Essays on corporate risk management and optimal hedging

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Chapter 4

THE EFFECT OF MANAGERIAL COMPENSATION ON OPTIMAL PRODUCTION AND HEDGING WITH FORWARDS AND PUTS

4.1 INTRODUCTION

This chapter considers the effect of managerial compensation on the desired production and hedging policies for a competitive commodity producer. Over the last three decades, optimal production and hedging policies by competitive commodity producers have been the object of considerable research. Furthermore, company stock and call options have become a very important component in managerial compensation plans. For instance, Hall and Murphy (2002) report that stock-based compensation for CEOs in the S&P 500 grew from 30% of total compensation in 1992 to 56% in 1998. This growth in equity-linked compensation is primarily due to a rise in call-option grants, growing from 23% in 1992 to 48% of total compensation in 1998. Hall (2002) argues that this proportion of equity-linked compensation further increases to 66% in 2001.[83] Within managerial payment programs, especially call compensation is highly criticized because managers are accused of enriching themselves at the expense of shareholders. Since the value of a call is a positive function of the volatility of the firm, managers may be inclined to increase firm risk in

[83] Presentation at the HBS Executive Compensation Workshop on October 10, 2002.

order to maximize the expected payoff on their calls. This argument, however, ignores that unlike outside investors, managers are usually not well-diversified since most of their capital is tied to the value of the firm. This is caused by the fact that managers are prohibited from trading in their own company's stock, short-selling this company stock, or hedging company call options. If they were able to do so, a manager could sell all firm-related wealth and invest optimally on the capital market line. However, since a large proportion of the manager's wealth is generally tied to the value of the firm, both in the case of company stock as well as call compensation, it is essential to assume that this manager is risk averse and maximizes his expected utility of future consumption. This implies that the manager faces a trade-off between increasing company risk (thereby increasing the value of his calls) and decreasing company risk because he is risk averse. This causes a possible conflict of interest between the stockholders and the managers of the firm, since diversified stockholders are concerned about maximizing the market value of the firm, whereas managers want to optimize their expected utility of consumption.

This chapter analyzes the effect of managerial risk aversion on both optimal production as well as risk management decisions by the use of forward contracts and put options. By combining the literature of optimal hedging with the literature on managerial compensation, we are able to evaluate the relationship between managerial compensation, risk taking and managerial behavior. Equity-linked pay, such as stock and call-option compensation, is usually introduced to mitigate the standard manager-shareholder conflict: securities are granted to inspire the manager of the firm to act in the interest of shareholders, by tying managerial compensation to the market value of the firm. In the context of this debate, we examine whether managers are given the right incentives for choosing the hedging policy preferred by the owner of a firm. If a manager is given the right incentives, his preferred production and hedging strategies will coincide with those of the owner, resulting in incentive compatibility between the optimal decisions for the manager and the shareholders of the firm.

The amount of compensation paid to managers has been a hot item in recent years. Opponents of equity-linked compensation often argue that managers receive exorbitantly high pay levels in case of stock and call compensation. Hall and Murphy (2000, 2002) and Meulbroek (2000), however, argue that due to the fact that managers are less diversified than the marginal investor on the stock market, the managerial value of equity-linked compensation is much less than the market value. First of all, since managers are undiversified and risk averse, the managerial discount rate is (partly) determined by undiversifiable risks, which increases this discount rate. Secondly, the fact that the options are not tradable, further decreases the managerial value relative to the market value. Jensen and Murphy (1990) argue that, for providing the right incentives, it is less important how much managers are paid than the way in which they are paid (i.e., it is the kind of compensation that matters).

Surprisingly, this type of managerial compensation analysis has not yet been introduced into the corporate hedging literature.[84] Therefore, this chapter extends the existing literature by analyzing the effect of the two most common compensation

[84] There is a sizable literature on hedging and production. Among many others, Danthine (1978), Holthausen (1979), and Feder, Just, and Schmitz (1980) examine optimal hedging and production decisions and show that, in case of unbiased forward prices, the well-known separation theorem holds and that the optimal hedge is given by a full hedge. Optimal production and put hedging has also been examined by Battermann, Braulke, Broll, and Schimmelpfennig (2000) and Benninga and Oosterhof (2004). They show that the optimal put hedge is an overhedge and that optimal production is higher with forward than with put hedging.

schemes on the optimal hedging policy by a competitive firm. We examine the optimal production and hedging strategies for the manager of a competitive commodity firm who is granted a cash-based salary in combination with either shares of stock or, as an alternative, at-the-money call options.[85] The manager can hedge commodity price risk with either forward contracts or put options, for both of which the pricing is assumed to be unbiased.[86] Within this analysis, we extend the existing line of literature by assuming that markets are incomplete in the sense that the implicit pricing system of individual agents may differ from the consensus in the market (see also Chapter 3).[87] Since a private implicit pricing system (i.e., the state prices from which all financial assets are perceived to be priced) is derived from individual utility functions, managerial state prices may differ from those of the primary owner of the firm (the latter will also be indicated as "the producer", in accordance with Chapter 3). Furthermore, both sets of prices may differ from the market state prices. As a consequence, any agent in the economy may agree or disagree with the market prices which, from an empirical point of view, is to be expected. Since all individuals have different preferences, the private value of a state-dependent security may differ among these individuals. However, by definition of competitive markets, the market price of any tradable asset is given and cannot be influenced by any individual, i.e., also not by the manager of the firm nor the producer himself. Optimal decision making is therefore driven by the differences between the fixed market prices and perceived

[85] Among many others, Hall and Murphy (2000, 2002) report that managerial stock options are usually issued at the money.

[86] See Chapter 3 for a detailed analysis of pricing unbiasedness.

[87] In general, in incomplete markets there is no unique implicit price system. However, for ease of exposition, we assume that the prevailing asset prices in the market are considered to be based on a consensus set of state prices, to be labeled as market state prices.

private prices (i.e., the price the manager or producer are willing to pay, given their utility functions).

In this chapter, we derive a number of new results. First, we show that if the manager can use unbiased forward contracts to hedge the firm's commodity price risk, then both stock as well as at-the-money call compensation will result in the well-known full hedging and separation theorems. Furthermore, since full hedging is also optimal for the primary owner of the firm, this implies that there is incentive compatibility between the owner and the manager of the firm. This result is independent of the assumption of market incompleteness. Second, if the manager uses unbiased put options as a hedging device, this results in overhedging the commodity price risk exposure for both cases of equity-linked compensation. Furthermore, given hedging with unbiased put options, the well-known separation theorem does not hold. The overhedge decreases as managerial equity-linked compensation increases and the overhedge also decreases as the manager's degree of risk aversion increases. Numerical exercises confirm the theorems and lemmas derived, and show that incentive compatibility between the manager and the primary owner of the firm rarely ever occurs. However, under reasonable parameter values, the desired strategies do not differ very much implying that the primary owner of the firm is not harmed too much by the hedging and production strategies, which are suboptimal from his point of view.

The remainder of this chapter is organized as follows. Section 4.2 sets out our model. In Section 4.3, we derive the optimal hedging strategies if the manager can hedge commodity price risk with forward contracts, whereas Section 4.4 analyzes the impact of managerial compensation on optimal hedging with put options. Section 4.5 provides a numerical analysis of the analytical results derived before. We summarize our findings in Section 4.6.

4.2 THE MODEL

We employ a two-date framework where today is denoted as time 0 and the future as time 1. At time 1, N possible states of the world can occur. The model's single asset has a spot price S_0 today and $\tilde{S} = \{S_1, S_2, ..., S_N\}$ prices in the states of the world tomorrow. The state probabilities are given by $\tilde{\pi} = \{\pi_1, \pi_2, ..., \pi_N\}$, and the implicit state prices by which financial assets are priced are denoted as $\tilde{q} = \{q_1, q_2, ..., q_N\}$. The essence of our analysis is that the market is assumed to be incomplete in the sense that there is no unique state-price vector, and private implicit state prices will, in general, differ across individual agents on the market. This will cause the manager to value financial claims according to his private personal state prices and – therefore – hedge commodity price risk according to these private values (see Benninga and Oosterhof, 2004). Recall from Chapter 3 that the private state prices are derived from his utility function and given by $q_j = \delta \pi_j \frac{U'(c_j)}{U'(c_0)}$, in which δ is the manager's pure rate of time preference. Consider a firm, which has already decided on its production technology and sells a single product in a competitive market. Henceforth, we will call the initial owner of this firm the producer. Consider the situation in which the producer has employed a manager who is responsible for the production and hedging policy of the firm. This manager is granted cash in combination with either shares of stock or call options in his compensation plan. The manager is, however, prohibited from selling his shares and call options at date 0.[88]

We assume that, at time 0, the manager has to derive the optimal production and risk management decisions and that output y is non-stochastic at time 1. The cost function $C(y)$ is strictly increasing and convex such that $C(0) \geq 0$, $C'(y) > 0$, and $C''(y) > 0$. The producer faces uncertainty because the future price of the products sold is random. At time 1, uncertainty regarding this commodity price is resolved. Furthermore, if the manager has decided to hedge, the forward and put contracts may pay off at this time. Assuming that the producer invests the initial wealth W_0 at the risk-free rate of interest, future firm value Y_j and stock price V_j are given by:

(4.1) $$Y_j = \left(W_0 - C(y)\right) \cdot \left(1 + r_f\right) + S_j \cdot y + n_F \cdot \left(S_j - F\right) + n_P \cdot \left(\left[X_P - S_j\right]^+ - P\left(X_P\right) \cdot \left(1 + r_f\right)\right)$$

(4.2) $$V_j = \frac{Y_j}{n_V}$$

where r_f is the risk-free rate of interest, n_F is the number of forward contracts with a forward price equal to F, n_P is the number of puts with an exercise price of X_P and a price equal to $P(X_P)$, whereas n_V represents the total number of shares outstanding.

