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## Monetary policy and financial stability in a banking economy

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**Chapter One:** *Monetary policy and financial stability in a banking economy: transmission mechanism and policy tradeoffs*

**Monetary Policy and Financial Stability in a Banking Economy: Transmission  
Mechanism and Policy Tradeoffs<sup>a,b</sup>**

Abstract

The 2008 global financial crisis demonstrated that monetary policy and financial stability policy are more highly interrelated than previously thought. This paper analyzes the interactions between these policies using a non-linear overlapping-generations model with financial frictions in the form of banking financial intermediation. The paper embeds negative externalities due to contagion effects in physical investments which creates the need for financial stability policy. We show how the monetary policy transmission mechanism depends on financial stability policy tools as well as on regulatory and institutional constraints.

We find policy tradeoffs in trying to accomplish both monetary and financial stability targets. The central bank must take these tradeoffs into account when selecting the tools in its policy toolbox. Another important finding is the interchangeability of price stability and financial stability policy tools.

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## 1. Introduction

The 2008 global financial crisis refuted views held prior to the crisis on the goals and tools of monetary policy. The key policy interest rate (KPR), was regarded to be *orthogonal* to financial stability policy tools, such as liquidity and capital adequacy requirements. The conventional view prior to the crisis was also that “there is no general trade-off between monetary and financial stability” (Issing, (2003)). Similarly, it was argued that a central bank “that was able to maintain price stability would also incidentally minimize the need for lender-of-last-resort” intervention (Schwartz, (1998)).

The motivation for this paper stems from the global financial crisis which demonstrated the need to deepen the understating of the complex connections between monetary policy and the financial system stability (Adrian and Shin, (2010)). The words of Paul Volcker (2010) echo what is now the prevailing position currently held by economists and financial regulators: "Monetary policy and concerns about the structure and condition of banks and the financial system more generally are inextricably intertwined".

A second motivation for exploring the intricacies of the interaction of monetary and financial stability policies is the reported significant weakening of the pass-through between the short-term central bank rate and banks' rates in many countries (e.g. Aristei and Gallo (2014)). For a review of the literature on the interaction between macro-prudential and monetary policies see Angelini, Nicoletti-Altamari and Visco (2012). We analyze the interchangeability of the policy tools in implementing the policies. To that end we construct a general equilibrium model with financial frictions in the form of a banking system, where endogenous systemic risk exists.

In this paper we study the interaction of monetary and financial stability policies where central bank operations are carried out solely through the banking channel.<sup>1</sup>

The groundbreaking works by Bernanke and Blinder (1988) and Bernanke et al. (1999) introduced credit market frictions into monetary policy models. More recently, Curdia and Woodford (2011), Gertler, Kiyotaki (2011), to mention a few, incorporated financial intermediation into a general equilibrium model, along with spreads between lending and borrowing rates, analyzing their impact on optimal monetary policy. Our framework enables us to highlight the tradeoffs between monetary policy and financial stability policy and derive the effects of financial stability tools on the effectiveness of monetary policy. While they may be complementary, conflicts often exist.

At times, financial stability instruments, such as capital requirements, may become an effective substitute for the KPR in affecting the price and availability of credit (Cecchetti and Kohler (2014)). At the same time, situations may arise in which the fine tuning of the KPR in response to adverse bank liquidity shocks is a more appropriate financial stability policy response than bank recapitalization (Diamond and Rajan (2012)).

In the spirit of the New Monetarist approach (see Lagos and Wright (2005) and Williamson (2012), to mention few)<sup>2</sup>, we employ a non-linear overlapping-generations model (OLG) model with agent heterogeneity which enables us to

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<sup>1</sup> Two variants of the credit channel of monetary policy transmission can be distinguished: a narrow bank lending channel, measured in terms of the supply of bank loans, and a broad credit channel focusing on the external finance premium in credit markets (Hendricks and Kempa (2011)).

<sup>2</sup> These models assume heterogeneity among economic agents and multiple sub-periods within each time period, in which restrictions are placed on undertaking exchange activities within a given sub-period.

examine intertemporal exchange.<sup>3</sup> The OLG model enables us to derive analytical solutions, facilitating comparative static analysis and simulations in the face of non-linearities.<sup>4</sup> Our framework shares features found in Diamond and Dybvig (1983). In our model, however, market interest rates are determined endogenously.

In our model, the central bank simultaneously pursues both price and financial stability. It implements monetary policy by using the KPR as its primary policy instrument. Financial stability policy is implemented primarily through capital adequacy requirements. We explicitly take into account the constraint that central bank resources are limited.

The banking system in our model is subject to policy constraints imposed by the central bank. The latter provides collateralized loans and partial deposit insurance to the banks. In our economy there are two types of households ("rich" and "poor"), which can take advantage of two channels to transfer purchasing power to next period: nominal bank deposits and physical investment.

Our paper fills several gaps left in the literature. Due to its rich set up we can address some unresolved issues encountered in previous papers dealing with the interaction of monetary policy and financial stability. First and foremost our model provides a detailed structure of the banking system. This structure includes constraints imposed by the central bank, a bank-initiated leverage (risk management) constraint and specification of the market structure. To the best of our knowledge no previous paper has simultaneously modeled these factors. This allows us to more thoroughly explore the "black box" of the transmission mechanism from the KPR to bank interest

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<sup>3</sup> Non-linear models are crucial in current research-based policy regarding the link between macro-prudential and monetary policies.

<sup>4</sup> Although OLG models have been used to explain long-term issues, such as intergenerational transfers and social security arrangements, they have also been used to explain shorter-term phenomena, such as transient bubbles (Martin and Ventura (2012)) and the affects of noise trading on stock prices (Delong, Shleifer, Summers, & Waldman(1990)).

rates and to explicitly identify the interaction between monetary and financial stability policy. Previous papers have opted for more simplistic frameworks, lacking detail in their description of the financial sector, rendering them unsuitable for a comprehensive policy analysis<sup>5</sup>.

Second, is the way systemic risk is integrated in the system via a negative externality arising from the expected return on the physical investment. This externality is a result of contagion associated with the scale of investment projects. It creates a wedge between the risk of investment as perceived by individuals and banks and the actual aggregate level of risk. It is this aggregate risk which defines the systemic risk taken into account by the central bank when implementing its policies. We specify an explicit measure of financial stability that is endogenous and measured by the probability of failure of the banking system. Previous papers have either ignored systemic risk entirely in their models or have incorporated it in an indirect and not fully satisfactory way. For example, the spread between the KPR and the lending rate has been used as a proxy for financial friction as well as a measure of financial stability to which the central bank reacts (Curdia and Woodford (2011), Woodford (2012), Checchetti and Kohler (2013)) do not consider deposit interest rates at all.

Third, our structured model is well suited to analyze the policy reactions of the central bank to shocks and the differential interactions of monetary and financial policies in wake of different types of shocks. Angelini, Neri and Panetta (2011) also analyze the policy implications of supply versus financial shocks in a less detailed model.

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<sup>5</sup> See for example Brunnermeier and Sannikov (2011).

These features enabled us to derive results that contribute to the literature and have important policy implications. Our main contributions are: i) we trace and specify a tradeoff between monetary and financial stability policies. ii) We find the extent to which monetary and financial stability tools are interchangeable in pursuing their respective objectives. iii) Our model enables us to untangle the various links of the monetary transmission mechanism. These findings deepen our understanding of how the monetary transmission mechanism operates, and enables us to suggest ways to mitigate impairments to it. The plan of the paper is as follows: Section 2 presents the stochastic OLG model, followed by an analysis of the equilibrium characteristics of the model in Section 3. The following section deals with the effectiveness of monetary policy and the transmission mechanism and in Section 5 we simulate our model to study the interaction between monetary policy and financial stability policy under various shock scenarios. Conclusions are drawn in Section 6. The derivation of the consolidated budget constraints of individuals in our model appears in Appendix A, followed by the specification of the banks' expected profit functions (Appendix B). The next appendix illustrates the characteristics of the model when a semi-log linear utility function is assumed. Proofs of the propositions and Lemmas are provided in the last appendix (Appendix D).

## **2. The Model**

We consider an OLG model in an economy consisting of households, commercial banks and a central bank (henceforth CB). There exists a storable good where in each period this good can either be consumed at a price of  $p_t$  or be stored as a capital good. In each period  $t$ , there are markets for the consumption good, commercial banks deposits and loans, capital goods, and CB collateralized loans to commercial banks.



## 2.1. Households

A new generation of  $N$  young people is born every period  $t$  and lives for two periods. There are two types of individuals who are identical except for their initial endowments. Half of the young individuals are of type 1 and the other are of type 2. Young individuals of type  $j$ ,  $j=1, 2$ , are each endowed with  $w_j$  units of the storable good, where  $w_1 < w_2$ . Old individuals (in their second period of life), rely in their consumption on the returns on assets that they had accumulated when they were young as well as on commercial bank dividends that are evenly distributed among them at the beginning of the period.

Let  $\beta$  denote the time preferences of the households<sup>6</sup>, and let  $c_{jt}^y, c_{jt}^o$  denote period  $t$  consumption of individuals of type  $j$  who are young,  $y$ , or old,  $o$ , respectively<sup>7</sup>. Returns on the real investment (specified below) are uncertain and we assume that all households are risk-neutral. Formally, the preferences of household  $j$  in period  $t$  are represented by the following expected utility function based on information available at period  $t$ :

$$E_t\{u(c_{jt}^y, c_{jt+1}^o)\} = v(c_{jt}^y) + \beta E_t\{c_{jt+1}^o\} \quad j = 1, 2, \quad (1)$$

where the function  $v: \mathfrak{R}^+ \rightarrow \mathfrak{R}^+$  satisfies the standard characteristics: continuous, twice differentiable with  $v_c > 0, v_{cc} < 0, \lim v_c = \infty$  as  $c \rightarrow 0$ .  $E$  is the mathematical expectations operator<sup>8</sup>.

