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Consent or Conflict: Coevolution of Coordination and Networks*

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Societies are sometimes divided into groups that behave in different ways or have strongly opposing opinions. At other times, everyone seems to behave according to similar principles and opinions. It is likely that individual decisions on behavior or opinions are affected by social networks through influence and selection processes. However, the outcomes are not necessarily optimal for the society as a whole. Two types of problems might arise: (a) polarization of the society into two camps that do not reach consensus, possibly leading to conflict; (b) actors choosing suboptimal behavior, because changing behavior is too risky if done unilaterally. Simulations show that if a society is rather segregated initially, there exists a heightened probability that this situation will worsen. The effect of network density is twofold. First, density has a positive effect on reaching a uniform opinion and, therefore, decreases the likelihood of polarization. Second, density increases the likelihood that actors do not change their behavior, worsening the inefficiency of already suboptimal initial situations.

Introduction

A long research tradition in sociology and social psychology has shown that social networks play an important mediating role in the diffusion of behaviors and opinions through a society. In many different contexts, people are influenced by those with whom they interact (Erickson, 1988; Marsden & Friedkin, 1993; Merton, 1968). Empirical examples of such processes include peer pressure among adolescents (Davies & Kandel, 1981), diffusion of innovations (Coleman, Katz & Menzel, 1957; Valente, 2005), rebellion, and collective action (Gould, 1991, 1993; Opp & Gern, 1993). These findings are relevant for the study of polarization, described as the social or ideological separation of a society into two or more groups (see also Esteban & Schneider, 2008), because the adaptation process might increase agreement within groups, while it deepens disagreement between groups. The extent to which a society will polarize into possibly opposing camps is likely to be influenced by the patterns of

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social relations through which members of the society influence each other, and through which opinions, behaviors, and ideologies diffuse.

It is important to realize that social networks are not always static but can be altered by actors consciously selecting their relations. At least in part, this selection process is based on behavioral traits of others; sociological research shows that people tend to choose their friends among those who behave and think in a similar way to themselves (a process known as ‘homophily’; see Lazarsfeld & Merton, 1954; McPherson, Smith-Lovin & Cook, 2001; Zeggelink, Stokman & van de Bunt, 1996). The combination of network influence and selective network-formation processes implies that, first, polarization may occur because behavior or opinions cluster locally within networks and, second, a society may segregate socially because people with different behaviors or opinions tend to avoid each other.

Our study of polarization takes into account that social and ideological alienation and behavioral alienation between groups develop interdependently. In other words, the degree to which polarization on a behavioral trait occurs depends on patterns of social ties, but this social structure itself is also influenced by behavioral choices. We aim at a theoretical understanding of the interplay between polarization of behavior and social structure. We develop a model in which actors are involved in interactions with others with whom they have social relations, while this social network is itself subject to change by the actors. This model predicts how the likelihood of polarization of behavioral outcomes depends on the social structure of a society at the time that actors have to decide on a certain form of collective action or have to develop an opinion on some issue that becomes salient. In addition to problems in which persistent disagreement constitutes a clear potential for conflict, the model captures other types of processes from which conventions emerge, such as lifestyle choices of pupils in school classes. In such networks, persistent disagreement is not problematic per se.

**Polarization, Conflict and Coordination**

Group mobilization and group formation have previously been modeled in different ways, for example, as social influence processes (Axelrod, 1997), as multiperson Prisoner’s Dilemmas (Takács, 2001), or as collective action problems (Gould, 1999). Identification with a group can also be considered a multiperson coordination problem, in the sense that belonging to a group and making the same choice as others are more important than what choice is actually made (Hardin, 1995). In group identification, one prefers to join a group if others do the same, because benefits can be expected from group membership itself. There is little sense in speaking English if everyone else speaks French; similarly, it may not be beneficial to identify as Serbian if everyone else identifies as Yugoslavian. However, if enough people start to call themselves Serbs, it becomes attractive to join this group.