We assume that the manager, who is responsible for the hedging and operating policies of the firm, maximizes a strictly concave Von Neumann-Morgenstern utility function U defined over his future consumption c_j. Since the manager is strictly risk averse, $U'(c_j) > 0$ and $U''(c_j) < 0$. We consider the two most common

[88] Since stock and call compensation is issued to motivate managers to act in the interest of shareholders, it is essential that they are non-tradable and that managers are not allowed to short-selling company stocks and options. If the manager can sell or hedge his equity-linked compensation, his personal wealth is not tied to the value of the firm, giving him no incentives at all to act in the interest of shareholders.

compensation schemes for the manager. We first analyze the case where the manager's salary is paid in cash and shares of stock. Secondly, we consider the effect of managerial compensation by cash and call options on the optimal hedging and production policies.

With respect to the first case, suppose for now that the manager owns a total number of n_S shares of stock. Besides making the optimal production decision, the manager can also choose the number of forward and put contracts to hedge price risk exposure. However, since his personal wealth depends on the value of the firm, he can use these derivatives in order to maximize his expected utility of consumption. In this case his maximization problem becomes:

$$\begin{aligned} &\underset{n_F, n_P, y}{Max}\, E\left[U\left(\tilde{c}\right)\right] = \delta \sum_{j=1}^{N} \pi_j U\left(c_j\right) \\ (4.3)\quad & s.t. \\ & c_j = \overline{W} + n_S \cdot V_j \end{aligned}$$

where δ is the manager's pure rate of time preference, c_j is consumption in state *j*, and $\overline{W}$ is his cash-based salary. The relevant first-order conditions for solving optimization problem (4.3) in case of forwards, puts, and production, respectively, are given below:

(4.4)
$$\begin{aligned}\frac{dE\left[U\left(\tilde{c}\right)\right]}{dn_F} &= \delta \cdot \frac{n_S}{n_V} \cdot \sum_{j=1}^{N} \pi_j U'\left(c_j\right)\left(S_j - F\right) \\ &= \delta \cdot \frac{n_S}{n_V} \cdot E\left[U'\left(c_j\right)\left(S_j - F\right)\right] \\ &= \delta \cdot \frac{n_S}{n_V} \cdot \left(E\left[U'\left(c_j\right)\right]E\left[\left(S_j - F\right)\right] + Cov\left(U'\left(c_j\right),\left(S_j - F\right)\right)\right) \\ &= 0\end{aligned}$$

(4.5)
$$\begin{aligned}\frac{dE\left[U\left(\tilde{c}\right)\right]}{dn_P} &= \delta \cdot \frac{n_S}{n_V} \cdot \sum_{j=1}^{N} \pi_j U'\left(c_j\right)\left(\left[X_P - S_j\right]^+ - P\left(X_P\right)\left(1+r_f\right)\right) \\ &= \delta \cdot \frac{n_S}{n_V} \cdot E\left[U'\left(c_j\right)\left(\left[X_P - S_j\right]^+ - P\left(X_P\right)\left(1+r_f\right)\right)\right] \\ &= \delta \cdot \frac{n_S}{n_V} \cdot \left(E\left[U'\left(c_j\right)\right]E\left[\left[X_P - S_j\right]^+ - P\left(X_P\right)\left(1+r_f\right)\right] + Cov\left(U'\left(c_j\right),\left[X_P - S_j\right]^+\right)\right) \\ &= 0\end{aligned}$$

(4.6)
$$\begin{aligned}\frac{dE\left[U\left(\tilde{c}\right)\right]}{dy} &= \delta \cdot \frac{n_S}{n_V} \cdot \sum_{j=1}^{N} \pi_j U'\left(c_j\right)\left(S_j - C'(y)\left(1+r_f\right)\right) \\ &= \delta \cdot \frac{n_S}{n_V} \cdot E\left[U'\left(c_j\right)\left(S_j - C'(y)\left(1+r_f\right)\right)\right] \\ &= \delta \cdot \frac{n_S}{n_V} \cdot \left(E\left[U'\left(c_j\right)\right]E\left[\left(S_j - C'(y)\left(1+r_f\right)\right)\right] + Cov\left(U'\left(c_j\right),\left(S_j - C'(y)\left(1+r_f\right)\right)\right)\right) \\ &= 0\end{aligned}$$

Recall from Chapter 3 that the forward price is said to be unbiased if $F = E\left[\tilde{S}\right] = \sum_{j=1}^{N} \pi_j S_j$, and that the put price is unbiased if $P\left(X_P\right) = \frac{E\left[X_P - S_j\right]^+}{\left(1+r_f\right)} = \frac{\sum_{j=1}^{N} \pi_j \left[X_P - S_j\right]^+}{\left(1+r_f\right)}$.[89] If the forward and the put price are unbiased, equations (4.4) and (4.5) reduce to:

(4.7)
$$\frac{dE\left[U\left(\tilde{c}\right)\right]}{dn_F} = \delta \cdot \frac{n_S}{n_V} \cdot Cov\left(U'\left(c_j\right),\left(S_j - F\right)\right) = 0$$

89 For details on unbiasedness of forward and put prices, see e.g., Benninga and Oosterhof (2004) (i.e., see Chapter 3 of this dissertation).

(4.8) $$\frac{dE\left[U\left(\tilde{c}\right)\right]}{dn_P} = \delta \cdot \frac{n_S}{n_V} \cdot Cov\left(U'\left(c_j\right), \left[X_P - S_j\right]^+\right) = 0$$

The second case to consider is when the manager is rewarded a cash-based salary and additionally receives a number of call options on the firm's stock price, exercisable at time 1. Again, the manager can choose the number of forward and put contracts, as well as total production of the firm. If the manager is awarded a total number of n_C calls with an exercise price equal to X_C, his maximization problem becomes:

(4.9) $$\begin{aligned} &\underset{n_F, n_P, y}{Max}\ E\left[U\left(\tilde{c}\right)\right] = \delta \sum_{j=1}^{N} \pi_j U\left(c_j\right) \\ &s.t. \\ &c_j = \overline{W} + n_C \cdot \left[V_j - X_C\right]^+ \end{aligned}$$

The relevant first-order conditions for solving the optimization problem (4.9) in case of the optimal number of forwards, puts and production, respectively, are:

(4.10) $$\begin{aligned} \frac{dE\left[U\left(\tilde{c}\right)\right]}{dn_F} &= \delta \cdot \frac{n_C}{n_V} \cdot \sum_{j=1}^{N} \pi_j U'\left(c_j\right)\left(S_j - F\right) \\ &= \delta \cdot \frac{n_C}{n_V} \cdot E\left[U'\left(c_j\right)\left(S_j - F\right)\right] \\ &= \delta \cdot \frac{n_C}{n_V} \cdot \left(E\left[U'\left(c_j\right)\right] E\left[\left(S_j - F\right)\right] + Cov\left(U'\left(c_j\right), \left(S_j - F\right)\right)\right) \\ &= 0 \end{aligned}$$

$$
\text{(4.11)}\quad
\begin{aligned}
\frac{dE\left[U\left(\tilde{c}\right)\right]}{dn_P} &= \delta\cdot\frac{n_C}{n_V}\cdot\sum_{j=1}^{N}\pi_j U'\left(c_j\right)\left(\left[X_P-S_j\right]^+ - P\left(X_P\right)\left(1+r_f\right)\right)\\
&= \delta\cdot\frac{n_C}{n_V}\cdot E\left[U'\left(c_j\right)\left(\left[X_P-S_j\right]^+ - P\left(X_P\right)\left(1+r_f\right)\right)\right]\\
&= \delta\cdot\frac{n_C}{n_V}\cdot\left(E\left[U'\left(c_j\right)\right]E\left[\left[X_P-S_j\right]^+ - P\left(X_P\right)\left(1+r_f\right)\right] + Cov\left(U'\left(c_j\right),\left[X_P-S_j\right]^+\right)\right)\\
&= 0
\end{aligned}
$$

$$
\text{(4.12)}\quad
\begin{aligned}
\frac{dE\left[U\left(\tilde{c}\right)\right]}{dy} &= \delta\cdot\frac{n_C}{n_V}\cdot\sum_{j=1}^{N}\pi_j\cdot U'\left(c_j\right)\cdot\left(S_j - C'(y)\left(1+r_f\right)\right)\\
&= \delta\cdot\frac{n_C}{n_V}\cdot E\left[U'\left(c_j\right)\cdot\left(S_j - C'(y)\left(1+r_f\right)\right)\right]\\
&= \delta\cdot\frac{n_C}{n_V}\cdot\left(E\left[U'\left(c_j\right)\right]E\left[\left(S_j - C'(y)\left(1+r_f\right)\right)\right] + Cov\left(U'\left(c_j\right),\left(S_j - C'(y)\left(1+r_f\right)\right)\right)\right)\\
&= 0
\end{aligned}
$$

If the prices of the forward contracts and the put options are unbiased, equations (4.10) and (4.11) reduce to:

$$
\text{(4.13)}\quad \frac{dE\left[U\left(\tilde{c}\right)\right]}{dn_F} = \delta\cdot\frac{n_C}{n_V}\cdot Cov\left(U'\left(c_j\right),\left(S_j - F\right)\right) = 0
$$

$$
\text{(4.14)}\quad \frac{dE\left[U\left(\tilde{c}\right)\right]}{dn_P} = \delta\cdot\frac{n_C}{n_V}\cdot Cov\left(U'\left(c_j\right),\left[X_P - S_j\right]^+\right) = 0
$$

Thus, if both the prices of forward contracts and put options are unbiased predictors of their future payoff, the optimal hedging policy is to choose the number of forward contracts and put options such that the covariance between the marginal utility of managerial consumption and the payoff on the derivative instruments is zero. Given unbiasedness of forward and put prices, this optimization problem holds for any compensation package. Specifically, in the case of this chapter it holds for equations (4.7), (4.8), (4.13), and (4.14).