To accomplish optimal consumption schedules young individuals can shift purchasing power from one period to another through two channels: i) a nominal

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<sup>6</sup> The terms households and individuals are used interchangeably in the paper.

<sup>7</sup> We will omit the subscripts and the superscripts unless we deem the notations necessary to avoid confusion.

<sup>8</sup> The assumption of risk neutrality is maintained in this specification as the utility function is linear in  $c_{t+1}$ .

deposit,  $d_t$ , at a commercial bank that bears a one-period nominal interest rate,  $i_{dt}$ . ii) a physical investment,  $k_t$  : young individuals can use all or part of their endowments to invest in a real technology project, which yields  $f(k_t)$  units at the beginning of period  $t+1$ . We assume that gross returns  $f(k_t)$  follow the following stochastic process<sup>9</sup>

$$f(k_t) = \begin{cases} Ak_t^\alpha & 0 < \alpha < 1, \quad \text{with prob. } \lambda(K_t) \\ 0 & \text{with prob. } 1 - \lambda(K_t) \end{cases} \quad (2)$$

where  $K_t$  is the aggregate investment of all individuals in period  $t$ , that is,  $K_t = \sum_{j=1}^N k_{jt}$ .

where  $k_{jt}$  denotes individual  $j$ 's investment in the real technology. We assume that  $\lambda$ , the probability of success of the physical investment, evolves according to the following process:

$$\lambda(K_t) = \gamma e^{-\gamma K_t}, \quad 0 < \gamma \leq 1,$$

(2a)

Satisfying  $\lambda'(K) < 0$ ,  $\lambda(0) = \gamma$ , and  $\lambda(K_t) \rightarrow 0$  as  $K \rightarrow \infty$ <sup>10</sup>.

The specification of (2a) demonstrates the diminishing marginal quality (increasing marginal risk) of investments. That is, we assume increasing risk embedded in the marginal investment, due to the risk being contagious, i.e. affecting total investment<sup>11</sup>

We further assume a threshold level of investment in the physical project  $k_{\min}$  that satisfies the inequality

$$w_1(1 + \tau) < k_{\min} \leq w_2, \quad (3)$$

<sup>9</sup> The uncertainty here is not related to the financing of the project, but rather to the physical characteristics of gross returns.

<sup>10</sup> Note that  $\gamma$  is the upper bound of the value of  $\lambda(K_t)$ .

<sup>11</sup> The contagion phenomenon is well documented in domestic markets as well as among economies. See for example Longstaff (2010) on the subprime credit crisis and contagion in financial markets and [Van Rijckeghem](#), C. and [B. Weder](#) (2001) on the sources of cross-border contagion.

where  $\tau w_j, j = 1,2$  is a banking risk-management constraint on the amount of loans an individual of type  $j$  can receive (introduced in equation (4g)). This  $k_{\min}$  is instrumental for the resulting separating behavior of individuals of different types. It prevents type-1 individuals from using the physical investment channel, inducing them to make commercial bank deposits<sup>12</sup>

We assume that individuals who deposit in the commercial banks enjoy a safety net provided by the CB in the form of partial deposit insurance for a predetermined share  $\theta, 0 \leq \theta \leq 1$ , of their deposit.<sup>13</sup>

In the following analysis we assume that a separating equilibrium exists, such that type-1 individuals invariably use bank deposits to transfer purchasing power to next period, while type-2 individuals for the same purpose use their endowments and bank loans to invest in the physical capital. We later prove that equilibrium with this separation does indeed exist. The individuals' respective budget constraints are presented in equations (A:1) and (A:2).

Given market prices, market interest rates, bank dividends, deposit insurance, and based on the perceived probability of bank default, a representative depositor chooses  $\{c_t, E_t c_{t+1}, d_t\}$  to maximize (1), subject to (A:1) and (A:9) (in appendix A). Similarly, a representative investor operating in the same environment and perceived  $\lambda$ , chooses  $\{c_t, E_t c_{t+1}, k_t, l_t\}$  to maximize (1), subject to (A:2) and (A:10) (in appendix A), where  $l_t$  denotes a bank loan.

## **2/2. Commercial banks**

<sup>12</sup> It also prohibits type-1 individuals from purchasing shares in the physical investment project and hence our model does not allow for external equity financing.

<sup>13</sup> The choice of  $\theta$  could reflect two opposing forces, which provide the backdrop for this model. A higher  $\theta$  enhances financial stability by reducing the correlation of shocks to deposits and a systemic shock, while at the same time it enhances risk taking by the commercial banks (moral hazard).

There are  $I$  identical infinitely-lived commercial banks<sup>14</sup> in the economy. Each bank offers one-period nominal deposits,  $D_t$ , which pay a nominal interest rate,  $i_{dt}$ <sup>15</sup>. We assume that the deposit market is *perfectly* competitive, while the loan market is *imperfectly* competitive. Banks offer investors one-period nominal loans,  $L_t$  at a nominal interest rate,  $i_{lt}$ . The collateral for this loan is the borrower's realized return  $f(k_t)$ . In the conduct of financial intermediation, each bank draws on its (beginning-of-period) equity capital,  $FK_t$  that has been accumulated from retained earnings of previous periods and it is therefore predetermined (for a similar structure see Barnea and Kim, 2014).

The young individuals own and manage the commercial banks and the profits of the banks are realized a period later, when these individuals become old<sup>16</sup>. In period  $t$ , the young individuals choose a fraction of bank earnings,  $\delta_{t+1}$ , to be distributed as dividends. The remainder  $(1 - \delta_{t+1})$  is retained and is accrued to the bank's equity capital<sup>17</sup>,  $FK_{t+1}$ . Accordingly, banks are expected to accumulate financial capital following the dynamic process:

$$E_t FK_{t+1} = FK_t - (1 - \lambda)L_t + (1 - \delta_{t+1})E_t \Pi_{t+1}, \quad (4a)$$

where  $L_t$  is the total outstanding representative bank loans, and where the second term on the RHS of (4a) is the bank loans that were written off after the realization of the return on the physical investment at the beginning of period  $t+1$ . The last term represents the bank's retained earnings, where  $\Pi_{t+1}$  is the representative bank profits.

<sup>14</sup> They are identical in the sense that they have the same shareholders and they each begin period  $t$  with the same equity capital.

<sup>15</sup> Since banks are identical, we refer in the model to a representative bank and omit a superscript indicating individual bank unless otherwise deemed necessary for clarity.

<sup>16</sup> Hence the young individuals are motivated to maximize profits since they will enjoy the fruits of their choices (the dividends) when they become old.

<sup>17</sup> Note that the choice of  $\delta_{t+1}$  could be strictly between zero and one, where the realized earnings in period  $t+1$  can also be used to maintain the required capital ratio.

Commercial banks can borrow a one-period (monetary) loan,  $l_{mt}$ , from the CB supplied at a nominal rate,  $i_{mt}$ <sup>18</sup> (KPR). The CB lending facility is extended against secure collateral, which, in our model consists of commercial bank reserves (denoted  $RR$ ) deposited at the CB. The CB lending facility increases the potential supply of commercial bank loans to households from a maximum of  $L_t = (1-rr_t)D_t + FK_t$ , when monetary loans are not available and where  $rr$  is the reserves/deposit ratio, to a maximum of  $L_t = D_t + FK_t$ , when the entire amount of monetary loans is extended. The reserves ratio,  $rr$ , in our model is a structural variable necessary for the implementation of monetary policy and not a monetary policy tool.

Each commercial bank faces the following collateral constraint:

$$l_{mt} \leq RR_t. \quad (4b)$$

We assume banks do not hold excess reserves, that is,

$$RR_t = rrD_t. \quad (4c)$$

The representative bank must maintain a minimum required capital ratio of the *expected* beginning-of-period  $t+1$  equity capital,  $E_t FK_{t+1}$ , relative to the total lending of the bank in period  $t$ ,  $L_t$ , of  $\kappa$  percent.<sup>19</sup> The minimum capital requirement ratio is, therefore:

$$\frac{E_t FK_{t+1}}{L_t} \geq \kappa. \quad (4d)$$

The resource constraint (balance sheet) of the bank prior to the realization of systemic risk is:

<sup>18</sup> The CB in many countries provides secured loans to commercial banks through reverse repurchase agreements. Such operations constitute the primary day-to-day monetary policy instrument in these countries.

<sup>19</sup> Note that  $FK_t$  is a state and predetermined variable at period  $t$ , thus we require the capital requirements to refer to  $E_t FK_{t+1}$  rather than to the current period stock of financial capital.

$$L_t + RR_t = D_t + FK_t + I_{mt}. \quad (4e)$$

If systemic risk is realized, such that the bank becomes insolvent and the total (period  $t$ ) financial capital of the representative bank is decimated, the CB reacts by paying off the insured a fraction  $\theta$  of the deposits to depositors through the commercial banks. In this situation the following condition is satisfied for the representative bank:

$$\lambda(K_t)(1 + i_{Lt})L_t + RR_t = \theta(1 + i_d)D_t + (1 + i_{mt})I_{mt}. \quad (4f)$$

Following a period in which the commercial bank becomes insolvent, it continues operating in the following period as a new financial intermediary<sup>20</sup>.

Finally, a maximum leverage constraint is placed on each borrower, which is set by the bank's management and is given by

$$\frac{l_t}{p_t w_j} \leq \tau, \quad j = 1, 2, \quad (4g)$$

where  $l_t$  denotes the amount extended to a single borrower by the representative bank as a loan. We refer to this constraint as the risk-management constraint. The risk-neutral bank management imposes this constraint because of anticipated bankruptcy costs.<sup>21</sup> As will be shown later, when binding, this constraint affects the monetary transmission mechanism. (Result 3, and Proposition 3 and its proof in appendix D)

The *state unconditional* expected profits of period  $t+1$ ,  $E_t \Pi_{t+1}$ , is the sum of conditional profits (B:1 and B:2 in Appendix B) weighted by their probabilities in all states. It is given by

<sup>20</sup> Note that all assets and obligations in our model are of single-period duration. Therefore, in the following period everything starts from nil. Alternatively, we can assume that a new bank is established in every period.

