Chapter Two: *Reflections on the failure of the Taylor principle under commitment*

Reflections on the Failure of Taylor Principle under Commitment[a,b]

Abstract

We offer an explanation of why optimal policy under commitment requires weaker reaction to supply shock, reflected in the failure of the Taylor principle. This lesson seems to be prevalent among central banks and yet has been analyzed incomprehensively in the economic literature.

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1. Introduction

The Taylor principle ruled monetary theory and practice for years and it is still the predominant view among economists even now (see, for example, Smets & Wouters, 2003) when the ability to commit has increased. However, recent authors have indicated that under a regime of commitment, the Taylor Principle fails; see, for example, Clarida et. al. (1999), henceforth CGG, who treat this finding suspiciously (as explained below) and Svensson & Woodford (2003), (henceforth SW), who imply this failure without stating it explicitly. The failure is in the sense that following a shock to expected inflation, the optimal policy is to raise nominal interest rate by less than the shock. In this paper we explain this phenomenon. We base our derivation on the model of SW in the framework of an equilibrium from a "timeless perspective".

The key feature is that under optimal policy in discretion expected inflation is negatively related to the *level* of the output gap, whereas under credible commitment it is negatively related to the *change* in the output gap level, including the initial period. With credible commitment the optimal policy reacts less vigorously following a supply shock.

2. Failure of the Taylor principle under commitment

Suppose the policymaker seeks to minimize

$$E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{1}{2}(\pi_t^2+\lambda x_t^2) \qquad (1)$$

where π_t is inflation, x_t is output gap (the difference between actual and potential output), and assuming both inflation's and output gap's targets are zero, λ is a positive parameter and E_{t_0} is expectations operator taken in the present period, $0<\beta<1$ is the discount factor.

There is a forward-looking aggregate supply (AS), or Phillips curve, given by

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t \text{ with } u_t = \rho u_{t-1} + \varepsilon_t, \; 0<\rho<1 \tag{2}$$

where u_t is a supply shock which follows an *AR*(1) process with ε being a white noise, and $\kappa>0$.

There is an *IS* function, given by

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^n), \; \sigma > 0, \tag{3}$$

where i_t is nominal interest rate and r_t^n is exogenous marginal product of capital which we treat as a constant.

Under *commitment* the first-order conditions (FOC) are

$$\lambda x_t + \varphi_t \kappa = 0 \tag{4}$$

$$\pi_t + \varphi_{t-1} - \varphi_t = 0. \tag{5}$$

for all t, where φ_t is the Lagrange multiplier[35] associated with (2). Eliminating the φ_ts and advancing one period, yield

$$E_t x_{t+1} - x_t = -\frac{\kappa}{\lambda} E_t \pi_{t+1}, \; t \geq t_0. \tag{6}$$

In *discretion* we set $\varphi_{t-1}=0$ for all t, and get instead of (6)

$$E_t x_{t+1} = -\frac{\kappa}{\lambda} E_t \pi_{t+1}, \; t \geq t_0, \tag{7}$$

where the level of the output gap replaces the change in it. Solving for i_t from the IS equation (3) we obtain

[35] Gali (2008) assumes that for the present period t_0, $\varphi_{t_0-1} = 0$, which contradicts SW.

$$i_t = E_t \pi_{t+1} + r^n + \frac{1}{\sigma}(E_t x_{t+1} - x_t)\,. \tag{8}$$

Under commitment we substitute (6) in (8) to have the optimal rule

$$i_t = (1 - \frac{\kappa}{\lambda\sigma})E_t \pi_{t+1} + r^n\,,\ \ t \geq t_0, \tag{9}$$

which shows that the nominal interest rate rises less than expected inflation (contrary to the Taylor principle). Here we follow SW in assuming that t_0 is the present period, but not the first one.

This rule (9) appears explicitly in CGG, who treat it with suspicion. They point out that "a rule of this type may permit self fulfilling fluctuations in output and inflation that are clearly suboptimal" (p.1683). They maintain that a rise in the aggregate demand, gives rise to an increase in inflationary expectations, and in this situation a drop in the real interest rate will further stimulate the demand.

SW (p. 22) show that rule (9) does not lead to indeterminacy in their model, and enables a unique and stable equilibrium, without mentioning the connection to the Taylor principle. Gali (2008) prefers to state his conclusions in terms of the levels of prices. His analysis, based on our equation (5) is satisfied at all $t \geq 0$, Gali (p.103), adjusted by us for $\varphi_{t_0 - 1} \neq 0$, implies also a rejection of the Taylor principle in a regime of commitment.

