# Seven-branes and instantons in type IIB supergravity

Hartong, Jelle

# Conclusions and discussion

This thesis aimed at providing a better understanding of the full set of one-half BPS branes that are present in the type IIB supergravity theory. The one-half BPS objects were listed in table 2.2.1 on page 40. Of all the branes contained in that table the $(p', q')$ $p$-branes with $p'$ and $q'$ relatively prime integers are well-established objects of the theory for which a microscopic interpretation is given in string theory as Dirichlet branes on which a $(p', q')$ string is ending. A $(p', q')$ string is an $SL(2, \mathbb{Z})$ transformation of the fundamental $(1, 0)$ string and so these branes can be considered to be the Dirichlet branes of the $SL(2, \mathbb{Z})$ transformed IIB string theory. Also coincident sets of identical $(p', q')$ $p$-branes can be understood using string theory. The location of a $(1, 0)$ p-brane in moduli space is at the point $\tau_0 = i\infty$ where the string coupling $g_s$ is small. The $(p', q')$ $p$-branes are then located at $SL(2, \mathbb{Z})$ transformed images of $\tau_0 = i\infty$, that are rational points on the real line $\mathrm{Im}\,\tau = 0$ where $g_s$ goes to infinity.

Table 2.2.1 also shows that there can be more than only $(p', q')$ branes. For 7-branes it was found to be possible to refer to them as $(p, q, r)$ 7-branes where the numbers $(p, q, r)$ parameterize the monodromy of $\tau$ when going around the 7-brane. The $(p', q')$ 7-branes as well as coincident identical $(p', q')$ 7-branes fall into the class of $(p, q, r)$ 7-branes for which $r^2 = 4pq$ (or $\det Q = 0$) with $p = p'^2$ and $q = q'^2$. What the analysis of chapter 3 has shown is that in type IIB supergravity one can introduce the notion of a Q7-brane, i.e. a 7-brane located at one of the orbifold points $\tau_0 = i, \rho$ (or any of their $SL(2, \mathbb{Z})$ transformed images) of the quantum moduli space $SO(2)\backslash PSL(2, \mathbb{R})/PSL(2, \mathbb{Z})$ for which $r^2 > 4pq$ (or $\det Q > 0$) and where $g_s$ is of order unity.

The properties of the Q7-branes such as their monodromy and mass have a natural interpretation in terms of coincident F-theory 7-branes that have monodromies such as $T$, $ST^{-2}$ and $T^2ST^{-4}$, i.e. coinciding $(p', q')$ 7-branes with different values for $p'$ and $q'$. The $T^{-1}S$ Q7-brane is formed by taking two mutually non-local F-theory 7-branes coincident whereas the case of an $S$ Q7-brane is formed by taking three F-theory 7-branes coincident of which two must be mutually non-local (so that two are identical). It was suggested that the gauge group of a $T^{-1}S$ Q7-brane is $(U(1))^2$ and that the gauge group of an $S$ Q7-brane is $SU(2) \times (U(1))^2$.

If the relative positions of the F-theory 7-branes that make up a Q7-brane are kept fixed so that they remain coincident and only the fluctuations associated to the center of mass motion are considered then the Q7-brane behaves effectively as a single brane that couples to an 8-form. By electro-magnetic duality it can be argued that there should exist a Q-instanton, i.e. a Q(-1)-brane that couples magnetically to the same 8-form to which a Q7-brane couples electrically. Indeed it has been shown in chapter 4 that such instantons exist.

The path integral approach to the Q-instantons shows the existence of new vacua and a new superselection parameter $\chi'_\infty$. Further, it was argued that the Q-instantons contribute to the $\mathcal{R}^4$ terms of the string effective action near the points $\tau_0 = i, \rho$ of the quantum moduli space $SO(2)\backslash PSL(2,\mathbb{R})/PSL(2,\mathbb{Z})$. The expansion of the generalized Eisenstein series around the points $\tau_0 = i, \rho$ contains terms that do not depend on $\chi'$ and for which an interpretation is yet to be found.

Further, it was mentioned in section 2.3 that there should exist such a thing as a Q3-brane that lives near the orbifold points $\tau_0 = i, \rho$. To make the list of possible Q-branes complete it was argued in section 3.12 that there may exist an open Q-string ending on a Q7-brane. The tension of a Q-string was suggested to be proportional to $(T^2 - 4\det Q)^{1/4}$ a quantity that also showed up in the Q-instanton contributions to the $\mathcal{R}^4$ terms around the points $\tau_0 = i, \rho$.

Even though the string coupling $g_s$ is only small near $\tau_0 = i\infty$ it is believed that the results on the Q7-branes and Q-instantons allows one to consider IIB supergravity (perhaps in a restricted sense) as providing a valid field theory approximation of some underlying quantum theory near each of the orbifold points of the quantum axidilaton moduli space of figure 3.9.1 (see page 81). In fact for each orbifold point of the moduli space one can define a tension that goes to zero at that particular orbifold point. For $\tau_0 = i\infty$ this is $g_s$ and for $SL(2,\mathbb{Z})$ transformations of $\tau_0 = i\infty$ the tension is the $SL(2,\mathbb{Z})$ transformed version of $g_s$ while for $\tau_0 = i, \rho$ (and their $SL(2,\mathbb{Z})$ transformed images) the tension is suggested to be proportional to $(T^2 - 4\det Q)^{1/4}$. In all cases the tensions have a geometrical interpretation as measuring the geodesic distance of a geodesic that goes through the point $\tau_0$. The geodesic along which $g_s$ measures the geodesic distance is the line $\chi = \text{cst}$, with $\chi$ the RR axion, that goes through the point $\tau_0 = i\infty$ and similarly for the $SL(2,\mathbb{Z})$ transformed images of $\tau_0 = i\infty$. The geodesic along which $(T^2 - 4\det Q)^{1/4}$ measures the geodesic distance is the line $\chi' = \text{cst}$ (or $\varphi = \text{cst}$, see figure 1.4.1 on page 33) that goes through the points $\tau_0 = i, \rho$ (or through one of their $SL(2,\mathbb{Z})$ transformed images).

Finally, for the case of the doublet of 10-form potentials there are some open questions as to what their status in the theory is. It was argued in section 2.5 that there could be an effective notion of an O9-plane present in the combined system of 9-branes that couple to the quadruplet and the doublet of 10-forms. In the case of the 7-branes something similar has been shown to be true. The O7-plane, figure 3.11.1 on page 96, can be understood as a special limit of different coinciding F-theory 7-branes.