<sup>21</sup> Note that  $\tau w_j$  is the upper bound on the amount of bank loans (in real terms) an individual of type  $j$  can obtain. Note, further, that type-1 individuals cannot acquire the  $k_{\min}$  required for the operation of the physical investment because of (3) and constraint (4g).

$$E_t \Pi_{t+1} = \lambda i_{Lt} L_t - (1 - (1 - \theta) q_{t+1}) i_{dt} D_t - i_{mt} l_{mt}, \quad (5)$$

where  $q_{t+1}$  is the probability of bank's failure.

Commercial bank shareholders seek to maximize their expected period  $t+1$  dividends,  $\delta_{t+1} E_t \Pi_{t+1}$ , where expected earnings are defined in (5). In period  $t$ , commercial banks choose  $i_{Lt}$ ,  $l_{mt}$ ,  $E_t FK_{t+1}$ , and  $\delta_{t+1}$  to maximize the expected dividends, subject to the terms and constraints articulated in (4a)-(4g) and given  $FK_t$ . Lastly the realized retained earnings of period of  $t+1$  belong to the young generation of period  $t+1$ , augmenting their predetermined financial capital at the beginning of that period.

### 2.3. Central bank

The CB pursues two goals: price stability in the form of an inflation target and financial stability, defined in terms of the probability of systemic bank defaults  $q_{t+1}$ . For simplicity we assume that both target levels are those which existed prior to the occurrence of the shocks we introduce. The CB undertakes its policy in period  $t+1$  after  $\lambda(K_t)$  has been realized, and it succeeds in restoring target levels.

In conducting its monetary policy, the CB provides an infinitely elastic supply of monetary loans to commercial banks at its policy interest rate,  $i_{mt}$ . We assume that the policy interest rate is set to achieve the inflation target. To safeguard financial stability, the CB imposes a minimum capital adequacy ratio,  $\kappa$ , which limits credit expansion and consequently, exposure to risk. The CB also provides a safety net in the form of partial deposit insurance for a share  $\theta$ ,  $0 \leq \theta \leq 1$  of commercial bank deposits, where  $\theta$  is predetermined. Deposit insurance is one way of dealing with

bank runs.<sup>22</sup> It is modeled for obtaining an equilibrium in the absence of a run and accordingly the fraction  $\theta$  is a structural parameter.

The CB accumulates real balances of seigniorage revenues ( $SR$ ) from the monetary loans, on which it charges a positive interest rate,  $i_{mt}$ . Given that deposit insurance has not been activated, the accumulated  $SR$  at the CB is:

$$SR_t = \sum_{i=1}^I \left( \frac{rr_t D_t^i - l_{mt}^i - (rr_{t-1} D_{t-1}^i - l_{mt-1}^i)}{p_t} + \frac{i_{mt-1} l_{mt-1}^i}{p_t} \right) + \frac{p_{t-1}}{p_t} SR_{t-1}. \quad (6)$$

Capital letters denote aggregation over all individuals and commercial banks. The CB real net worth is increased by the flow of  $SR_t - \frac{p_{t-1}}{p_t} SR_{t-1}$  and is depleted when deposit insurance is activated. To meet its deposit insurance obligations, the CB needs to generate sufficient  $SR$ . Consequently, the following incentive-compatible constraint must be satisfied

$$\sum_{i=1}^I \left( \theta(1+i_{dt}) \frac{D_t^i}{p_t} - (\lambda(K_t)(1+i_{Lt}) \frac{L_t^i}{p_t} + \frac{RR_t^i}{p_t} - (1+i_{mt}) \frac{l_{mt}^i}{p_t} \right) \leq SR_t, \quad (7)$$

Constraint (7) reflects the requirement that the  $SR$  accumulated up to period  $t$  should be no less than the sum (over all banks) of the value of the banking system's net worth when systemic risk is realized. We limit ourselves to an equilibrium in which the inequality in (7) holds. Finally, for simplicity we assume that the amount of  $SR$  accumulated is kept as reserves at the  $CB$ , i.e. it accrues no interest. This accumulation helps consolidate the credibility of the  $CB$ .<sup>23</sup>

### 3. Equilibrium characteristics

<sup>22</sup>Another possible way to prevent a run on banks is to relax the collateral requirements imposed on banks when borrowing from the  $CB$ .

<sup>23</sup> Since we assume that  $CB$  resources are finite, there is always a positive probability that its financial resources will be perceived as insufficient to back its deposit insurance obligations. The greater the accumulated seigniorage reserves, the higher the public's confidence in the integrity of the deposit insurance arrangement will be.



In this section we analyze results emanating from the model, which characterize the model's equilibrium.

### **3.1. Information setup, market clearing and equilibrium**

Since the returns on the physical investment undertaken in period  $t$  is realized at the beginning of period  $t+1$ , individual and commercial bank choices for period  $t$  cannot be contingent on the realization of  $\lambda(K_t)$ . Since the choices of some of these variables depend on period  $t$  expectations of  $t+1$  price levels,  $p_{t+1}$  too cannot be contingent on the realization of  $\lambda(K_t)$ . These variables depend on a predetermined or perceived  $\lambda$  and  $q_{t+1}$ , which in turn are contingent on the realizations in the previous period. The consumption of old individuals in period  $t+1$  can be contingent on the realization of  $\lambda(K_t)$ .

As mentioned above the CB reacts to period  $t$  shocks at the beginning of period  $t+1$  after the realization of  $\lambda(K_t)$  and prior to the decision-making of the young individuals in period  $t+1$ .

In period  $t$ , type-2 young individuals make their decision regarding the physical investment based on their perceived  $\lambda$ , irrespective of the contagion effect of their choice on  $\lambda(K_t)$  (see (2a)). To explain this externality we assume that the individual is aware of the contagion effect (2a) but has no incentive to act upon it. In fact he/she benefits if all other investors adjust their investments in accordance with the impact of  $K$  on  $\lambda(K_t)$ , while he/she does not. Since this holds for all individuals, ultimately no one will adjust their investment. Without the interference of the CB, this externality will result in over-investment and lower the probability of success  $\lambda(K)$ .

To maintain financial stability, the CB reacts by adjusting  $\kappa$  in period  $t+1$ , following the realization of  $\lambda(K_t)$ , such that it will successfully manage to have the  $\lambda$

and  $q$  perceived by type-2 young individuals for period  $t+1$  equal the period  $t$  levels. Note that this is not a *ceteris paribus* situation, since changing  $\kappa$  may affect inflation expectations as well as other aspects of the macroeconomic equilibrium.

In equilibrium the realized probability of a bank failure,  $q_{t+1}$ , should be consistent with macroeconomic fundamentals, since these determine the conditions under which commercial banks fail. Note that as  $q_{t+1}$  rises, financial stability is worsened. Hence  $q_{t+1}$  must satisfy the following:

$$q_{t+1} = H\{FK_t + (1+i_{Lt})\lambda L_t + RR_t - (1+i_{dt})D_t - (1+i_{mt})I_m \leq 0\}, \quad (8)$$

where  $H$  is the cumulative probability distribution of the default on loans granted by the representative bank in the preceding period<sup>24</sup>, and it is a monotonic function with  $H' < 0$ . The term in the curly brackets is the set of all states in which the representative bank's equity capital is non-positive.

Substituting the resource constraint (4e) and conditional expected earnings when systemic risk has not materialized (B:1 and B:2), and inserting (B:1) into (8) yields the following:

$$q_{t+1} = H\{FK_t + E_t \Pi_{t+1}^{1-q} - (1-\lambda)L_t \leq 0\} \quad (8a)$$

where  $E_t \Pi_{t+1}^{1-q}$  is defined in (B:1) in Appendix B. The expression within the curly brackets indicates all states in which the beginning-of-period  $t$  bank equity capital and the following period's expected earnings are not sufficient to cover expected loan losses. Since  $FK_t$  is predetermined in period  $t$ ,  $q_{t+1}$  is determined by the expected conditional bank earnings in the following period and expected loan defaults.

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<sup>24</sup> The expression  $(1+i_{Lt})\lambda(K_t)L_t + RR_t$  is the nominal value of the commercial bank's total assets, expected to be realized in period  $t+1$ , while  $(1+i_{dt})D_t + (1+i_{mt})I_m$  is the nominal value of its total liabilities (to the individuals and the CB), expected to be realized in period  $t+1$ . When the latter exceeds the former, the addition to bank equity is negative.

Consolidating individual budget constraints, commercial bank retained earnings and CB resource accumulation gives rise to the following per capita equilibrium clearing condition in the consumption goods market:

$$c_{1t}^y + c_{2t}^y + c_{1t}^o + c_{2t}^o + k_t + \frac{FK_t}{\left(\frac{N}{2}\right)p_t} + (1-\theta)q_t \frac{d_{t-1}}{p_t} + \frac{SR_t}{N/2} - \frac{p_{t-1}}{p_t} \frac{SR_{t-1}}{N/2} = w_1 + w_2 + \lambda(K_t)f(k_{t-1}) \quad (9)$$

The last term in the first row represents the uninsured deposits of which a share  $q_t$  is lost.

### 3.2. Households: separating equilibrium

We use the first-order conditions (FOC) to characterize an individual's choices in our economy. Let  $v_{c_t}$  denote the marginal utility with respect to  $c_t$ . As investors regard the perceived  $\lambda$  as given, the condition characterizing their choice is

$$\frac{v_{c_t}}{\beta} = \lambda f_k, \quad (10a)$$

where  $f_k$  denotes the marginal productivity of the physical investment  $k_t$ . Thus the marginal rate of substitution in consumption (the *LHS* of (10a)) equals the expected return on the physical investment. We further obtain for the investors that

$$\lambda f_k = \lambda \frac{1+i_{Lt}}{p_{t+1}/p_t} \equiv R_{Lt}, \quad (10b)$$

$R_{Lt}$  is defined as the expected real interest rate on loans (RHS of (10a)) and it equals the expected return on the physical investment (LHS of (10a)).