In *discretion* we substitute (7) in (8) and use the FOC to obtain

$$i_t = \gamma E_t \pi_{t+1} + r^n\,,\ \gamma = 1 + \frac{(1-\rho)\kappa}{\rho\sigma\lambda},\ \text{and}\ \ E_t \pi_{t+1} = \rho\pi_t = \rho\lambda q u_t,\ \ q > 0, \tag{10}$$

which is consistent with the Taylor principle (CGG).

3. Explanation of the Failure

We have to explain why in discretion the optimal policy is based on the level of the output gap, while in commitment it is based on its rate of change. Let us examine equations (4) and (5). In discretion it is not allowed for the policy making to rely on lagged values (φ_{t-1}=0 in (5)). This prevents the reference of the policy to the *change* in level in t_0. By contrast, the credible commitment regime is based on lagged values (the essence of commitment). So it enables the support of the past to have the change of output gap enter the optimal rule. Consequently under commitment the optimal reaction to supply shock is less vigorous.

Alternatively, we observe (from (9)) that under commitment $E_t\pi_{t+1} = -\frac{\lambda\sigma}{\kappa}(r_t - r^n)$ where r_t is the *ex-ante* real interest rate. An increase in r_t lowers inflation expectations. Reversing this relationship we have that under commitment an increase in expected inflation lowers the real rate of interest (opposite to the Taylor principle). In discretion an increase in the real interest rate is associated with a rise in inflationary expectations (equation (10)). This, in turn, raises the real interest rate (γ >1 in (10)).

The upshot is that commitment enables the policymaker to have a lower inflation in future periods at the cost of limiting his freedom of action at present.

4. Impulse Response[36]

So far we considered the impact effect in period *t* without examining the dynamics in future periods. Is the Taylor principle violated in commitment only on impact or is this property relevant also for future periods? What in this context is the

[36] A similar analysis is carried by Gali (2008) pp. 99-100. However, our analysis is directed to the Taylor principle.

role of serial correlation? To deal with these questions we find it useful to conduct an impulse response experiment.

For this illustration we use parameters' values (Table 1) that are commonly used in calibrated models of macroeconomics[37]. Now suppose a positive temporary shock, u_t, afflicts the economy in period t, assuming that this is the only shock that occurs (past and future) so that u_t is identical with ε_t in (2). What are the dynamics that emerge in the inflation expectations and the output gap under discretion vs. under commitment? We answer these questions in Figures 1-3.

The parameter	Value
β	0.98
κ	0.20
λ	0.50
σ	0.60
r^n	0.02
π^*	0
x^*	0
derived[a] c	0.76

[a] as is derived in SW.

Table 1. The values of the parameters and the steady-state variables used for the impulse response simulations.

In discretion it follows from the FOC ((7) and (10)) that an increase in u_t will cause x_t to fall. The policymaker has an incentive to use surprise inflation to reverse the effect of the shock. Accordingly, the inflationary expectations will rise and by (10), both the nominal and the real rates of interest will increase. The Taylor principle is upheld (Figure 1).

[37] We did not impose conditions for uniqueness of equilibrium for discretion.

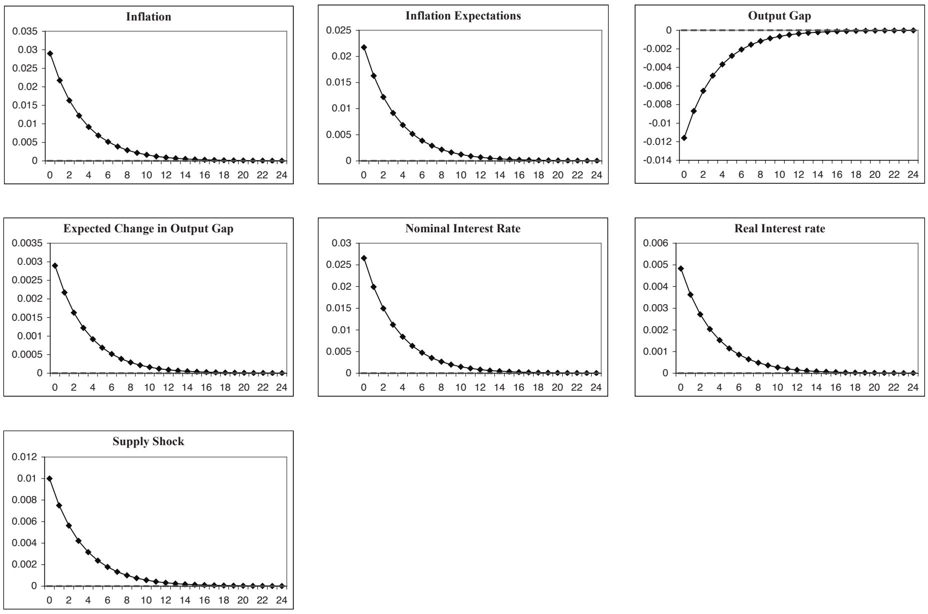

Figure 1. The Impulse Responses to a 1% Positive Nominal Shock to Aggregate Supply Under Discretion (ρ=0.75).