4.3 OPTIMAL HEDGING AND PRODUCTION WITH FORWARD CONTRACTS

In this section, we derive the effect of stock and call compensation on the optimal production and hedging policy, given the availability of unbiased forward contracts. We consider the case where the manager is awarded a cash-based salary equal to $\overline{W}$ and additionally receives a total number of n_S shares of stock or, alternatively, n_C call options.[90]

Theorem 4.1: *If the forward price is unbiased, a manager who is compensated with either shares of stock or at-the-money call options will fully hedge the firm's commodity price risk. Furthermore, there will be separation between the hedging and the production decision.*

Proof: Consider the case where the manager has been awarded call options in his compensation plan. Since the call options have been issued at the money, the exercise price of the call options, X_C, equals:

$$(4.15)\quad X_C = V_0 = \sum_{j=1}^{N} q_j V_j = \frac{W_0 + S_0 \cdot y - C(y)}{n_V}$$

Since consumption for the manager is given by $c_j = \overline{W} + n_C \cdot \left[V_j - X_C\right]^+$:

[90] Note that we implicitly assume that the present value managerial compensation payments – whether this concerns cash, call options, or stocks – is already incorporated into current firm value (i.e., firm value V_0, by means of W_0, has been decreased by the present value of these payments).

$c_j = \overline{W}$

if $V_j \leq X_C$

$$c_j = \overline{W} + n_C \cdot \left(V_j - X_C\right) = \overline{W} + n_C \cdot \frac{\left(W_0 - C(y)\right)\cdot\left(1 + r_f\right) + S_j \cdot y + n_F \cdot \left(S_j - F\right)}{n_V}$$

if $V_j > X_C$

If we consider the benchmark case of a full hedge, in which $n_F = -y$, then the time-1 stock price is given by:

$$\text{(4.16)}\quad \begin{aligned} V_j &= \frac{\left(W_0 - C(y)\right)\cdot\left(1 + r_f\right) + S_j \cdot y + y \cdot \left(S_j - F\right)}{n_V} \\ &= \frac{\left(W_0 - C(y)\right)\cdot\left(1 + r_f\right) + F \cdot y}{n_V} \end{aligned}$$

Hence, since the call options are issued at the money, the payoff on a call option always equals:

$$\text{(4.17)}\quad \begin{aligned} \left[V_j - X_C\right]^+ &= \left[\frac{\left(W_0 - C(y)\right)\cdot\left(1 + r_f\right) + F \cdot y - \left(W_0 - C(y) + S_0 \cdot y\right)}{n_V}\right]^+ \\ &= \left[\frac{\left(W_0 - C(y)\right)\cdot r_f + S_0\left(1 + r_f\right)\cdot y - S_0 \cdot y}{n_V}\right]^+ \\ &= \frac{r_f \cdot \left(W_0 - C(y) + S_0 \cdot y\right)}{n_V} \end{aligned}$$

which shows that, upon exercise, the call is always in the money. Since the payoff is strictly constant, it is independent of S_j. This means that utility, and therefore marginal utility, is also constant which implies that the optimal hedge is a full hedge since the covariance term $Cov\left(U'\left(c_j\right),\left(S_j - F\right)\right)$ is zero. This satisfies first-order condition (4.13). A full hedge is therefore the optimal policy for a risk-averse utility-maximizing manager. Now consider the optimal production decision. Multiplying first-order condition (4.12) by $\frac{n_V}{n_C}$ and dividing by $U'\left(c_0\right)$ yields:

$$\begin{aligned}\frac{dE\left[U\left(\tilde{c}\right)\right]}{dy}\cdot\frac{n_V}{n_C U'\left(c_0\right)} &= \delta\sum_{j=1}^{N}\pi_j\cdot\frac{U'\left(c_j\right)}{U'\left(c_0\right)}\cdot\left(S_j - C'\left(y\right)\left(1+r_f\right)\right)\\ &= E\left[\tilde{S} - C'\left(y\right)\left(1+r_f\right)\right]E\left[\delta\frac{U'\left(\tilde{c}\right)}{U'\left(c_0\right)}\right] + Cov\left(\tilde{S} - C'\left(y\right)\left(1+r_f\right),\delta\frac{U'\left(\tilde{c}\right)}{U'\left(c_0\right)}\right)\end{aligned}$$

However, since the manager engages in a full hedge, $U'\left(\tilde{c}\right)$ is constant (i.e., $U'\left(\tilde{c}\right) = U'\left(\bar{c}\right)$) which allows us to rewrite the first-order condition as:[91]

$$\begin{aligned}\frac{dE\left[U\left(\tilde{c}\right)\right]}{dy}\cdot\frac{n_V}{n_C U'\left(c_0\right)} &= E\left[\tilde{S} - C'\left(y\right)\left(1+r_f\right)\right]E\left[\delta\frac{U'\left(\bar{c}\right)}{U'\left(c_0\right)}\right] + Cov\left(\tilde{S} - C'\left(y\right)\left(1+r_f\right),\delta\frac{U'\left(\bar{c}\right)}{U'\left(c_0\right)}\right)\\ &= \frac{E\left[\tilde{S}\right] - C'\left(y\right)\left(1+r_f\right)}{1+r_f}\\ &= \frac{F}{1+r_f} - C'\left(y\right)\\ &= S - C'\left(y\right)\end{aligned}$$

This completes the proof in the case of call compensation. The proof for the case of stock compensation is similar.[92] ||

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[91] Henceforth, a bar (¯) is used to denote a non-random variable.

[92] See e.g., Holthausen (1979), Feder, Just and Schmitz (1980), and Benninga and Oosterhof (2004). See also Theorem 3.2 in Chapter 3 of this dissertation.

By Theorem 4.1, production is extended up to the point where the marginal costs of production equal the current spot price and the optimal forward position is to fully hedge the firm's price risk exposure. Thus, the risk preferences of the manager do not influence his optimal production decision, nor the optimal forward hedge. This implies that the optimal hedge is a full hedge and that there is separation between the production and hedging decision. Furthermore, note that Theorem 4.1 shows that the optimal hedging policy for the manager is exactly the same as if he were the owner of the firm. This implies that the manager exactly hedges in the way the owner of the firm should want him to (i.e., there is incentive compatibility). This implication deserves some further attention. If the owner of a firm can hedge the firm's price exposure with unbiased forward contracts, this results in a risk-free gain since the forward price is greater than the current spot price. Naturally, this is the optimal result for any risk-averse agent in the economy, so this result also applies directly to the manager in case he becomes a partial shareholder. The same argument holds in the case of granting the manager with at-the-money call options. Since the exercise price of the call options equals the current spot price of the firm, hedging the price risk of the firm with unbiased forward contracts leads to a risk-free gain since future firm value is always greater than current firm value (which equals the exercise price of the calls). Again, a full hedge is the optimal position for any risk-averse agent in the economy. Thus, even though the manager can be awarded the two most common kinds of compensation, in both cases, he will engage in a full hedge and there is separation between the production and hedging decision.

4.4 OPTIMAL HEDGING AND PRODUCTION WITH PUT OPTIONS

4.4.1 Optimal hedging and production decisions

In this section, we analyze the effect of managerial compensation on the optimal production and hedging policy if the manager can hedge the firm's commodity price exposure with put options. First consider the case where the manager is awarded a cash-based salary and a total number of n_S shares of stock. If the manager uses puts for hedging purposes, the time-1 stock price equals:[93]

$$(4.18)\quad V_j = \frac{\left(W_0 - C\left(y\right)\right)\cdot\left(1+r_f\right)+S_j\cdot y+n_P\cdot\left(\left[X_P-S_j\right]^+ - P\left(X_P\right)\cdot\left(1+r_f\right)\right)}{n_V}$$

and, assuming unbiased put prices, the manager has to satisfy first-order equation (4.8).[94]

Theorem 4.2: *If the manager is compensated with shares of stock, hedging with unbiased put contracts results in overhedging the firm's commodity price risk.*

[93] Like in Theorem 4.1, we again implicitly assume that the present value of managerial compensation payments – whether this concerns cash, call options, or stocks – is already incorporated into current firm value. See also footnote 90.

[94] A more or less similar analysis can be found in Battermann, Braulke, Broll, and Schimmelpfennig (2000) and Benninga and Oosterhof (2004). See also Chapter 3 of this dissertation.

Proof: First note that the value of the firm V_j equals:

$$(4.19)\quad \begin{aligned} V_j &= \frac{\left(W_0 - C(y)\right)\cdot\left(1+r_f\right) + S_j\cdot y + n_P\cdot\left(X_P - S_j - P(X_P)\cdot\left(1+r_f\right)\right)}{n_V} \\ &= \frac{\left(W_0 - C(y)\right)\cdot\left(1+r_f\right) + n_P\cdot\left(X_P - P(X)\cdot\left(1+r_f\right)\right) + S_j\cdot\left(y - n_P\right)}{n_V} \end{aligned}$$

if the put expires in the money, and that:

$$(4.20)\quad V_j = \frac{\left(W_0 - C(y) - n_P\cdot P(X_P)\right)\cdot\left(1+r_f\right) + S_j\cdot y}{n_V}$$

if the put expires out of the money. Consider the benchmark case of a full hedge. In this case, equations (4.19) and (4.20) reduce to:

$$(4.21)\quad V_j = \frac{\left(W_0 - C(y)\right)\cdot\left(1+r_f\right) + y\cdot\left(X_P - P(X_P)\cdot\left(1+r_f\right)\right)}{n_V} \qquad \text{if } S_j \le X_P$$

$$(4.22)\quad V_j = \frac{\left(W_0 - C(y)\right)\cdot\left(1+r_f\right) + y\cdot\left(S_j - P(X_P)\cdot\left(1+r_f\right)\right)}{n_V} \qquad \text{if } S_j > X_P$$