From the FOC of depositors' choices, we then obtain:

$$\frac{v_{ct}}{\beta_1} = (1 - (1 - \theta)q_{t+1}) \frac{1 + i_{dt}}{p_{t+1}/p_t} \equiv R_{dt}, \quad (11a)$$

$R_d$  is defined as the expected real return on bank deposits (RHS of (11a)), and it equals the marginal rate of substitution in consumption. (LHS of (11a)). Rearranging (10b) and solving for the inflation rate, we arrive at the following equilibrium condition

$$\frac{p_{t+1}}{p_t} = \frac{1 + i_{Lt}}{f_{kt}}. \quad (11b)$$

This equilibrium condition guarantees the satisfaction of the Fisher relationship between the nominal (gross) interest rate,  $1 + i_{Lt}$ , the real (gross) rate of return,  $f_k$  and the (gross) expected rate of inflation. In our model, inflation is determined through asset portfolio selection. This approach to modeling inflation is consistent with Sims (2009).

These equilibrium conditions imply a *separating* equilibrium:

**Result 1.** *If inequality (3) is binding and if the perceived probabilities  $\lambda$  and  $1 - (1 - \theta)q_{t+1}$  are such that the incentive compatible inequality  $R_{Lt} > R_{dt}$  holds, and if there exists an equilibrium for our economy with  $D_t > 0$ , then we obtain a separating equilibrium in which the lowly-endowed individuals (type 1) deposit funds in the banks, and do not invest in physical capital projects, while the highly-endowed individuals (type 2) invest in physical capital, and do not deposit at the banks.*

(See proposition 1 and its proof in appendix D).

### 3.3. Commercial banks: choices

In this section we derive the conditions characterizing the representative bank's choices. We eliminate  $D_t$  from (5) by utilizing (4c) and the resource constraint

(4e), and then use constraints (4a)-(4b), (4d) and (4g) to set the Lagrangian for the representative bank as follows

$$\begin{aligned}
\Psi &= \delta_t E_t \Pi_{t+1} \\
&+ \frac{N}{2} \varphi_{1t} (\varpi_t w_2 - l_t) + \varphi_{2t} (E_t FK_{t+1} - \kappa L_t) \\
&+ \varphi_{3t} (rr(L_t - FK_t) - l_{mt}) + \varphi_{4t} (FK_t - (1-\lambda)L_t + (1-\delta_t)E_t \Pi_{t+1} - E_t FK_{t+1}) \\
&= \delta_t [\lambda i_{L_t} L_t - (1 - (1-\theta)q_{t+1}) \frac{i_{dt}}{1-rr} (L_t - FK_t - l_{mt}) - i_{mt} l_{mt}] \\
&+ \frac{N}{2} \varphi_{1t} (\varpi_t w_2 - l_t) + \varphi_{2t} (E_t FK_{t+1} - \kappa L_t) \\
&+ \varphi_{3t} (rr(L_t - FK_t) - l_{mt}) + \varphi_{4t} (FK_t - (1-\lambda)L_t + (1-\delta_t)E_t \Pi_{t+1} - E_t FK_{t+1})
\end{aligned} \tag{12}$$

where  $\varphi_j, j=1,2,3,4$ , are the Lagrange multipliers associated with constraints (4g), (4d), (4b) and (4a), respectively, and  $N/2$  denotes the number of individuals who borrow from the commercial bank. As before capital letters indicate aggregate quantities.

We begin with the *FOC* derived for the payout ratio  $\delta$  and obtain:

$$E_t \Pi_{t+1} - \varphi_{4t} E_t \Pi_{t+1} = 0. \tag{13a}$$

If  $0 < \delta < 1$ , such that (13a) is binding, then  $\varphi_{4t} = 1$ , all  $t$ , where  $\varphi_{4t}$  is the shadow price (in terms of expected profit) of the financial capital accumulation constraint. Note that both (4a) and (4d) restrict  $E_t FK_{t+1}$  and they are in fact linear, so in the case where they are both binding we have:

$$\varphi_{2t} = \varphi_{4t} = 1, \tag{13b}$$

where  $\varphi_{2t}$  is the shadow price of the minimum capital requirement constraint<sup>25</sup>.

### 3.4. The use of CB monetary loans

<sup>25</sup> The financial capital accumulation (4a) is always binding, while the minimum capital ratio (4c) may not be binding, in which case banks hold financial capital in excess of the mandatory amount.

**Result 2.** *There is no partial use of commercial bank collateral for the CB monetary loans. Either collateral is not used at all or the banks utilize all of their collateral. such that  $l_{mt} = rrD_t$ .*

(See Proposition 2.)

### **Corollary**

Result 2 implies that equation (6) for the  $SR$  can be reduced to the following:

$$SR_t = \sum_{i=1}^I \frac{i_{mt-1} L_{mt-1}^i}{p_t} + \frac{p_{t-1}}{p_t} SR_{t-1} \quad (6a)$$

Namely, the accumulation of seigniorage revenues by the CB is contingent on the interest payments on the monetary loans and the inflation tax. Hence, in equilibrium where banks use all of their collateral such that  $l_{mt} = rrD_t$ , the resource constraint of the commercial bank (4e) implies that:

$$L_t = D_t + FK_t, \text{ all } t. \quad (4e1)$$

## **4. The transmission and effectiveness of monetary policy**

In this section we analyze the implementation of monetary policy and derive the pass-through from the CB interest rate to bank interest rates. To demonstrate the implications of our results we analyze unconventional monetary policies by the CB. We also examine the effectiveness of the monetary policy, taking into consideration the constraints imposed on commercial banks.

### **4.1. Inside the Transmission Mechanism "Black Box": Pass-through from the CB policy interest rate to the banking interest rates**

The pass-through from the CB policy interest rate to the banks deposit and loan rates is derived from the commercial banks' FOC. We first obtain the equilibrium relationship between the deposit rate and the monetary policy rate:

$$(1 - (1 - \theta)q_{t+1}) \frac{i_{dt}}{1 - rr} = i_{mt} + \varphi_{3t}. \quad (13c)$$

Next, we derive the relation between the loan interest rate and the deposit interest rate from the FOC of  $i_{Lt}$ , obtaining

$$\lambda \left(1 + \frac{1}{\eta_t}\right) i_{Lt} = (1 - (1 - \theta)q_{t+1}) \frac{i_{dt}}{1 - rr} + \varphi_{1t} + \kappa - rr\varphi_{3t} + (1 - \lambda), \quad (13d)$$

where  $\eta_t = \frac{i_{Lt}}{L_t} \frac{\partial L_t}{\partial i_{Lt}}$  is the interest rate elasticity of the demand for loans.

Finally we combine (13c) and (13d) to eliminate  $(1 - (1 - \theta)q_{t+1})$  and obtain the transmission mechanism from the CB rate  $i_{mt}$  to the bank loan rate  $i_{Lt}$  as follows:

$$\lambda \left(1 + \frac{1}{\eta_t}\right) i_{Lt} = i_{mt} + \varphi_{1t} + \kappa + (1 - rr)\varphi_{3t} + (1 - \lambda). \quad (13e)$$

Equations (13c) - (13e) contain some important insights. First, the transmission mechanism from the central bank policy rate to lending and deposit rates is affected by financial stability tools as well as the probability of bank default. In particular, the pass-through from the  $i_{mt}$  to  $i_{dt}$  (see (13c)) depends on the soundness of the banking system ( $q_{t+1}$ ), and on the extent of deposit insurance coverage, as well as on the reserves ratio. In the case of the pass-through from  $i_{mt}$  to  $i_{Lt}$  (see (13e)) it depends on  $\lambda$ , on the capital ratio and on the reserve ratio.

Second, equation (13c) clearly demonstrates the manner in which the policy interest rate and capital adequacy requirements may be used by the CB to reach both

price stability and financial stability targets. If, for example<sup>26</sup>, in an attempt to stave off deflationary pressures, the CB implements an expansionary monetary policy (by reducing  $i_{mt}$ ) and simultaneously raises the minimum capital requirement ratio  $\kappa$  for financial stability purposes, the combined impact on  $i_L$  could offset one another (see 13e). These results clearly demonstrate the need for coordinated monetary and financial stability policies.

Third, the shadow price of the collateral constraint,  $\varphi_3$ , plays an important role in the monetary transmission mechanism. When the CB tightens its monetary policy by raising  $i_{mt}$ , for example, the demand for monetary loans is reduced, lowering in turn the shadow price of the collateral constraint  $\varphi_3$ . It is apparent from (13c) and (13e) that the reduction in  $\varphi_3$  mitigates, *ceteris paribus*, the positive impact of the increase in  $i_{mt}$  on deposit and lending rates,  $i_{dt}$  and  $i_{Lt}$  respectively. Importantly, the decrease in  $\varphi_3$  more than offsets the positive effect of  $i_{mt}$  on the deposit rate, such that the expected return on deposits (the LHS of (13c)) is actually lower. (See Lemma 1 and Figure 4).

#### 4.2. Unconventional monetary policy

To demonstrate the implications of the above analysis, let us suppose that the CB enacts an "unconventional" monetary policy, whereby it relaxes the collateral requirements of its lending facility, allowing the CB loans to be not fully collateralized or accepting collateral in the form of risky corporate securities<sup>27</sup>. This policy is reflected in our model in the parameter  $\chi$ , where  $\chi < 1$ . We insert it into (4b) and obtain:

<sup>26</sup> This example is taken from reality as is reflected in the following two cases: (1) The Bank of Israel recently undertook a similar combination of measures described in the example, where instead of increasing  $\kappa$ , it reduced the maximum Loan-to-Value allowed on mortgages. (2) Regulators in some European countries set lower as well as upper limits on  $\kappa$ . The latter is designed to prevent a deviation of actual from potential output.

