.5,\ Q^M_{out,3} = 0.8$ Keeping in mind the abuse of notation with u_i and $w_{T,i}$ the disturbances are piece wise constant in the set $\mathbf{W} := [2,3] \times [2,2] \times [5,7]$ resembling the heat production from pigs, in a stable corresponding to the given zone sizes. We refer to [7] for details on heat production from animals. We set the heating capacities to: $u_1 = 3,\ u_2 = 2,\ u_3 = 3$. The thermal zones are defined to: $x_{1m} = x_{3m} = 14,\ x_{1M} = x_{3M} = 16,\ x_{2m} = 12,\ x_{2M} = 14$.

The initial state is set in the thermal zone, with the following initial controller actions: The controller for zone 1 is heating up, while the controllers for zone 2 and zone 3 are cooling down. Figure 4 show the result of a simulation using the presented controllers pointing out that the controllers maintain the state within the thermal region. We omit graphs for the control signals, knowing that when a zone is heating up $u_i = u_i^M$ and when cooling down $Q_{i,out} = Q^M_{i,out}$.

A key feature of the presented controllers is the capability of using internal flow as a heating mechanism. Figure 5 shows the occurrence of internal flow between the zones. As we would expect, Figure 5 shows that internal flow only occurs from zone 1 and 3 to zone 2. The reason is that the thermal region for zone 2 is lower than the thermal region for zone 1 and zone 3: $x_{1m} \geq x_{2M} \leq x_{3m}$. This means that whenever Zone 2 is heating up and either one of Zones 1 or 3 is cooling down, internal flow takes place.

VI. Conclusion

The paper has discussed a control strategy for a multi zone climate unit capable of maintaining the state within a safe set (*thermal region*), by management of internal flow between zones. The control law is inherently hybrid and decentralized in the sense that it only changes action when certain boundaries are met and/or when neighboring conditions change, and that the information requirements are limited to neighboring zones. Our motivation for considering the devised control strategy is the possible implementation in a resource constrained environment using wireless battery powered climatic sensors. Hence, we were after a solution to the problem which allowed to reach the control goal by transmitting feedback information only sporadically. We observe that the controller takes on values in a finite set, thus allowing for a potentially robust information transmission encoded using a finite number of bits. We have showed that the control law handles internal flow efficiently e.g. by using warm air from a neighbor zone to heat up, whenever certain conditions are met. An experimental facility to test our strategy has been constructed, and is currently being adjusted for minor details before we can move to real life experiments. We assume to be able to report on this ongoing work soon.

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