Examination of equations (4.21) and (4.22) shows that the stock price is constant until the exercise price is reached, and strictly increasing in $\tilde{S}$ afterwards. Managerial marginal utility of consumption is therefore constant until the exercise price is reached, and strictly decreasing in $\tilde{S}$ thereafter. Since the put payoff is – as a consequence – a constant and decreasing function in $\tilde{S}$, this implies a positive covariance between managerial marginal utility of consumption and the put payoff (i.e., $Cov\left(U'\left(c_j\right),\left[X_P - S_j\right]^+\right) > 0$). If the producer has full access to the bond market, the covariance term must be zero (see Theorem 3.3 in Chapter 3). The manager must therefore increase the put position in order to maximize his personal utility of consumption, which results in overhedging. This completes the proof. ||

Theorem 4.3: *If the manager's compensation package consists of cash and at-the-money call options on the time-1 stock price then, given unbiasedness of the put price, the optimal hedge is an overhedge using at-the-money put options.*

Proof: If the manager hedges the producer's price risk with at-the-money put options, the exercise price of the put equals $X_P = \sum_{j=1}^{N} q_j \cdot S_j = S_0$. Furthermore, since the put price is unbiased $P\left(X_P\right)\cdot\left(1+r_f\right) = \sum_{j=1}^{N} \pi_j \left[X_P - S_j\right]^+$. This results in the time-1 stock price equaling:

$$(4.23)\quad V_j = \frac{\left(W_0 - C\left(y\right)\right)\cdot\left(1+r_f\right) + S_j \cdot y + n_P \cdot \left(\left[X_P - S_j\right]^+ - P\left(X_P\right)\cdot\left(1+r_f\right)\right)}{n_V}$$

Assume that the manager fully hedges total production, implying that $n_P = y$. Depending on whether the put expires in or out of the money, there are four possibilities for future firm value and, therefore, managerial consumption. Take a look at the situation where the put expires in the money, i.e., $S_j < X_P$. In this case the stock price is constant and given by:

(4.24)
$$\begin{aligned} V_j &= \frac{\left(W_0 - C(y)\right)\cdot\left(1+r_f\right) + S_j \cdot y + n_P \cdot \left(\left[X_P - S_j\right]^+ - P(X_P)\cdot\left(1+r_f\right)\right)}{n_V} \\ &= \frac{\left(W_0 - C(y)\right)_0\cdot\left(1+r_f\right) + S_j \cdot y + y \cdot \left(\left[S_0 - S_j\right] - P(X_P)\cdot\left(1+r_f\right)\right)}{n_V} \\ &= \frac{\left(W_0 - C(y)\right)\cdot\left(1+r_f\right) + S_0 \cdot y - y \cdot P(X_P)\cdot\left(1+r_f\right)}{n_V} \\ &= \frac{\left(W_0 - C(y) - y\cdot P(X_P)\right)\cdot\left(1+r_f\right) + S_0 \cdot y}{n_V} \end{aligned}$$

Given that $X_C = \dfrac{W_0 + S_0 \cdot y - C(y)}{n_F}$, it can be concluded that $V_j \underset{>}{\overset{<}{-}} X_C$.

So, if the put ends in or at the money, it is not immediately evident whether the call options expire in, at, or out of the money. The payoff on a call option, which is given by:

(4.25)
$$\begin{aligned} \left[V_j - X_C\right]^+ &= \left[\frac{\left(W_0 - C(y) - y\cdot P(X_P)\right)\cdot\left(1+r_f\right) + S_0\cdot y - \left(W_0 + S_0 \cdot y - C(y)\right)}{n_V}\right]^+ \\ &= \left[\frac{r_f \cdot\left(W_0 - C(y)\right) - y\cdot P(X_P)\cdot\left(1+r_f\right)}{n_V}\right]^+ \end{aligned}$$

can therefore be both positive as well as zero, depending on the initial wealth of the firm and the commodity price at time 1. In order to analyze the covariance term $Cov\left(U'(c_j), \left[X_P - S_j\right]^+\right)$, we have to consider the following 2 cases:

Case 1: The first case to consider is when initial firm wealth $W_0 > C(y) + \dfrac{y\cdot P(X_P)\cdot\left(1+r_f\right)}{r_f}$. Given initial firm wealth W_0 being larger than this threshold, we have to consider the effect of the put payoff on managerial consumption:

a) If the put option expires in the money, the call options pay off a strictly positive and fixed amount equaling $\left[V_j - X_C\right]^+ = V_j - X_C = \frac{r_f \cdot \left(W_0 - C(y)\right) - y \cdot P(X_P) \cdot \left(1 + r_f\right)}{n_V}$. Managerial consumption c_j is therefore constant and equals $\bar{W} + n_C \cdot \left(\frac{r_f \cdot \left(W_0 - C(y)\right) - y \cdot P(X_P) \cdot \left(1 + r_f\right)}{n_V}\right)$.

b) If the put option expires out of the money, the call options pay off a strictly increasing and positive amount which equals $\left[V_j - X_C\right]^+ = V_j - X_C = \frac{\left(W_0 - C(y)\right) \cdot r_f + y \cdot \left(S_j - S_0 - P(X_P) \cdot \left(1 + r_f\right)\right)}{n_V}$.

Managerial consumption is therefore also strictly positive and increasing and equals $\bar{W} + n_C \cdot \left(\frac{\left(W_0 - C(y)\right) \cdot r_f + y \cdot \left(S_j - S_0 - P(X_P) \cdot \left(1 + r_f\right)\right)}{n_V}\right)$.

Case 2: The second case to consider is where the initial wealth of the firm $W_0 \le C(y) + \frac{y \cdot P(X_P) \cdot \left(1 + r_f\right)}{r_f}$. Again, given that W_0 is smaller than this threshold, we have to consider the following possibilities for future managerial consumption:

a) If the put expires in the money, the call options expire out of the money, meaning that $\left[V_j - X_C\right]^+ = 0$. Managerial consumption is therefore equal to the cash-based salary $\bar{W}$.

b) If the put expires out of the money, then, depending on the value of S_j, the call options expire in or out of the money.

i. If the commodity price $S_j \le \frac{y\cdot\left(P\left(X_P\right)\cdot\left(1+r_f\right)+S_0\right)-\left(W_0-C\left(y\right)\right)\cdot r_f}{y}$ the call options expire out of the money, i.e., $\left[V_j-X_C\right]^+=0$. Therefore, in this case, managerial consumption is constant and equals the cash-based salary $\overline{W}$.

ii. If the commodity price $S_j > \frac{y\cdot\left(P\left(X_P\right)\cdot\left(1+r_f\right)+S_0\right)-\left(W_0-C\left(y\right)\right)\cdot r_f}{y}$, the call options expire in the money and will, therefore, payoff a strictly increasing and positive amount which is equal to $\left[V_j-X_C\right]^+=V_j-X_C=\frac{\left(\left(W_0-C\left(y\right)\right)\cdot r_f+y\cdot\left(S_j-S_0-P\left(X_P\right)\cdot\left(1+r_f\right)\right)\right)}{n_V}$.

In this case, the manager's total consumption at time 1, c_j, equals $\overline{W}+n_C\cdot\left(\frac{\left(W_0-C\left(y\right)\right)\cdot r_f+y\cdot\left(S_j-S_0-P\left(X_P\right)\cdot\left(1+r_f\right)\right)}{n_V}\right)$.

From this we can conclude that, given a full put hedge, managerial consumption is strictly constant until a threshold is reached and strictly increasing afterwards. In case 1, where the call always expires in the money, this threshold equals the exercise price of the put. As long as the put is in the money, firm value and, as a consequence, managerial consumption is constant. When the exercise price is reached, firm value is a strictly increasing function in $\tilde{S}$, and since the call is always in the money, it is also strictly increasing for states of the world where $S_j > X_P$. In case 2, where the call options do not always payoff, the situation is a little less obvious. If the put option expires in the money, the payoff on the calls is always zero. This payoff remains zero until this call ends in the money, which will happen for a threshold value of S_j. Managerial consumption is strictly increasing after this threshold. These results imply that the marginal utility of managerial consumption is constant and strictly decreasing after the threshold. Since the put payoff, in either case, is also a strictly decreasing function in S_j until the exercise price is reached and constant thereafter, the covariance term $Cov\left(U'\left(c_j\right),\left[X_P - S_j\right]^+\right)$ is positive. This implies that the manager must increase the put position to maximize his personal utility of wealth, resulting in *overhedging* to satisfy first-order condition (4.14). This completes the proof. ||

From Theorems 4.2 and 4.3, it is clear that the optimal put position is to overhedge total production. The following theorem, therefore, analyzes this optimal production decision.[95]

Theorem 4.4: *Hedging with unbiased put contracts leads to a decrease in production relative to hedging with unbiased forward contracts*.