To deal with commitment we need introductory calculations. Substituting from (6) into (2) yields the following second order difference equation

$$x_{t+1} - \hat{a}x_t + \frac{1}{\beta}x_{t-1} = \frac{\kappa}{\lambda\beta}u_t \quad \text{with} \quad \hat{a} = 1 + \frac{\kappa^2}{\beta\lambda} + \frac{1}{\beta}. \tag{11}$$

SW show (p 17) that the characteristic equation corresponding to (11) possesses two real roots: $0 < c < 1$ denotes the smaller real root of the characteristic equation, and a larger root that is given by $\frac{1}{\beta c} > 1$. The destabilizing effect of the latter is eliminated by setting its coefficient equal to zero. So the dynamic system is stable and its stability depends only on the homogeneous part and hence is independent of the shocks.

With $\rho > 0$ in (2), the standard solution for x_t in (11), assuming saddle path[38] stability, is

$$x_t = -\frac{\kappa}{\lambda} c \sum_{j=1}^{\infty} (\beta c)^{j-1} u_{t+j} + c x_{t-1} \tag{12}$$

Taking expectations at t and using $E_t u_{t+j} = u_t \rho^j$ we obtain

$$E_t x_t = -\frac{c\kappa}{\lambda} \frac{\rho}{1-\beta\rho c} u_t + c x_{t-1}. \tag{13}$$

Denoting $\frac{c\kappa}{\lambda} \frac{\rho}{1-\beta\rho c} \equiv A > 0$ we get

$$E_t x_t = -A u_t + c x_{t-1} = x_t. \tag{14}$$

Using (14) and $u_t = \rho u_{t-1} + \varepsilon_t$ to get

$$E_t x_{t+1} = -A\rho u_t + c x_t. \tag{15}$$

From (13) and that initially $x_{t-1} = x^* = 0$, we then obtain

$$x_t = - A u_t. \tag{16}$$

From (6) we get $\pi_t = -\frac{\lambda}{\kappa}(x_t - x_{t-1})$ and thus for the initial period t, assuming that initially $\pi_{t-1} = 0$, we have

$$\pi_t = -\frac{\lambda}{\kappa} x_t = \frac{\lambda}{\kappa} A u_t. \tag{17}$$

Finally, from (6), (14) and (15) we get for t+j, j≥0

$$E_t \pi_{t+1+j} = -\frac{\lambda}{\kappa}(E_t x_{t+1+j} - x_{t+j}) = \frac{\lambda}{\kappa}((1-(\rho+c))x_{t+j} + \rho c x_{t+j-1}). \tag{18}$$

For j=0 when the shock occurs and $x_{t-1}=0$ we have

[38] See for example Obstfeld-Rogoff (1996), 726-741.

$$E_t \pi_{t+1} = \frac{\lambda}{\kappa}(1-(\rho + c))x_t \,, \tag{18a}$$

where the sign of (1-(ρ+c)) is ambiguous and there are two possibilities: (i) A low ρ where (1-(ρ+c)) is positive and (ii) A high ρ where it is negative.

Under commitment, the public trusts the policymaker to pursue the declared policy of low inflation. Based on this trust, the policymaker knows he can reduce the nominal interest rate. The equilibrium solution involves lowering the interest rate by less than the fall in the inflation expectations and thereby raising the real interest rate to regain the steady-state equilibrium.