<sup>27</sup> This was implemented by the Fed in the recent crisis as part of its quantitative easing policy.



$$i_{mt}^i \leq \frac{1}{\chi} RR_t^i. \quad (4b1)$$

In this case the Euler's condition (13c) remains intact, while (13d) and (13e) change, becoming:

$$\lambda \left(1 + \frac{1}{\eta_t}\right) i_{Lt} = (1 - (1 - \theta)q_{t+1}) \frac{i_{dt}}{1 - rr} + \varphi_{1t} + \kappa - rr \frac{\varphi_{3t}}{\chi} + (1 - \lambda) \quad (13d1)$$

$$\lambda \left(1 + \frac{1}{\eta_t}\right) i_{Lt} = i_{mt} + \varphi_{1t} + \kappa + (1 - rr) \frac{\varphi_{3t}}{\chi} + (1 - \lambda). \quad (13e1)$$

Relaxing the collateral requirements by lowering  $\chi$  increases the excess supply of monetary loans and as a result reduces the shadow price of the collateral constraint  $\varphi_3$ . The commercial bank's demand for public deposits weakens so that the expected return on deposits falls (see (13c)).

The result in *Lemma 2* demonstrates how quantitative easing can affect the monetary transmission mechanism, resulting in a reduction of expected bank interest rates.

**Lemma 2:** *If  $\varphi_{1t}=0$ , the perceived  $\lambda$  remains constant and constraint (4b) is binding, then reducing  $\chi$  will lower the expected return on deposits,  $(1 - (1 - \theta)q_{t+1}) \frac{i_{dt}}{1 - rr}$  and the expected loan rate,  $\lambda \left(1 + \frac{1}{\eta_t}\right) i_{Lt}$ .*

(For the proof, differentiate (13c), (13d1) and (13e1) with respect to  $\chi$ ).

### 4.3. Effectiveness of the transmission mechanism

We measure the effectiveness of the monetary transmission mechanism by the deviations from 1 of the partial derivatives of the expected marginal revenue from loan and the expected marginal cost of deposits with respect to  $i_m$  (see equations

(13c1) and (13e2)). The smaller the absolute value of these deviations is, the more effective the transmission mechanism.<sup>28</sup>

Differentiating the Euler conditions for the deposit and loan rates (13c) and (13e) with respect to  $i_{mt}$  ((13c) and (13e) respectively) results in the following system of relationships, which reflects the effectiveness of the monetary policy:

$$\frac{\partial((1-(1-\theta)q_{t+1})i_{dt})}{\partial i_{mt}} = 1 + \frac{\partial \varphi_{3t}}{\partial i_{mt}} \quad (13c1)$$

$$\frac{\partial(\lambda(1+\frac{1}{\eta_t})i_{Lt})}{\partial i_{mt}} = 1 + (1-rr) \frac{\partial \varphi_{3t}}{\partial i_{mt}}, \quad (13e2)$$

where  $\frac{\partial \varphi_{3t}}{\partial i_{mt}} < 0$ .

The last inequality stems from the following dynamics. When the CB increases its interest  $i_m$ , making the use of its lending facility more expensive and therefore the commercial bank demands for these loans as well as the shadow price of the collateral constraint  $\varphi_3$  are reduced.

An important trade-off emerges from this finding. On the one hand, requiring full collateralization for CB loans safeguards central bank resources. On the other hand, stricter collateral requirement reduces the effectiveness of the monetary transmission mechanism as it increases the absolute value of  $\frac{\partial \varphi_{3t}}{\partial i_{mt}}$ .

Another insight gained from equations (13c-13e) is that when the CB interest rate reaches the zero bound, the CB can affect  $i_{Lt}$  by adjusting capital adequacy

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<sup>28</sup> Note that monetary transmission becomes particularly effective when there is full deposit insurance ( $\theta=1$ ). But this in turn may, *ceteris paribus*, expose the banking system to more systemic risk, as the willingness of commercial banks to expand lending increases. This is yet another example of the tradeoff between the effectiveness of monetary policy and systemic risk.

requirements. Note that the direction of this effect is not straightforward, as can be seen below. We differentiate (13c) and (13e) with respect to  $\kappa$  to obtain:

$$\frac{d((1 - (1 - \theta)q_{t+1})\frac{i_{dt}}{1 - rr})}{d\kappa} = \frac{d\varphi_{3t}}{d\kappa} < 0 \quad (13c2)$$

and

$$\frac{d(\lambda(1 - \frac{1}{\eta_t})i_{Lt})}{d\kappa} = 1 + (1 - rr)\frac{d\varphi_{3t}}{d\kappa}. \quad (13e3)$$

Increasing  $\kappa$  reduces the supply of bank loans and consequently reduces the demand for CB loans. This in turn reduces the shadow price  $\varphi_3$ , i.e.  $\frac{d\varphi_{3t}}{d\kappa} < 0$ . As can be seen from (13e3), the sign of the impact of  $\kappa$  on the expected lending rate depends on the marginal impact of  $\kappa$  on the shadow price  $\varphi_{3t}$ . Thus, for example, the expansionary effect from the easing of capital requirements on lending rates weakens as the absolute value of  $\frac{d\varphi_{3t}}{d\kappa}$  increases.

Hence, when a reduction in the lending rate is warranted to offset a deflationary shock, for example, what is considered an instrument of financial stability under normal circumstances can become a viable tool for implementing monetary policy. This clearly demonstrates how inextricably interlocked the tools of monetary policy and financial stability indeed are<sup>29</sup>.

#### ***4.4. The effect of the risk-management (leverage) constraint on the monetary policy transmission mechanism***

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<sup>29</sup> While the recent relaxation of collateral requirements by many CBs may have increased the pass-through from the CB interest rate to the deposit rates, it entails a cost of putting taxpayers' money at greater risk and raises questions as to the extent to which CB should have policy independence.

Finally we examine the impact of the risk-management constraints (4g) on the effectiveness of monetary policy. For this we make constraint (4g) on the endowment of type-2 individuals (investors) binding. We have also assumed that  $k_{\min}$  is such that only type-2 individuals can receive loans to finance their investments.

***Result 3.** If the maximum leverage constraint is binding at the equilibrium banking interest rates for all individuals, then the monetary policy (changes in  $i_m$ ) has no effect on either  $L_t$ ,  $D_t$ ,  $k_t$ , or  $i_{Lt}$  and therefore, has no effect on its target of inflationary expectations.*

(See proposition 3 and its proof in appendix D)

## **5. Interaction between monetary policy and financial stability**

In this section we simulate our model to study the interaction and the reciprocity of monetary policy and financial stability policy. We resort to simulations because the model contains non-linearities, rendering the analysis complicated and prone to ambiguous results. In particular, we examine the impact of two shocks: (i) a permanent positive shock to the return on physical investment which is translated in our model to a shock to expected inflation, to which the CB reacts by adjusting its policy interest rate, overlooking the potential impact of the shock on financial stability; (ii) a negative shock to  $\lambda(K_t)$ , which is interpreted as a shock to financial stability to which the CB reacts by adjusting the capital adequacy requirement  $\kappa$ , ignoring the impact on expected inflation.

In the simulations we introduce persisting<sup>30</sup> shocks and then perform a comparative statics analysis following the convergence of the economy to its new equilibrium. The simulations are based on the relationships derived from the model

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<sup>30</sup> Persistence in our model environment means the two-period life of the individuals repeats itself indefinitely. This is also the definition of steady-state in our model.

and selected values for both the model's parameters and state variables (presented in Table 1). The initial values of the policy variables  $i_m$  and  $\kappa$  are chosen to be consistent with our model's initial steady-state solution, and the latter then serves as a benchmark for the policy response to the shocks in the comparative statics analysis.

**Table 1:**  
**The parameter's values used in the simulations**

Parameter	Value	Steady State value	Parameter	Value	Steady State value
$A$	0.32	0.32	$k_{min}$	1.22	1.22
$A$	5.7-6.6	6.2	$\frac{FK_t}{P_t}$	0.0051*	0.0051
$\lambda$ (perceived)	0.72	0.72	$i_{mt}$	0.045-0.15	0.06
$q_{t+1}$ (perceived)	0.25	0.25	$\tau$	Non-binding	Non-binding
$B$	0.975	0.975	$\gamma$	0.9, 0.95	0.95
$w_1$	1.2	1.2	<i>Inflation target</i>	0.1136*	0.1136
$w_2$	2.0	2.0	$SR_{t-1}$	0.0205*	0.0205
$\theta$	0.25	0.25	$rr$	0.4	0.4
$N$	2	2	$K$	0.05-0.06	0.05
$I$	1	1			

\* indicates that the initial value (in period  $t-1$ ) was taken to equal the endogenous period  $t$  value.

Given the value ranges of the model's parameters (specified in Table 1), we use the MATHLAB8 software to numerically solve for the model's equilibrium. The derived equilibrium characteristics include several incentive compatible constraints: inequality (3); equation (4e1) with  $d_t > 0$ ; and  $R_{Lt} > R_{dt}$ . We display the relevant equilibrium values of these constraints in Table 2.

From the simulations we derive the model relationships which are displayed in Figures 1-8. The simulations were conducted on a wide range of the values of the model parameters and the results were found to be robust.

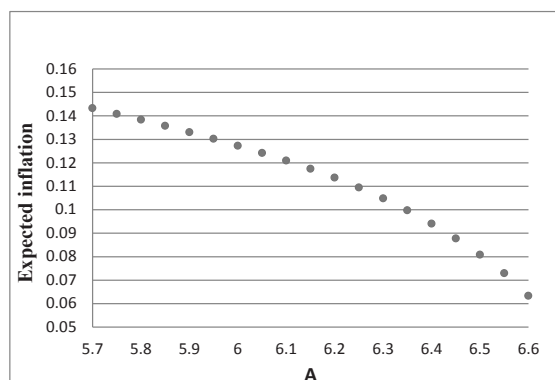
**Table 2:**  
**Incentive Compatible Variables  
in the Steady-State Equilibrium**

Variable	Value
$\frac{d_t}{p_t}$	0.0971
$\frac{l_t}{p_t}$	0.1022
$\frac{FK_t}{p_t}$	0.0051
$R_{lt}$	1.222
$R_{dt}$	0.93
$w_1$	1.2
$w_2$	2.0
$k_t$	1.2575
$k_{\min}$	1.24

Based on proposition 1, when numerically solving for equilibrium in our model economy, while assuming separating behaviors for type-1 and type-2 individuals, we obtain a solution (an equilibrium) in which the incentive-compatible conditions are all satisfied (see Table 2). It implies that a separating equilibrium with financial intermediation in our model economy does exist.