Proof: Consider the case of compensation with call options. The first-order condition that needs to be satisfied by the manager equals $\frac{dE\left[U\left(\tilde{c}\right)\right]}{dy} = \delta \cdot \frac{n_C}{n_V} \cdot \sum_{j=1}^{N} \pi_j \cdot U'\left(c_j\right) \cdot \left(S_j - C'\left(y\right)\left(1+r_f\right)\right) = 0$. Multiplying by $\frac{n_V}{n_C}$ and dividing by $U'\left(c_0\right)$ yields:

[95] Note that we assume that risk-free lending and borrowing is not possible. See also footnote 82.

$$\frac{dE\left[U\left(\tilde{c}\right)\right]}{dy}\cdot\frac{n_V}{n_C U'\left(c_0\right)}=\sum_{j=1}^{N}\pi_j\delta\frac{U'\left(c_j\right)}{U'\left(c_0\right)}\left(S_j-C'\left(y\right)\left(1+r_f\right)\right)$$
$$=\sum_{j=1}^{N}\pi_j\delta\frac{U'\left(c_j\right)}{U'\left(c_0\right)}S_j-C'\left(y\right)$$

So, the first-order condition is satisfied if and only if $C'\left(y\right)=\sum_{j=1}^{N}\pi_j\delta\frac{U'\left(c_j\right)}{U'\left(c_0\right)}S_j$. Remember that the state prices q_j are defined as $q_j=\delta\pi_j\frac{U'\left(c_j\right)}{U'\left(c_0\right)}$. From Theorem 4.3 it is clear that the optimal put position is long. A long position in a put option increases future consumption in the states of the world where the put expires in the money which causes, because of declining marginal utility, $U'\left(c_j\right)$ to decrease. Furthermore, by paying for the put options today, $U'\left(c_0\right)$ increases. The implies that buying puts will make the implicit state prices go weakly down for every state of the world. Now suppose we have an equilibrium in which the manager does not buy puts, so that $n_P=0$. Recall that the first-order condition is satisfied if $C'\left(y\right)=\sum_{j=1}^{N}\pi_j\delta\frac{U'\left(c_j\right)}{U'\left(c_0\right)}S_j$. Consequently, if the manager engages in a long position in puts by increasing n_P, this decreases the implicit state prices and therefore leads to a reduction in output *y*. This completes the proof. The proof for stock compensation is similar. ||

4.4.2 Properties of the optimal hedging decisions

The Theorems 4.2, 4.3, and 4.4 in the previous subsection show that if a manager is rewarded equity-linked compensation and can hedge with unbiased put contracts, this results in overhedging. Furthermore, there will be a decrease in production. This shows that, contrary to hedging with unbiased forward contracts, hedging with puts and equity-linked compensation does not lead to the well-known full hedging and separation principles. Since the amount of the overhedge is dependent on several variables we now turn to the properties of this overhedge.[96]

Lemma 4.1: *If the put price is unbiased, the overhedge decreases as the fraction of stock or call compensation increases.*

Proof: Consider the case of stock compensation. From Theorem 4.2 we know that, given full hedging and unbiasedness of the put price, the covariance term $Cov\left(U'\left(c_j\right),\left[X_P - S_j\right]^+\right)$ is strictly positive. Assume for now that the manager's total wage is fixed. An increase in stock compensation therefore leads to a reduction in his cash wage. The manager's total time-1 consumption therefore equals:

$$
\begin{aligned}
c_j &= \left(\overline{W} - n_S \sum_{j=1}^{N} q_j V_j\right) + n_S V_j \\
&= \overline{W} + n_S\left(V_j - V_0\right)
\end{aligned}
$$

Since, for analyzing the covariance term $Cov\left(U'\left(c_j\right),\left[X_P - S_j\right]^+\right)$ the fixed part (i.e., $\overline{W} - n_S V_0$) is unimportant, we can rewrite consumption as $c_j^* = n_S V_j$. If the manager receives relatively more shares of stock, his equity-linked consumption c_j^* increases for all states of the world. Given the strict convexity of the utility function, and

[96] Note that in the following two lemmas, the results hold for a given level of production. If production volume is assumed to be endogenous, it is necessary to analyze the joint impact of the compensation package on hedging and production.

therefore, the strict concavity of managerial marginal utility of consumption, the covariance term $Cov\left(U'\left(c_j^*\right),\left[X_P - S_j\right]^+\right)$ decreases. Therefore, the manager must increase the put position less to force the covariance term to be zero, resulting in less overhedging. Since optimal decision making is independent of the amount of cash compensation, this result holds irrelevant of whether we assume total compensation to remain constant. This implies that any increase in the fraction of stock compensation (i.e., as a proportion of firm value), decreases the optimal overhedge. This completes the proof. The proof for call compensation is similar ||

Lemma 4.2: *If the put price is unbiased, the overhedge decreases as the manager becomes more risk averse.*

Proof: The proof is similar to that of Lemma 4.1. Given unbiasedness of the put price and full hedging with puts, the covariance term $Cov\left(U'\left(c_j\right),\left[X_P - S_j\right]^+\right)$ is strictly positive. If a manager is more risk averse, marginal utility will be less increasing, which causes the covariance term $Cov\left(U'\left(c_j\right),\left[X_P - S_j\right]^+\right)$ to be less positive. The manager must therefore increase the put position by a smaller amount, resulting in less overhedging. This completes the proof. ||

4.4.3 Conclusions

In Theorems 4.2, 4.3, and 4.4, as well as in the previous two lemmas, it is shown that incentive compatibility between the manager and the primary owner of the firm can normally not be expected. This can be explained by the fact that – contrary to hedging with unbiased forward contracts – the optimal decisions in case of hedging with put options depend on individual utility functions and, therefore, on the implicit private pricing system. Because the utility functions of the manager and the producer will normally differ and, as a result, also the private state prices will not coincide, only in incidental cases will the optimal decisions be the same. To give a logical case of incentive compatibility: if the primary owner as well as the manager of the firm both own half of the shares and have the same utility function with the same level of risk aversion, the desired hedging and production policies are exactly the same. In this very specific case, incentive compatibility occurs.

Given compensation with call options in addition to cash, incentive compatibility between the manager and the owner also rarely ever occurs. As analytically shown in this chapter's lemmas, the overhedge is a strictly decreasing function to the degree that the manager is rewarded more calls and/or to his degree of risk aversion. It can therefore be expected that it is very unlikely that the optimal hedging policy for the manager and the primary owner of the firm is the same. Again, as will be shown in the next section, only in some incidental cases, incentive compatibility will occur.

Since, in the case of hedging with put options, optimal hedging and production decisions depend on the specifications of the utility function it is not possible to derive analytical closed-form solutions. As shown in the lemmas derived in this chapter, the overhedge and the total amount of production depend on these specifications. If analytical analyses cannot derive immediate conclusions on the exact amount of production and put hedging, numerical exercises provide a tool to get – at least some – insight in the theorems and lemmas derived. In the next section we, therefore, turn to the second-best approach of numerical exercises to analyze the sensitivity of the model parameter values. In this section, we critically analyze situations in which incentive compatibility does occur.

4.5 NUMERICAL RESULTS WITH RESPECT TO PUT HEDGING

4.5.1 Introduction

In this section we present the results from numerical exercises regarding the optimal put hedge given partial stock and call compensation (i.e., in addition to cash).[97] Since the driving factor behind our analytical results is market incompleteness, the manager of the firm and its primary owner (the producer) may have different private pricing systems, which may also differ from the market pricing system. This causes differences in optimal decision making. As shown in Section 4.3, given the availability of linear hedging instruments, there is separation between the hedging and production decision resulting in a full hedge of production. The optimal decisions are independent of utility functions and, as a consequence, of the subjective state prices. This holds for both (partial) compensation with shares of stock as well as compensation with call options. Given incompleteness of financial markets, discrepancies between optimal hedging and production policies arise in case hedging is based on unbiased put options. This is caused by the fact that, in this case, the optimal risk management and production decisions do depend on the specifications of individual utility functions. If the primary owner and the manager have a different utility function, their implied pricing systems will differ causing conflicts of interest

[97] The numerical exercises have been performed using the mathematic program *Mathematica*.

regarding optimal decision making. As a result, they are likely to disagree on the optimal hedge as well as the optimal production decision. Since the analytical model does not provide closed-form solutions on the differences in optimal decision making, numerical analyses provide a second-best alternative to analyze the conditions influencing these decisions.

The objective of this section is threefold. First of all, for obvious reasons, it is important to check whether or not the theorems and lemmas derived in this chapter are numerically confirmed. The second goal is – given numerical confirmation of the theoretical results – to analyze the situations in which incentive compatibility between the manager and the owner of the firm occurs. We hypothesize that the optimal decisions do not coincide frequently, given the underlying assumptions in our analysis. If markets are incomplete, the implicit pricing systems of the manager and the owner of the firm are likely to differ, resulting in different optimal solutions. In such cases, it is important to know how much variation there will be in the optimal solutions. If these optimal solutions are more a less alike, the owner of the firm will not be harmed too much by the decisions made by the manager, which are suboptimal from the owner's point of view. The third goal, finally, relates to the second one. If incentive compatibility does occur (or, alternatively, if the optimal decisions do not differ substantially), it is important that changes in the exogenous model parameters do not lead to extreme changes in optimal decision making. If optimal decision making is highly sensitive to changes in important model parameters, little value can be placed on the numerical results as well as the theoretical model derived. In other words, the robustness of the numerical solutions must be checked by changing the model input. If the numerical results appear to be robust, this will extend our previous theoretical results.

It is, however, important to realize that we do not intend to derive "real-life" predictions for optimal decision making since the model presented is far too simple to predict the real world. What is important is to check whether or not the theorems and lemmas derived are confirmed and how sensitive these results are for changes in the parameter values. As mentioned before, in this section we concentrate on put hedging. Given the simple model from which the normative rules are derived, we analyze the sensitivity of optimal decision making by changing some major input variables. In practice, this implies changing the level of equity-linked compensation as well as the risk-aversion coefficient of the utility function. It is reasonable to argue that these are the most important inputs in the decision model. In addition, we also consider the impact of changes in production and state prices. Finally, we provide two extreme scenarios in order to investigate the joint impact of changes.

The remainder of this section is organized as follows: Subsection 4.5.2 presents the results from the benchmark case, in which we derive the optimal numerical hedge ratio for the primary owner as well as for the manager given different levels of equity-linked compensation. In the subsequent Subsections 4.5.3 to 4.5.5, we analyze the numerical sensitivity of the results in the benchmark case to changes in some additional model inputs. Subsection 4.5.6, finally, concludes.