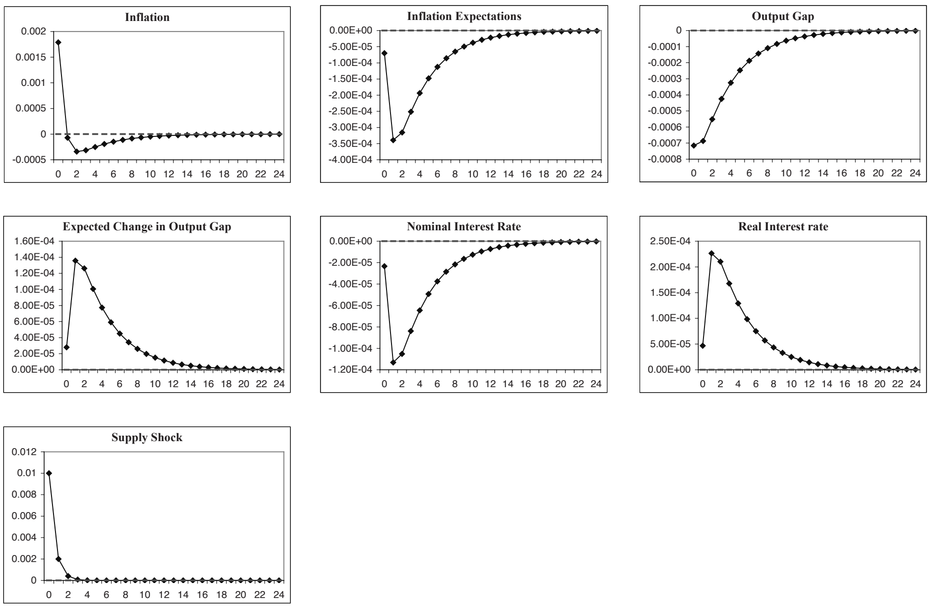

Figure 2. The Impulse Responses to a 1% Positive Nominal Shock to Aggregate Supply Under Commitment (ρ=0.2).

The shock is a surprise and therefore in both commitment and discretion there is a rise in current inflation and a drop in current output gap (see equations (16) and (17)). The *expected* inflation for j>0 is influenced by the lagged output gap and is given by (18). Under discretion the surprise rise in inflation in t is carried over to *expected* inflation, while under commitment the latter may take an opposite turn (Figure 2).

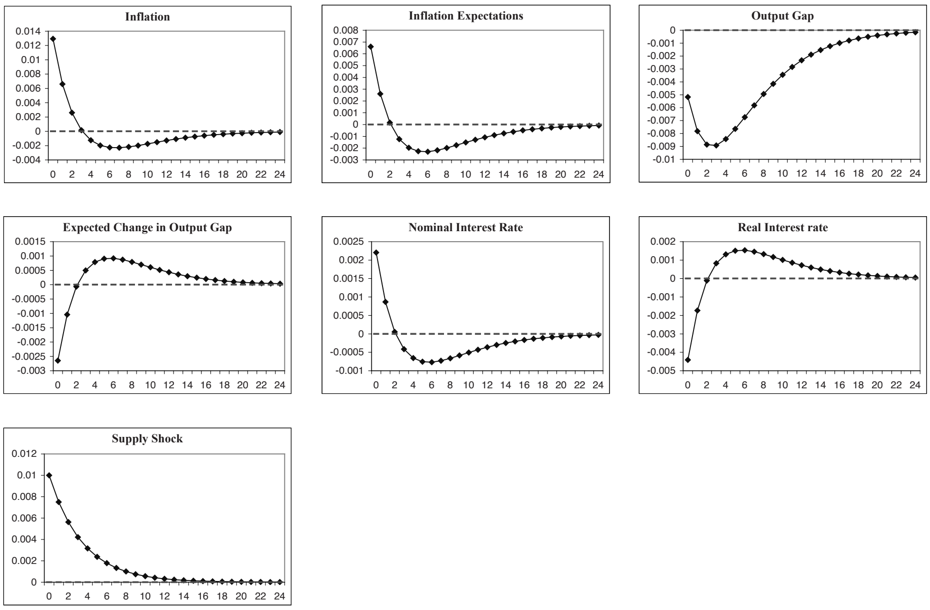

Figure 3. The Impulse Responses to a 1% Positive Nominal Shock to Aggregate Supply Under Commitment (ρ=0.75).

The difference between high and low ρ is in the initial periods, the high ρ yields after a while the same process as the low one (compare Figures 2 and 3). The output gap has saddle path stability and accordingly will rise after some time asymptotically towards its long run steady state. The real rate of interest (ex-ante) is proportional, by (3), to the expected change in the output gap and will therefore rise on impact in low ρ, and subsequently decline monotonically and approach asymptotically its long term value. Along the convergence path the real rate will be higher than its long run rate. With high ρ this process takes place after some delay.

5. Conclusions

Credible commitment allows a less vigorous optimal reaction of the central bank to supply shock, and it enables enjoying low inflation in the future. It produces the negative association of real interest rate with inflation expectations along the stable saddle path. These results are reflected in the failure of the Taylor principle.

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