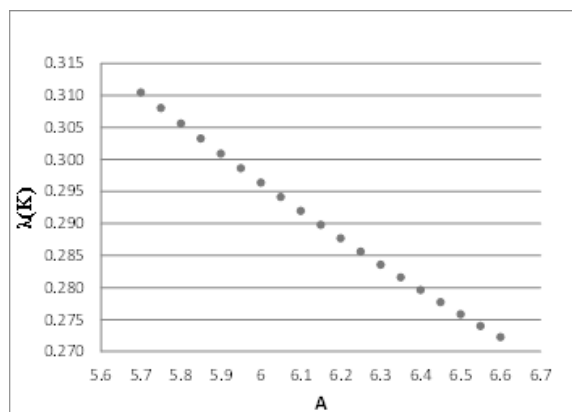
### **5.1. Shock to the return on physical investment**

We begin with the economic impact of a permanent positive shock to the return on the physical investment (an increase in  $A$  in eq. (2)). It permanently affects inflation expectations and thus can be considered a permanent shock to inflation expectations, to which the CB reacts.



**Figure 1. The effect of  $A$  on the expected inflation**

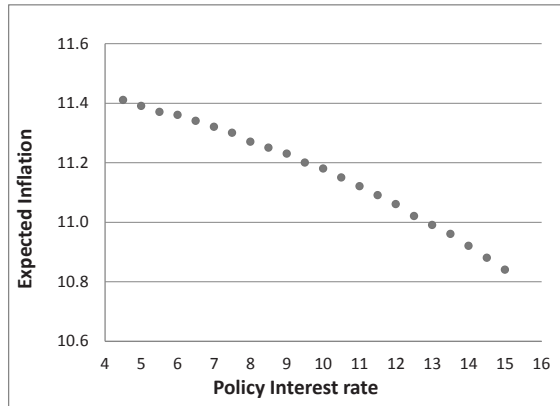
As can be seen in Figure 1, such an increase in  $A$  will lower inflation expectations, causing it to be below its target, and reduce the probability of success of the physical investment (see Figure 2), resulting in a negative deviation of financial stability from its target.



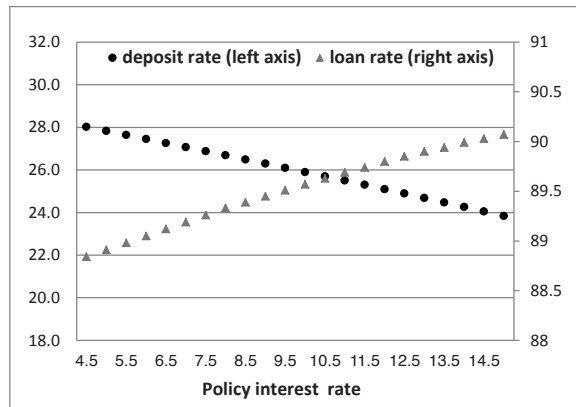
**Figure 2. The effect of  $A$  on  $\lambda(K)$**

Suppose now that the CB policy restores inflationary expectations back to their target level, while ignoring the deterioration in financial stability. The CB reacts by decreasing its policy interest rate. Figure 3 displays the *negative* effect of the CB policy rate on expected inflation, enabling the CB to reduce or eliminate the deviation of inflation expectations from the target. In Figure 4 we display the monetary

transmission mechanism in our model from the policy interest rate to the deposit and loan rates (see Lemma 1)<sup>31</sup>.



**Figure 3. The effect of the policy interest rate on the expected inflation**

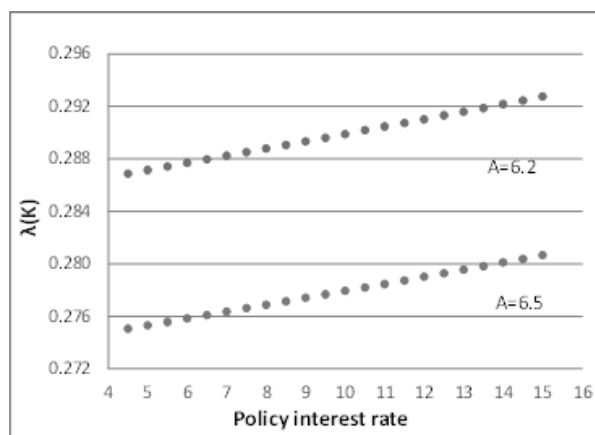


**Figure 4. The effect of the policy interest rate on deposit and loan rates**

From Figure 5 we learn that, since investment rises as a result of lowering the policy interest rate, it will further decrease the probability of success in the physical investment and exacerbates deviation of financial stability from its target induced by the shock to  $A$ . That is, in facing this shock to  $A$ , the CB is able to eliminate the deviations of expected inflation while worsening the financial stability stance

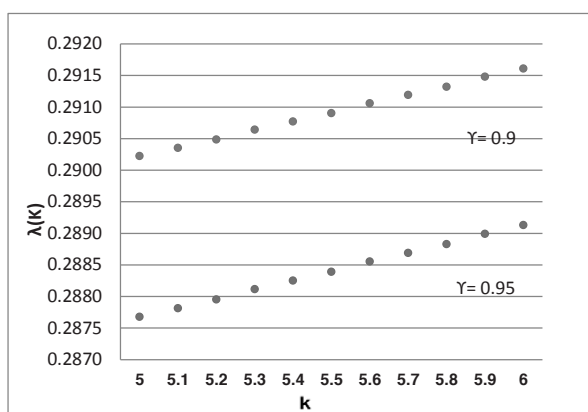
<sup>31</sup> In Figure 4 we find the *negative* impact of the policy rate on the deposit rate, alongside with the *positive* impact of the policy rate on the loan rate.



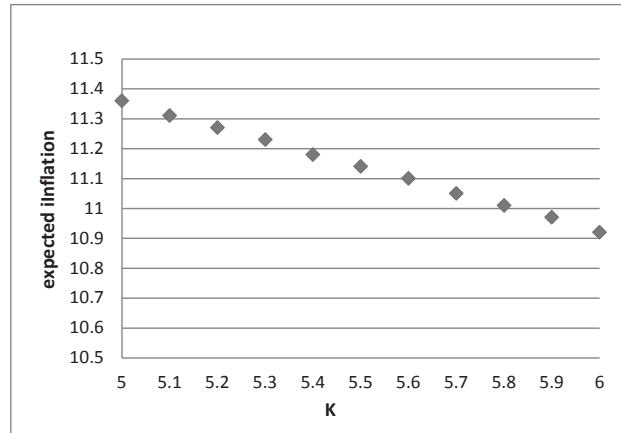


**Figure 5. The effect of the policy interest rate on  $\lambda(K)$  in different values of A**

Suppose now that following the aforementioned shock to  $A$ , the CB reacts by decreasing the capital ratio  $\kappa$  (Figure 7) rather than by using its policy interest rate in order to get the inflation expectations back to its target. According to Figures 6 and 7, decreasing  $\kappa$  will lower  $\lambda(K_t)$  while increasing expected inflation. Thus creating a policy *tradeoff* in this case as well. In fact the tools of monetary policy and financial stability may complement each other in dealing with deviations from both targets provided they are used in conjunction.



**Figure 6. The effect of  $\kappa$  on  $\lambda(K)$  in different values of  $\gamma$**



**Figure 7. The effect of  $\kappa$  on the expected inflation**

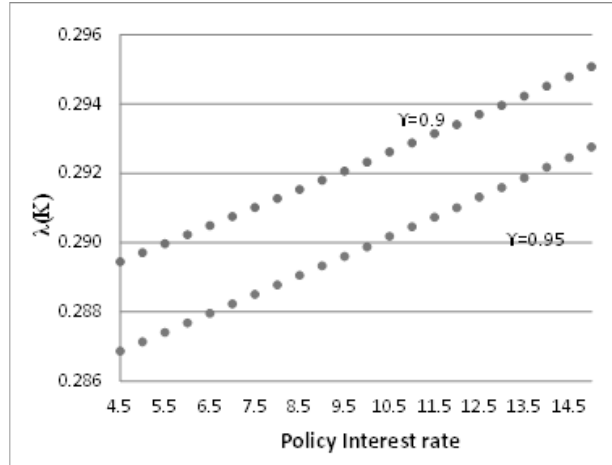
### 5.2. Shock to the probability of success of the physical investment

Next we study the impact of a permanent negative shock to the probability of success of the physical investment,  $\lambda(K_{it})$ , i.e. a permanent increase of  $\gamma$  in equation (2a), which can be considered a shock to financial stability<sup>32</sup>. As explained before, this shock is realized after the households and the commercial banks have already made their decisions of the current period. Their *ex-ante* perceived  $\lambda$  in period  $t$  is unaffected. The CB however reacts to this shock only in period  $t+1$  following the realization of  $\lambda$ .

In order to examine the impact of a permanent increase in  $\gamma$  we turn to the simulations in which an increase in  $\gamma$  reduces  $\lambda(K_{it})$  (Figure 6), while holding the policy interest rate and  $\kappa$  constant. This shock causes financial stability  $\lambda(K_{it})$  to fall below its target. *Expected* inflation at  $t$  however is unaffected and remains at its target since the *ex-ante*  $\lambda$  is unaffected. From the relationships displayed in Figures (6) and (8) it can be seen that the CB can either increase  $\kappa$  (Figure (6)) or increase its policy

<sup>32</sup> Note that when evaluated at the equilibrium  $\frac{\partial \lambda(K_t)}{\partial \gamma} < 0$ , because  $\gamma K_t > 1$ .

interest rate (Figure (8)) to reduce the deviation of financial stability indicator  $\lambda(K_t)$  from its target.