4.5.2 Optimal put hedging in the benchmark case

The numerics in this section are based on a simple model with 4 possible states of the world for which the commodity price $\tilde{S} = \{S_1, S_2, S_3, S_4\}$ with $S_1 < S_2 < S_3 < S_4$. The state probabilities are given by $\tilde{\pi} = \{\pi_1, \pi_2, \pi_3, \pi_4\}$. Since managerial utility is assumed to be strictly concave it can be stated that that for time-additive utility the managerial stochastic discount factors are characterized by $\frac{q_1}{\pi_1} > \frac{q_2}{\pi_2} > \frac{q_3}{\pi_3} > \frac{q_4}{\pi_4}$.[98]

In the benchmark case, we assume that the manager maximizes a power utility function, characterized by $U\left(c_j\right) = \frac{c_j^{1-\gamma}}{1-\gamma}$. This utility function is chosen for its attractive features of decreasing absolute risk aversion (ARA) and constant (i.c., zero) relative risk aversion (RRA). Decreasing ARA implies that absolute changes in wealth or consumption have less impact on wealthier managers which empirically may be expected. A constant RRA implies that the manager will have a constant risk aversion to a proportional loss of wealth, even though the absolute loss increases as total wealth does. This means that, for instance, a 10% loss in wealth is equally harmful for different levels of wealth. Friend and Blume (1975) have estimated changes in ARA and RRA as a function of the wealth of investors. Using sophisticated econometric techniques they find that a power utility function with the parameter for the level of risk aversion equaling $\gamma = 2$ is consistent with the empirical results. We will therefore use these specific model specifications in our benchmark case.

The state-dependent commodity prices are given by $S_1 = 1;\ S_2 = 2;\ S_2 = 3;\ S_4 = 4$, with equal probabilities. Furthermore, in the benchmark case, the state prices range from 0.3 for the worst state of the world to 0.2 for the best state of nature. Given equal state probabilities, the managerial stochastic discount factor is by definition strictly decreasing. The pure rate of time preference, δ, is

98 See Chapter 3 of this dissertation.

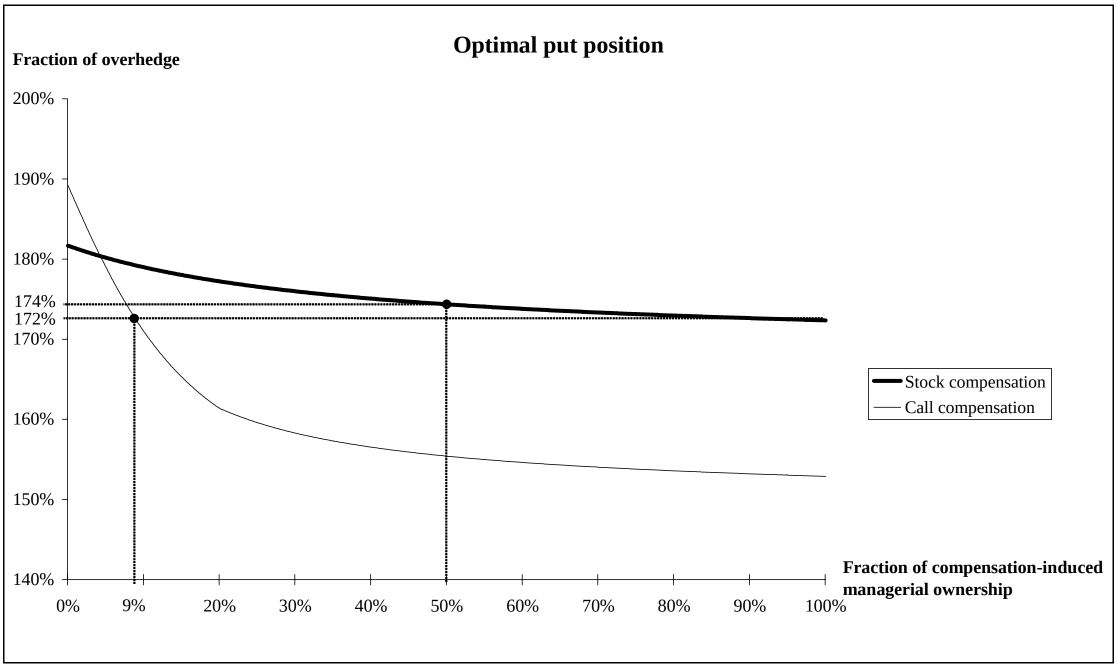
assumed to equal 1, the risk-free rate of interest is set at $r_f = 0$, and the put price is, of course, assumed to be unbiased. Managerial cash-based salary $\overline{W} = 1$, whereas the strictly convex cost function is given by $C(y) = 0.1 \cdot y^2$.[99]

Given these initial assumptions, the benchmark case is the optimal put hedge for the producer (i.e., the primary owner of the firm). Theorem 4.2 argues that hedging with unbiased put contracts results in overhedging the firm's commodity price risk, which is confirmed by the numerical results. If the producer can hedge commodity price risk with unbiased put options, he will hedge 172% of total production. Now suppose the producer hires a manager to make the hedging decisions. The manager is compensated with shares of stock or, alternatively, call options on the terminal stock price.[100] The calls are issued at the money, such that the exercise price equals the current stock price. Figure 4.1 presents the optimal put hedge as a function of equity-linked compensation.

[99] In *Mathematica* it is impossible to jointly solve for optimal production and risk management decisions. Production is therefore assumed to be fixed. In Subsection 4.5.4, we analyze the sensitivity of optimal put hedging, for alternative levels of production.

[100] We solely concentrate on equity-linked compensation and, therefore, abstract from the managerial non-firm-value related wage. As argued in Lemma 4.1, we only need to analyze compensation which is linked to firm value, since for the manager's fixed wage the covariance term $Cov\left(U'\left(\overline{W}\right), \left[X_P - S_j\right]^+\right) = 0$. A zero covariance does not influence the optimal decisions.

Figure 4.1: The optimal put position as a function of equity-linked compensation



In Figure 4.1, the fraction of compensation represents the value of equity-linked compensation divided by total firm value. For stock compensation, this is the resulting fraction of managerial stock ownership; for call compensation this is the fraction of (potential) future stock ownership. As can be seen from Figure 4.1, the overhedge decreases the more equity-linked compensation the manager receives, which holds for both stock as well as call compensation.

Given managerial compensation with shares of stock, the overhedge ranges from 182% if the manager possesses a very little fraction of the stocks to 172% if he becomes the full owner of the firm.[101] From the numerics we can conclude that incentive compatibility rarely ever occurs. Incentive compatibility occurs if the

[101] If the manager owns no stocks, the optimal hedge ratio is indeterminate. In this specific case, managerial compensation does not depend on firm value which implies that changing the number of put options has no impact on his expected utility.

producer and the manager both own 50% of the stocks, and have the same utility function with the same parameter values.[102] Given this specific case, the optimal put position for both the primary owner as well as the manager is to overhedge 174% of production. For alternative levels of stock compensation, the optimal put positions do not differ very much, which implies that the disoptimal put position, from the producer's point of view, is not great. These numerical results confirm Theorem 4.2, which states that if a manager can hedge commodity price risk using unbiased put options, the optimal risk management decision is to overhedge total production. Furthermore, as stated in Lemma 4.1, the overhedge decreases the more the manager becomes financially dependent on the value of the firm (i.e., his fraction of stock ownership increases). However, as mentioned before, the overhedge does not differ greatly from the optimal decision to the producer himself. It can be concluded that, by increasing the manager's salary in stocks, incentive compatibility is increased. Furthermore, the disoptimal overhedge, from the owner's point of view, is not too great for the whole range of managerial stock compensation.

Also for compensation by call options on the terminal stock price, the optimal hedging strategy for the manager is to overhedge total production (see the thin line in Figure 4.1). As is the case with stock compensation, the overhedge decreases as the fraction of call options in his salary increases. However, contrary to stock-linked compensation, when call compensation grows to a larger fraction (and, therefore, potential future managerial ownership gets relatively large), the overhedge becomes less than that optimal to the producer. In the numerics above, for an option value that

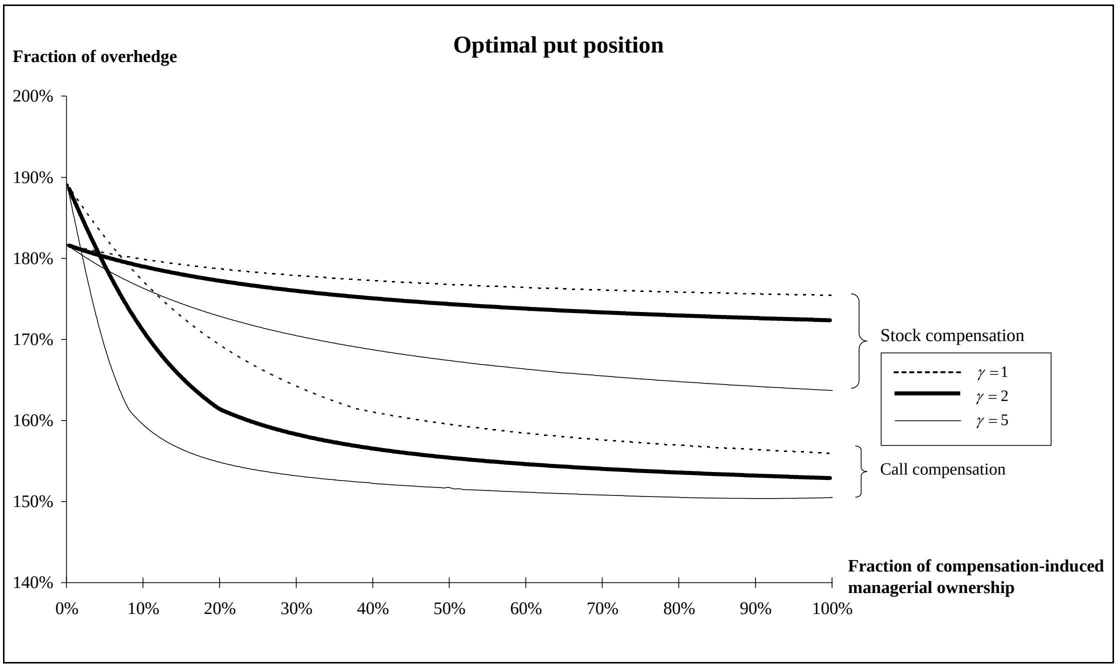
[102] Of course, for different combinations of parameter values of the utility function and compensation, incentive compatibility may accidentally occur.

relates to approximately 9% of potential managerial ownership, the overhedge is less than optimal to the producer (i.e., the overhedge is less than 172%). In this specific case of 9%, there is incentive compatibility between the manager and the owner of the firm, since for both it is optimal to have an overhedge of 172%. From the initial owner's point of view, for higher levels of call compensation, the manager overhedges too little, for lower levels too much.