It can be argued that not only identification with a group, but also group action are mainly matters of coordination. Usually, group mobilization and other forms of collective action are seen as free-rider problems. According to the ‘logic of collective action’ (Olson, 1971), individual group members should not be expected to contribute to collective efforts unless they have individual (selective) incentives that compensate their efforts. Hence, collective action should not occur in most cases because every individual has reasons to free-ride on the other group members. This led some scholars to problems. According to the ‘logic of collective action’ (Olson, 1971), individual group members should not be expected to contribute to collective efforts unless they have individual (selective) incentives that compensate their efforts. Hence, collective action should not occur in most cases because every individual has reasons to free-ride on the other group members. This prediction seems at odds with the real-world observation that group action does occur in many instances, from voting to mass demonstrations and collective violence. This led some scholars to
argue that coordination rather than cooperation is the basic strategic interaction that underlies group action. According to Hardin (1995), the power resulting from mass action can diminish the costs of joining to a level that is sufficiently low to reduce the free-rider problem to a problem of coordination (see Heckathorn, 1996; Macy, 1991; Marwell & Oliver, 1993). Others (Chwe, 2001; Gould, 1995; Klandermans, 1988; Lohmann, 1994) have also pointed at the importance of coordination in collective phenomena such as rebellion, uprisings, and union participation. Therefore, by modeling collective action as a coordination problem, we abstract from free-riding problems and focus on settings in which actors have an incentive to join if enough others do so as well.

Modeling group identification and group mobilization as a coordination problem is not to say that actors are indifferent between behavioral alternatives as long as they coordinate with others. The coordination game that is the backbone of our model accommodates for ranking of behavioral alternatives, while coordination with others still has priority. Consider the simple coordination game as displayed in Figure 1, with \( c = 8 \). This game has two Nash-equilibria, \((X, X)\) and \((Y, Y)\), in each of which actors do not wish to deviate as long as the other actor does not deviate. However, \((Y, Y)\) yields higher payoffs for both actors and is termed efficient or payoff-dominant. The other equilibrium \((X, X)\) is attractive in the sense that it is less risky: if an actor assumes that the other actor chooses \( X \) with equal probabilities, the expected payoff of choosing \( X \) is higher than that of choosing \( Y \). Therefore, \((X, X)\) is risk-dominant (Harsanyi & Selten, 1988). For our applications, choosing \( Y \) may represent joining an uprising in order to accomplish a regime change, while choosing \( X \) is to stick to the status quo. Joining the uprising is risky if you are not sure that others will also do so.

A consequence of choosing a multiperson version of the coordination game as described above implies that we can provide predictions not only on the likelihood and extent of polarization, but also on the extent to which actors coordinate on the efficient equilibrium. However, the model does not predict how the likelihood of the emergence of violent conflict depends on the polarization that might arise in a population. Rather, the theory assumes that polarization into separated but internally coordinated groups provides potential for violent group conflict, while the model specifies the conditions under which a polarized situation is more or less likely to occur.

As a measure of polarization, we use the two-group version of the index for qualitative variation IQV (Mueller & Schuessler, 1961: 177–179; see also Agresti & Agresti, 1978), which is defined as \( 4 \ p(Y)(1 - p(Y)) \), where \( p(Y) \) is the proportion of actors in the population choosing \( Y \). The measure implies that polarization is 0 if \( p(Y) = 0 \) or \( p(Y) = 1 \), while it equals its maximal value 1 for \( p(Y) = 0.5 \). This measure is the standardized version of a much older version of a diversity measure that dates back to Gini (see Lieberson, 1969, for an overview). We focus on polarization as defined above because, with only two groups, we cannot distinguish it from, for instance, fractionalization (see Montalvo & Reynal-Querol, 2005; Reynal-Querol, 2002).

The maximum of our polarization measure is reached if both groups are of equal size, which corresponds with the maximum for conflict potential according to Esteban & Ray (1999) for the general polarization measure (Esteban & Ray, 1994; see also Esteban & Schneider, 2008).