**Figure 8. The effect of the policy interest rate on  $\lambda(K_t)$  in different values of  $\gamma$**

Suppose then that the CB increases  $\kappa$  and restores financial stability. As can be seen in Figure 7, expected inflation will fall below its target. We get a similar result (in term of the deviation of the expected inflation from its target) in the case in which the CB decides to raise its policy interest rate rather than increase  $\kappa$  to increase  $\lambda(K_t)$  (see Figure 8).

Hence, similar to the impact of the shock to  $A$ , a policy tradeoff will emerge here too. A CB that is able to restore financial stability would not be able to simultaneously maintain price stability without taking in addition active policy measures aimed explicitly at this target. These results emphasize the need to coordinate price stability and financial stability policies.

## 6. Conclusions

In this paper we develop an OLG model to analyze the interactions between monetary policy designed to achieve inflation target and macro-prudential policy targeted to safeguard financial stability.

The model identifies how and in which circumstances the tools of monetary policy can be used to achieve financial stability and vice versa. For example, when the CB interest rate reaches the zero bound, it can affect banking interest rates by adjusting capital requirements, which under normal circumstances is an instrument for regulating financial stability. This has important policy implications, since it gives an additional degree of freedom to central bank in its policy considerations.

Our results indicate that in a banking economy, following either a shock to expected inflation or to financial stability, the CB faces policy tradeoff when it uses either the policy interest rate or the capital requirement but not both to mitigate or eliminate the effects of the shock. Thus monetary policy aimed at restoring price stability should consider as well its effect on financial stability and vice versa. These effects must be taken into account by the CB when selecting the tools in its policy toolbox.

Additional findings which contribute to the literature and have important policy implications include:

i) Monetary transmission mechanism from the CB policy rate to the bank lending and deposit rates is affected by financial stability policy tools (required reserves and capital ratios). Financial stability policy, such as a change in capital adequacy requirements may affect the effectiveness of monetary policy.

(ii) The collateral required from banks borrowing from the CB affects the outcome and effectiveness of monetary policy. It may in some cases completely offset or even reverse the impact of monetary policy on its targets.

(iii) Leverage constraints adopted voluntarily by banks or imposed by regulators within the framework of risk management, will mitigate and may even negate the

impact of monetary policy on bank loans, deposits and the banking interest rates and could impair the management of inflationary expectations.

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### Appendix A: *Households' budget constraints*

The households' budget constraints of young and old individuals in periods  $t$  and  $t+1$  are specified for depositors, denoted  $j=1$  (low endowment "poor" individuals), and for investors,  $j=2$  (high endowment "rich" individuals), as follows.

i) For the young individuals:

$$c_t + \frac{d_t}{p_t} = w_1, \text{ ("poor")}, \quad (\text{A.1})$$

where  $d_t$  are the bank deposits of the representative poor young and  $p_t$  is the price of the consumption good in period  $t$ .

$$c_t + k_t - \frac{l_t}{p_t} = w_2, \text{ ("rich")}. \quad (\text{A.2})$$

where  $l_t$  are the bank loans of the representative rich young.

Next we derive the old individuals' budget constraints. The state conditional budget constraints for the old individuals are as follows:

ii) For old individuals of type  $j=1$ , who have deposited with the bank and the latter has not failed (i.e. with probability  $1-q_{t+1}$ ) we have

$$c_{t+1} = \delta_{t+1} \frac{\Pi_{t+1}}{Np_{t+1}} I + \frac{(1+i_{dt})}{p_{t+1}} d_t \quad (\text{A.3})$$

where  $\Pi_{t+1}$  are bank profits of period  $t+1$ ,  $\delta_{t+1}$  is the dividend payout share of bank profits,  $I$  denotes the number of banks and  $N$  denotes the number of individuals in each generation.

(iii) For old individuals of type  $j=1$ , who have deposits with the bank and the latter has failed (i.e. with probability  $q_{t+1}$ ) we have



$$c_{t+1} = \delta \frac{\Pi_{t+1}}{Np_{t+1}} I + \theta \frac{(1+i_{dt})}{p_{t+1}} d_t \quad (\text{A.4})$$

where  $\theta$  is the fraction of the insured deposits. We assume that in making their choices at period  $t$ , individuals base their decisions on  $\theta$  as well as on the perceived  $\lambda$  and  $q_{t+1}$ .

(iv) For old individuals of type  $j=2$ , who successfully get the return on their real investment  $k_t$  (with probability  $\lambda$ ), we have

$$c_{t+1} = f(k_t) + \delta \frac{\Pi_{t+1}}{Np_{t+1}} I - \frac{(1+i_{Lt})}{p_{t+1}} l_t. \quad (\text{A.5})$$

(v) The old individuals of type  $j=2$ , who get zero in return for his/her real investment, and thus default on their loans (with probability  $1-\lambda$ ), we have

$$c_{t+1} = \delta \frac{\Pi_{t+1}}{Np_{t+1}} I. \quad (\text{A.6})$$

Note that there is no collateral which the commercial bank can recover ex-post in this case.

Next we pre-multiply equations (A:3)-(A:6) by their respective perceived probabilities and then consolidate constraints (A:3) with (A:4), and (A:5) with (A:6) to get the two period  $t+1$  state unconditional expected budget constraints of the old individuals

$$E_t c_{t+1} = \delta \frac{\Pi_{t+1}}{Np_{t+1}} I + (1 - (1-\theta)q_{t+1}) \frac{(1+i_{dt})}{p_{t+1}/p_t} \frac{d_t}{p_t} \quad (\text{for depositors}), \quad (\text{A.7})$$

$$E_t c_{t+1} = \delta \frac{\Pi_{t+1}}{Np_{t+1}} I + \lambda \left( f(k_t) - \frac{(1+i_{Lt})}{p_{t+1}/p_t} \frac{l_t}{p_t} \right) \quad (\text{for investors}). \quad (\text{A.8})$$

Note that the perceived probability  $q_{t+1}$  is known and is determined at equilibrium<sup>33</sup>.

To simplify the notation let  $R_{dt}$  be defined by  $R_{dt} \equiv (1 - (1 - \theta)q_{t+1}) \frac{1 + i_{dt}}{p_{t+1} / p_t}$

and  $R_{Lt}$  be defined by  $R_{Lt} \equiv \lambda \frac{1 + i_{Lt}}{p_{t+1} / p_t}$ . We can rewrite (A:7) and (A:8) as follows

$$E_t c_{t+1} = \delta \frac{\Pi_{t+1}}{N p_{t+1}} I + E_t R_{dt} \frac{d_t}{p_t} \quad (\text{for depositors}), \quad (\text{A.9})$$

$$E_t c_{t+1} = \delta \frac{\Pi_{t+1}}{N p_{t+1}} I + \lambda f(k_t) - E_t R_{Lt} \frac{l_t}{p_t} \quad (\text{for investors}). \quad (\text{A.10})$$

Note that  $p_{t+1}$  in (3c) and (3d) is the expected price of the consumption good, given the perceived values of  $\lambda_t$  and  $q_{t+1}$ . For both types of individuals the ex-post period  $t+1$ 's consumption may deviate from the expected consumption due to the difference between the perceived and actual values of  $\lambda$ .

### Appendix B: Expected profit functions

The *conditional* expected period  $t+1$  bank's profits when the realization of the systemic risk has not occurred (with probability  $1 - q_{t+1}$ ) is given by

$$E_t \Pi_{t+1}^{1-q} = \lambda i_{Lt} L_t^i - i_{dt} D_t^i - i_{mt} l_{mt}. \quad (\text{B.1})$$

The *conditional* expected period  $t+1$  bank's profits/losses when the realization of the systemic risk does occur (with probability  $q_{t+1}$ ) and the bank becomes insolvent is given by<sup>34</sup>

$$E_t \Pi_{t+1}^q = \lambda i_{Lt} L_t^i - i_{dt} \theta D_t^i - i_{mt} l_{mt}. \quad (\text{B.2})$$

<sup>33</sup> See equation (8a) for the determination of  $q_{t+1}$ .

<sup>34</sup> We assume here that even when there is a realization of systemic risk and the deposit insurance has been activated, still due to the "haircut" of the depositor return the commercial bank may realize positive profits, which are then distributed to the shareholders.

The term  $i_{dt}\theta D_t$  in (5b) is part of the obligations of the CB to depositors when deposit insurance is activated.

### Appendix C: Semi-Log linear consumer's preferences

In order to analytically trace better some of the equilibrium characteristics in our model economy, we assume in this appendix that the consumers' preferences are semi-log linear that is, take the form

$$u(c_t, c_{t+1}) = \log c_t + \beta c_{t+1} \quad (\text{C.1})$$

In this case the MRS is  $\frac{1}{\beta c_t}$ . Individuals with these preferences are indeed *risk neutral*. For simplicity and without loss of generality we further assume that the number of banks is one,  $I=1$ .