4.5.3 Sensitivity for changes in risk aversion

To check the robustness of the numerics as presented in Subsection 4.5.2, the sensitivity of the results for changes in levels of risk aversion is very important. Lemma 4.2 states that the overhedge decreases as risk aversion increases. Since the amount of overhedge depends on the covariance between marginal managerial utility and the put payoff, differences in levels of risk aversion have an impact on the optimal put position. As argued in Subsection 4.5.1, if alternative levels of risk aversion do not change this optimal put position substantially, the strength of our theoretical analysis is extended by the numerical analysis. Figure 4.2 presents the results of the numerical analysis for changes in the level of risk aversion.

Figure 4.2: The optimal put position for different levels of risk aversion



In the benchmark case, in which we analyze optimal put positions given a power utility function with a parameter for risk aversion $\gamma = 2$, the optimal put hedge for stock-based compensation is, of course, identical to Figure 4.1 ranging from an overhedge of 182% to 172% of total production. As can be concluded from Figure 4.2 above, the results do not change very much for different parameters of risk aversion. If the manager is far more risk averse (i.e., his parameter for relative risk aversion increases to $\gamma = 5$), the put overhedge ranges from 182% for the case of very little stock ownership to 164% in case of full ownership. The maximum decrease in overhedge is therefore approximately 5%, which is the case for full ownership. A lower level of risk aversion (i.e., $\gamma = 1$) shows similar results.

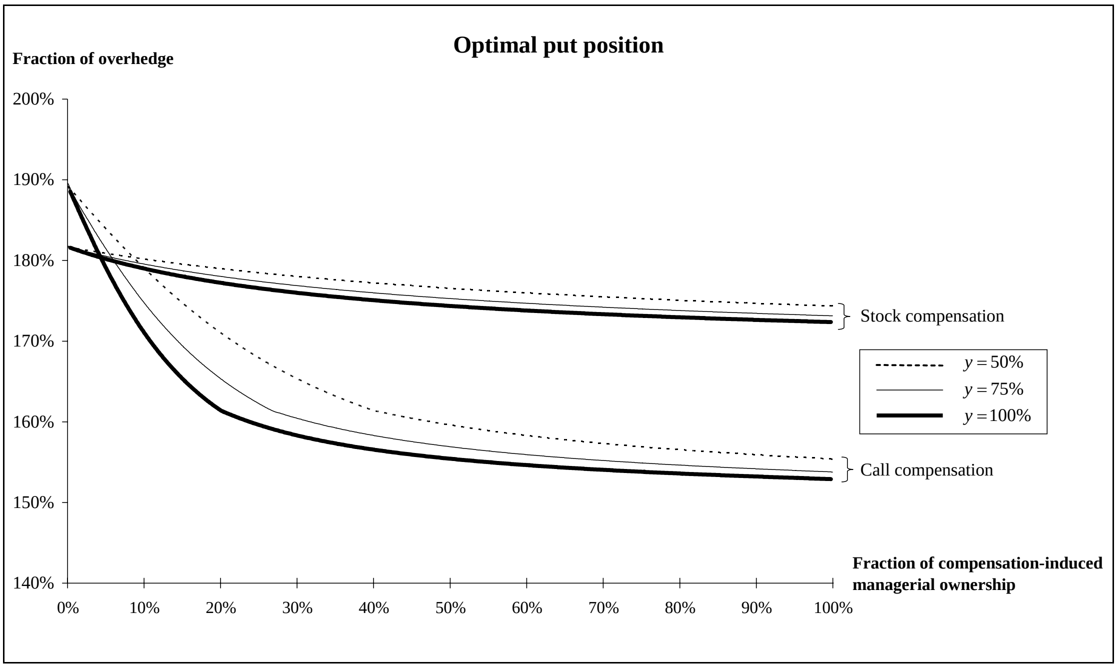
With respect to compensation with call options, similar results apply. However, the divergence in the optimal put positions, for different levels of risk aversion, sometimes differs more than for the case of stock compensation. This occurs for low and intermediate fractions of compensation. The largest difference with respect to the benchmark case is, in this specific example, for a value of call compensation which corresponds to approximately 8.1% of firm value. In this case, the optimal overhedge for the manager, given a high level of risk aversion, in which $\gamma = 5$, is 161%. In the benchmark case the optimal hedge for the manager, given 8.1% of call compensation, is to overhedge 174% of production; a difference of 7%. Again, similar results apply for the optimal put positions given a lower level of risk aversion. The differences, however, do not seem to be very large, which implies that the optimal overhedge is quite robust to changes in the level of risk aversion.

Concluding, as stated in Lemma 4.3, the optimal overhedge decreases as risk aversion increases. However, the results are quite robust given the small differences between the optimal hedges for different levels of risk aversion. Therefore, despite the case that incentive compatibility rarely occurs, the initial owner of the firm does not seem to be harmed too much by the, from his point of view, suboptimal put positions.

4.5.4 Sensitivity for changes in production

Subsections 4.5.2 and 4.5.3 analyze the sensitivity of optimal put hedging for different levels of compensation and risk aversion. Another parameter that needs to be investigated is the effect of total production on the optimal hedge ratios. As shown in Theorem 4.4, hedging with puts leads to a decrease in production. Within *Mathematica*, it is impossible to jointly solve optimal production and put positions. In order to investigate the joint effect, this subsection analyzes the optimal put positions for different given levels of production. Figure 4.3 below, graphically shows the optimal put positions for changes in the level of production.

Figure 4.3: The optimal put position for different levels of production



In Figure 4.3, the thick lines represent the optimal hedge ratios for the benchmark case ($y = 100\%$, with $\gamma = 2$ again). As can be seen from this figure, different levels of production do not change the optimal put positions very much. The overhedge increases minorly for lower values of production. In practice, however, it may not be expected that total production will decrease to, for instance, a level of 50%. Even given such a large reduction in production, the optimal put positions are quite similar. As can be seen from the graph, the optimal overhedge for production levels between 75% and 100% is almost identical, which is the case for both kinds of compensation.

This shows that optimal hedge ratios are also quite robust to changes in the optimal level of production.

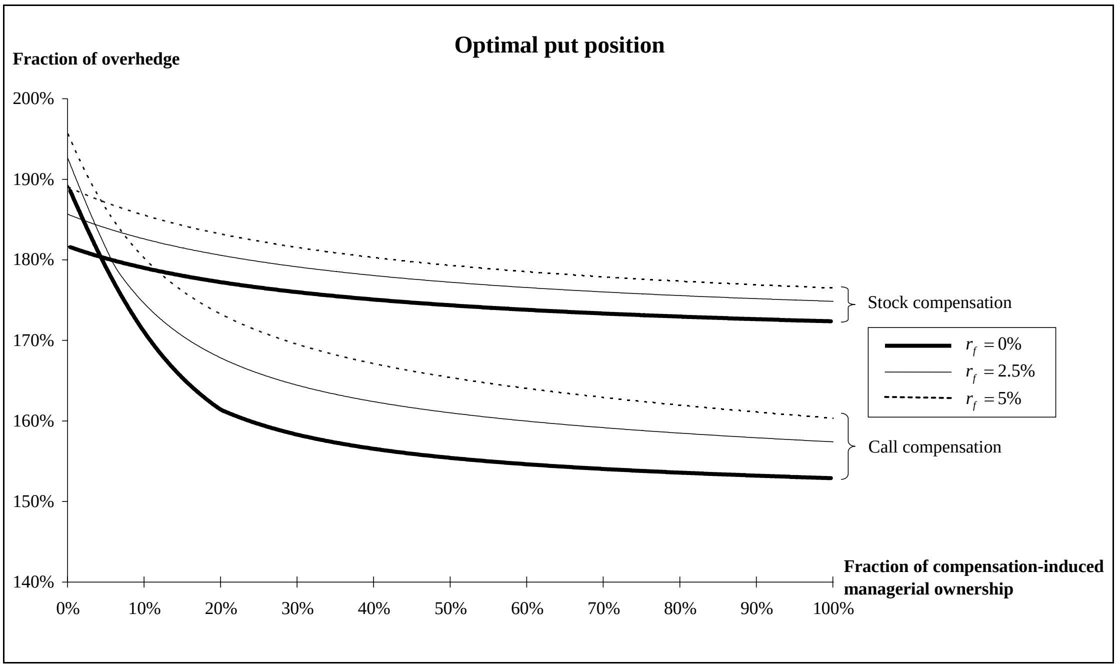
4.5.5 *Sensitivity for changes in the state prices*

In Theorem 4.4 it is shown that hedging with unbiased put contracts will make the implicit state prices go down minorly, for every state of the world. If the state prices decrease for every state of nature the implied risk-free rate of interest, which is defined as $r_f = \frac{1}{\sum_{j=1}^{N} q_j} - 1$, weakly increases. As shown in Theorem 4.4, since the optimal put position is long, future consumption increases for the bad states of the world since the put ends in the money for these states of nature. This causes $U'\left(c_j\right)$ to decrease for these states of the world. Furthermore, by paying for the put options today, $U'\left(c_0\right)$ increases. The combined effect is that the state prices decrease for every state of nature (remember that the state prices are defined as $q_j = \delta\pi_j \frac{U'\left(c_j\right)}{U'\left(c_0\right)}$).

Since increasing future consumption only occurs in the bad states of the world, the decrease in state prices is bigger for these states of nature (only for these states of the world the marginal utility of consumption, $U'\left(c_j\right)$, goes down). To analyze the effect of decreasing state prices for every state of the world, we manipulate these state prices such that the implied risk-free rate of interest goes up from a zero level to 2.5% and 5%. Since, for the bad states of the world the decreasing effect for the state prices is stronger, the proportional decrease in these state prices is larger than the proportional decrease for the state prices in the good states of nature. For instance, given an increase in the implied risk-free rate of interest from 0% to 2.5%, q_1 decreases with

3.3% whereas the decrease in q_4 is only 0.9%. The average decrease, by definition, causes the implied risk-free rate of interest to go up to 2.5%. The effect of changing state prices is presented in Figure 4.4.

Figure 4.4: The optimal put positions for changes in the state prices



As can be seen from Figure 4.4, decreases in the private state prices and, consequently, increases in the implied risk-free rate of interest, will make the optimal hedge ratios go up. The fact that the overhedge increases makes sense. Since the state prices decrease, the perceived put price decreases, which gives the manager an incentive to increase the put position.

The numerical effect, however, is again rather limited. For instance, for the case of stock compensation and an increase of the implied risk-free rate of interest to 2,5%, the optimal overhedge ranges from 186% to 175%, as compared to 182% to

172% in the benchmark case. Approximately similar changes apply for the case of call compensation as well as a further implied increase of the risk-free rate of interest to 5%. Again, the robustness of the optimal ratios in the benchmark case, by changing the state prices, is quite strong.

4.5.6 A joint effect of the parameters

Of course, we also need to check extreme scenarios in which the joint effect of the parameters is analyzed. Since the optimal overhedge increases as 1) the level of risk aversion decreases, 2) production decreases, and 3) the implied risk-free rate of interest goes up, we analyze the situation in which the parameter for risk aversion $\gamma = 1$, production decreases to 50%, and the implied risk-free rate of interest r_f goes up from 0% to 5%. We will call this scenario the upside case. The second extreme scenario, in which the overhedge decreases by a maximum amount, is given for a high level of risk aversion, a high level of production, and a low implied risk-free rate of interest (i.e., for parameter values $\gamma = 5$, $y = 100\%$, and $r_f = 0\%$).[103] We will call this scenario the downside case. Figure 4.5 plots the optimal put positions for the extreme cases as well as the benchmark case.

[103] Note that this case has already been analyzed in Subsection 4.5.3.

Figure 4.5: The joint effect of the parameters on the optimal put position

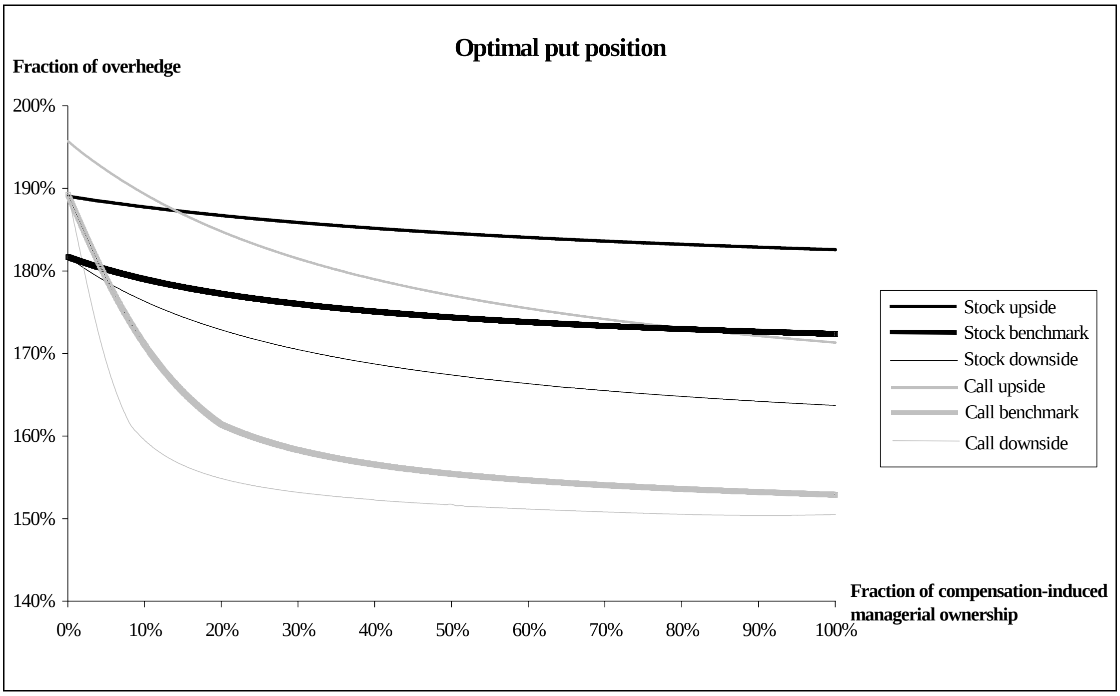


Figure 4.5 shows that, given stock compensation, a joint effect of the different factors affects the overhedge in a minor fashion. In the upside case, in which risk aversion is low, production is low, and the implied increase in the risk-free rate of interest is high, the overhedge ranges from 189% to 183%. Compared to the range in the benchmark case, in which the overhedge ranges from 182% to 172%, the average increase equals 5.6%. In the downside case, the overhedge decreases on average by 3.6%. Given stock compensation, therefore, the conclusion is that the numerical results in the benchmark case are quite robust to "extreme" changes in the most important model parameter values.

For the case of call compensation, in the upside scenario the optimal overhedge is affected moderately. Given parameter values of $\gamma = 5$, $y = 100\%$, and $r_f = 0\%$, the overhedge ranges from 196% to 171%, whereas the benchmark case presents a range of 189% to 153%, which results in an average increase of 12.8%. However, it should be noted that this moderate average increase is largely driven by intermediate and high values of call compensation. In practice, it cannot be expected that a manager will receive a number of calls that will give him the right to attain, for instance, more than 10% of the shares. For a potential future stock ownership lower than 10%, the average increase in the overhedge is approximately 7%. Furthermore, as can be seen from the graph, in the downside scenario the optimal overhedge changes minorly, with an average decrease of 3%.

Summarizing, the results from a joint effect of less realistic model parameter values leads to the conclusion that the results are quite robust to these extreme scenarios.

4.5.7 Conclusions

The numerical exercises in this section confirm the theorems and lemmas presented in this chapter. First of all, for both kinds of compensation the optimal put position is to overhedge total production, which confirms Theorems 4.2 and 4.3. This overhedge decreases as the proportion of equity-linked compensation increases (Lemma 4.1), however, this effect is not dramatic. Secondly, changes in risk aversion and the implied risk-free rate of interest (i.e., changing the state prices) lead to different levels of optimal hedging, confirming Lemma 4.2. However, again, the optimal overhedge is quite robust given the little variance as compared to the benchmark case. Third of all, as total production goes down, the overhedge increases with a very limited fraction, which has been theoretically derived in Theorem 4.4. All these results together show that the optimal hedge ratio is quite robust to changes in the most important parameters of the model. Also in extreme scenarios, in which the factors co-act together, changes in the optimal put positions do not change dramatically. A final conclusion to be drawn is that, as has been expected, incentive compatibility occurs in very little accidental situations. However, given the fact that the optimal overhedge is quite robust to – even unrealistic – changes in the parameters, the owner of the firm will, probably, not be harmed too much. Even thought the manager will not hedge commodity price exposure by the same amount as the owner of the firm would want him to, this suboptimality is quite small from which we conclude that the incentives are reasonably compatible. We conclude by noting that future research should extend the simple model presented, by capturing more realistic assumptions in order to predict optimal decisions for more realistic real-world scenarios.

4.6 CONCLUSIONS

This chapter examines the effect of managerial compensation on the optimal hedging strategies with forward contracts and put options. The manager is rewarded a combination of cash with either shares of stock or at-the-money call options and uses the put and forward market to maximize his expected utility of consumption. The underlying rationale for differences in optimal decision making is that we extend existing literature by assuming market incompleteness in the sense that individual agents may disagree the market pricing of financial assets. This is driven by the fact that the implicit (private) state prices are derived from individual utility functions which, by definition, differ among individual agents. We present a number of new results. First of all, we show that, if the manager of a firm can hedge commodity price risk using unbiased forward contracts, this leads to the well-known full hedging and separation principle. For both stock as well as at-the-money call compensation, the optimal hedge is to fully hedge total production and the optimal production and hedging decision can be separated. Given stock compensation, this result has been widely described in the optimal hedging literature. The result for call compensation, however, is new. This result even holds, given the assumption of market incompleteness, which is also the case in Chapter 3. A second result is that, using unbiased put contracts for hedging purposes, the optimal position is to overhedge total production. For stock compensation, this result has recently been described by Battermann, Braulke, Broll, and Schimmelpfennig (2000) as well as Benninga and Oosterhof (2004). We extend the existing literature by showing that the optimal put position in case of call compensation also leads to overhedging total production. A final extension of the existing literature is that we show that the overhedge decreases as the fraction of stock or call compensation increases and when the manager's degree of risk aversion increases. Numerical exercises confirm the generality of the theorems and lemmas derived in this chapter. These numerics further show that changes in the most important model parameters do not change the optimal decisions very much, implying that the results are quite robust. Incentive compatibility, in which the optimal decisions for the manager and the owner coincide, occurs in accidental situations. However, given the small changes in optimal hedge ratios, the primary owner of the firm is not harmed too much by the decisions made by the manager, which are suboptimal from his point of view.