Next we utilize (3a) and (11a) to get the individual's supply of bank deposits

$$\frac{d_t}{p_t} = w_1 - \frac{1}{\beta_1 R_{dt}}, \quad (\text{C.2})$$

We solve for the investor's demand for the capital stock, we get from the definitions of the functions  $f(k_t)$  and (10b)

$$k_t = (A\alpha)^{\frac{1}{1-\alpha}} \frac{1}{\left(\frac{1+i_{Lt}}{p_{t+1}/p_t}\right)^{\frac{1}{1-\alpha}}}, \quad \text{all } t. \quad (\text{C.3})$$

With this solution we use (2) to solve for the return on the physical investment

$$f(k_t) = A(A\alpha)^{\frac{\alpha}{1-\alpha}} \frac{1}{\left(\frac{1+i_{Lt}}{p_{t+1}/p_t}\right)^{\frac{\alpha}{1-\alpha}}}, \quad \text{all } t. \quad (\text{C.4})$$

We can now use (3b), (10a) and (10b) to get the investor's demand for bank loans as follows

$$\frac{l_t}{p_t} = \begin{cases} -w_2 + \frac{1}{\beta_2 R_{L_t}} + k_t & \text{if } \varphi_{L_t} = 0 \\ \tau w_2 & \text{otherwise} \end{cases} \quad (\text{C.5})$$

And finally equating the  $R_{L_{t+1}}$  to the expected marginal productivity of capital, as is required from (10b) yields

$$\lambda \alpha \frac{f(k_t)}{k_t} = R_{L_t} \quad (\text{C.6})$$

#### Appendix D: Proofs of Lemmas and Propositions

**Lemma 1:** *If constraint (4b) binds, then a necessary condition for the inflation expectations to be negatively affected by the CB monetary rate  $i_{mt}$ , is that the expected return on deposits,  $(1 - (1 - \theta)q_{t+1})i_{dt}$ , decreases when the CB increases  $i_{mt}$ .*

##### Proof of Lemma 1.

Suppose constraint (4b) binds and that inflation expectations are negatively affected by the CB  $i_{mt}$ . By way of contradiction suppose that the CB increases  $i_{mt}$  and thereby  $(1 - (1 - \theta)q_{t+1})i_{dt}$  increases as well. Using (13c) and (13d) to eliminate  $\varphi_3$  we get

$$\lambda \left(1 + \frac{1}{\eta}\right) i_{L_t} = r r i_{mt} + (1 - (1 - \theta)q_{t+1})i_{dt} + \kappa(1 + r r) + (1 - \lambda). \quad (\text{D.1})$$

It is apparent from (D:1) that if the CB increases  $i_{mt}$  and by supposition the expected *nominal* return on deposits  $(1 - (1 - \theta)q_{t+1})i_{dt}$  increases as well, then the *nominal* lending rate  $i_{L_t}$ , will follow suit and increases too.

By (4b) and (4e) and since  $FK_t$  is predetermined, we have that in equilibrium extending more banking credit must be accompanied by raising more public deposits

and vice versa. Therefore to support these changes in the quantity demanded and supplied in equilibrium, the *real* lending rate and the expected *real* deposit rate must be changed in opposite directions. It means that when the CB increases  $i_{mt}$  the inflation expectations must go up to make one of the real rates to fall, which contradicts the supposition.

Q.E.D

**Lemma 3.** *In an equilibrium with  $D_t > 0$ , the following inequality must hold*

$$\lambda i_{L_t} - (1 - (1 - \theta)q_{t+1})i_{dt} > 0. \quad (D.2)$$

**Proof of Lemma 3.**

Assume inequality (3) is satisfied and in addition that there exists an equilibrium with  $D_t > 0$ . Since constraint (4b) is binding we have that  $l_{mt} = rrD_t$ . From the commercial bank resource constraint we then have  $L_t = D_t + FK_t$ . Substituting this for  $L_t$  in the expected  $t+1$  profit of the commercial bank (5) yields

$$E_t \Pi_{t+1} = (\lambda i_{L_t} - (1 - (1 - \theta)q_{t+1})i_{dt})D_t - rri_{mt}D_t + \lambda i_{L_t}FK_t \quad (D.3).$$

Since  $FK_t$  is predetermined, it is immediate from (D.3) that unless (D:2) holds commercial banks will not accept the individuals' deposits which contradicts the supposition  $D_t > 0$ , which completes the proof.

**Proposition 1.** Given in our economy that inequality (3) is satisfied and if the probabilities  $\lambda$  and  $1 - (1 - \theta)q_{t+1}$  are such that the following inequality  $R_{L_t} > R_{dt}$  holds, and if there exists an equilibrium for our economy with  $D_t > 0$ , then it is a separating equilibrium where individuals of type 1 deposit funds in the banks and do not invest in the physical capital, while the individuals of type 2 invest in the physical capital and do not in the banks' deposits.

**Proof of Proposition 1.** Assume inequality (3) is satisfied and further that  $R_{L_t} > R_{d_t}$ .

Assume in addition that there exists an equilibrium with  $D_t > 0$ . By assumption we have that

$$\lambda \frac{1+i_{L_t}}{p_{t+1}/p_t} > (1-(1-\theta)q_{t+1}) \frac{1+i_{d_t}}{p_{t+1}/p_t}. \quad (\text{D.4})$$

For the rest of the proof we rely on the first-order conditions of individual of type  $j$

pertaining to the choice of  $\frac{d_t}{p_t}, \frac{l_t}{p_t}, k_t$ , respectively:

$$-\Xi_{l_t} + \Xi_{2t+1}(1-(1-\theta)q_{t+1}) \frac{1+i_{d_t}}{p_{t+1}/p_t} \leq 0, \quad (\text{D.5})$$

where  $\Xi_{l_t}, \Xi_{2t+1}$  are the Lagrange multipliers of period  $t$  budget constraint when young and period  $t+1$  budget constraint when old, respectively.

$$\Xi_{l_t} - \Xi_{2t+1} \lambda \frac{1+i_{L_t}}{p_{t+1}/p_t} \leq 0, \quad (\text{D.6})$$

$$-\Xi_{l_t} + \Xi_{2t+1} \lambda f_k \leq 0. \quad (\text{D.7})$$

Consider an individual who deposits at the bank,  $d_t > 0$ . Given her choice (D:5) is satisfied with strict equality. From (D:4) and (D:5) we have that (D:6) is strictly negative, which implies that the individual can do better off by reducing her choice of  $l_t$  to zero. Since by construction inequality (3) is also satisfied, individual of type 1 who deposits at the bank hasn't got enough endowment to invest in  $k_t$ .

Finally, for individual of type 2, who invests in  $k_t$  and borrows  $l_t$ , condition (D:6) is satisfied with strict equality. Therefore by (D:4) condition (D:5) is strictly negative, which implies that the individual can do better off by reducing  $d_t$  to zero, which completes the proof.

Q.E.D

**Proposition 2.** There is no partial use of the monetary loans the CB extends to the commercial banks. Either there is no use at all of these loans or the bank exhausts all of its collaterals and  $l_{mt} = rrD_t$ .

**Proof of Proposition 2.** Let the expected real dividends paid out by the banks to the representative old consumer be denoted by  $\Phi_{t+1} \equiv \delta \frac{E_t \Pi_{t+1}}{N_t p_{t+1}} I$  and without loss of generality we assume  $I=1$  (there is one representative commercial bank). Note that we can combine the expression for the expected profits (5c) and the resource constraint for the bank (4e) to get the following expression for the expected dividends payment

$$\begin{aligned} \Phi_{t+1} = & \delta_t \frac{p_t}{p_{t+1}} \left\{ [\lambda i_{L_t} - (1 - (1 - \theta)q_{t+1}) \frac{i_{dt}}{1 - rr}] \frac{l_t}{p_t} \right. \\ & + [(1 - (1 - \theta)q_{t+1}) \frac{i_{dt}}{1 - rr} - i_{mt}] \frac{l_{mt}}{N p_t} \\ & \left. + (1 - (1 - \theta)q_{t+1}) \frac{i_{dt}}{1 - rr} \frac{FK_t}{N p_t} \right\} \end{aligned} \quad (D.8)$$

where the optimal  $\delta_t$  is chosen to have constraint (4a) satisfied with equality. There are two expressions in the RHS of (D:8) that appear in squared brackets. The first one can be analyzed using equilibrium condition (13d) and the second one can be analyzed using condition (13c). For now we deal only with the latter, that is, substituting from (13c) into (D:2) to yield

$$\begin{aligned} \Phi_{t+1} = & \delta_t \frac{p_t}{p_{t+1}} \left\{ [\lambda i_{L_t} - (1 - (1 - \theta)q_{t+1}) \frac{i_{dt}}{1 - rr}] \frac{l_t}{p_t} \right. \\ & + [-\kappa \frac{rr}{1 - rr} + \frac{\varphi_{3t}}{1 - rr}] \frac{l_{mt}}{N p_t} \\ & \left. + (1 - (1 - \theta)q_{t+1}) \frac{i_{dt}}{1 - rr} \frac{FK_t}{N p_t} \right\} \end{aligned} \quad (D.9)$$

From which we get Result 2 regarding the commercial bank use of the CB monetary loans.

From the second line in (D:9) it appears that either  $l_{mt}=0$  (there is no use of the monetary loans) or constraint (4b) is binding,  $\varphi_{3t} > 0$ . Suppose to the contrary that the commercial bank maximizes its profits, constraint (4b) is not binding  $\varphi_{3t} = 0$  and  $l_{mt}>0$ . Recalling that the market for bank deposits is competitive (i.e.  $i_{dt+1}$  is given for the commercial bank) and since  $\frac{\kappa r r}{1 - r r} > 0$ , we get from the second line of (D:3) that the combination of lowering  $l_{mt}$  simultaneously with increasing  $d_t$  such that  $l_t$  remains unchanged will increase the expected profit of the bank, which constitutes a contradiction. This completes the proof.

Q.E.D

**Proposition 3.** If the maximum leverage constraint is binding at the equilibrium banking interest rates for all individuals, then the monetary policy shocks (changes in  $i_m$ ) have no effects on  $L_t, D_t, k_t$ , and no effects on  $i_{L_t}$  and therefore have no effects on expected inflation.

**Proof of Proposition 3.** If the maximum leverage constraint is binding at all interest rates and for all individuals and given that only individuals of type 2 use bank's loans then the loans outstanding real balances will be constant at the amount of  $\tau w_2$  per individual (see (4g)). By the bank's resource constraint (14a) the derived amount of deposits will also be constant. Therefore the bank which maximizes the dividends payout will set its lending rate,  $i_{L_t}$ , as high as possible so long as the leverage constraint is binding, irrespective of the monetary policy rate,  $i_{mt}$ . As for the choice of  $k_t$  by individuals of type 2 and the equilibrium expected inflation rate,  $\frac{p_{t+1}}{p_t}$ , they are



exclusively and simultaneously solved from the set of equations (11b) and (C:4) in appendix C, as function of  $i_{Lt}$  which is given. This completes the proof.

Q.E.D