Coordination and Social Networks

Earlier theoretical studies of coordination in large groups consider models in which actors interact with all other actors in the population, without assuming any social structure (e.g. Kandori, Mailath & Rob, 1993; Young, 1993). However, this assumption seems highly
unrealistic for many applications. Actors can often observe the behavior only of those with whom they interact directly; they observe only their own personal network. This is especially true for cases in which no public information is available about the distribution of behavior in the larger population, and so actors really have to rely on their close surroundings for information. They may even use the behavior of the members of their personal network as an approximation of the behavior in the larger population. Opp & Gern (1993) and Lohmann (1994), for example, discuss the importance of personal networks for the ‘Monday Demonstrations’ in Leipzig, 1989, which is a typical case in which public information was highly restricted. Gould (1991, 1993, 1995), Scott (1990), Hardin (1995), and Chwe (2001) similarly emphasize the role of networks in instances of (violent) collective action.

Theoretical models dealing with local interaction have been formulated by (among others) Ellison (1993), Young (1998), and Berninghaus, Ehrhart & Keser (2002), although these authors do not consider the possible heterogeneity of the network structure. Buskens & Snijders (2005) explicitly deal with coordination in heterogeneous social network structures, where heterogeneity refers to actors having different positions in the network, that is, they do not necessarily have the same number of relations. Considering static networks only, Buskens & Snijders (2005) show that denser and less segmented networks reach consensus more easily than less dense and more segmented networks. In addition, segmentation leads to more actors choosing the risky option, while density results in fewer actors choosing the risky option. Thus, segmentation renders less efficiency in the emerging behavior, while density has the opposite effect.

However, networks are created by people and change over time (Flap, 2004). Therefore, we consider a model in which actors are organized in a dynamic social network, obtaining ‘high’ payoffs for relations with actors who behave in the same way and ‘low’ payoffs for relations with actors who behave differently. Assuming that maintaining social relations is costly, relations have to be chosen cautiously, which implies that relations with actors who behave differently may be terminated.

Recently, the study of dynamic networks with strategic decision-making of actors in the network has developed quickly (for an overview, see Dutta & Jackson, 2003; Newman, Barabási & Watts, 2006). Specific models for coordination games played on dynamic networks are studied by Skyrms & Pemantle (2000), Jackson & Watts (2002), Goyal & Vega-Redondo (2005), and Berninghaus & Vogt (2006). These models focus on which networks are stable and on how behavior is distributed in stable networks. Typically, many configurations are possible. Therefore, we take a different approach and focus on how preconditions determine the emergence of a stable network and the related distribution of behavior. Hence, we
study the extent to which polarization in segregated groups emerges and inefficient behavior persists as a result of the initial network and the initial distribution of behavior. More specifically, we aim to answer the following research question: How do the polarization and efficiency of the emerging distribution of behavior depend on the initial distribution of behavior, the initial network, the tie costs, and the payoffs in the coordination games?

In the next section, we present the dynamic model and analytic results on stable states. Subsequently, we describe simulations and derive predictions on the effects of initial conditions on polarization and efficiency. In the final section, we conclude and interpret the results in terms of the probability that conflicts may emerge.

The Model

Actors are organized in a network of $n$ actors with the $n \times n$ adjacency matrix $N = (n_{ij})$, that is, $n_{ij} = 1$ if two actors are connected and $n_{ij} = 0$ otherwise. We assume that relationships are undirected, so $n_{ij} = n_{ji}$. Relationships have both benefits and costs. Actors have to choose between two types of behavior (or opinions, attitudes), and their benefits or payoffs depend on their own behavior and the behaviors of the actors with whom they have relationships. They cannot differentiate their behavior depending on specific relationships. In every existing relationship, the payoff is related to the actor’s own behavior and the behavior of the other person in correspondence with the coordination game depicted in Figure 1. From actors with whom actor $i$ does not have a relationship, $i$ obtains a payoff 0. In our multi-person coordination game, this implies that the payoff of an actor $i$ choosing $X$ equals $\sum_{j(X)} n_{ij} a + \sum_{j(Y)} n_{ij} c$, where $j(Z)$ runs over other actors who choose $Z$, $Z = X, Y$. If actor $i$ chose $Y$, the payoff would be $\sum_{j(X)} n_{ij} b + \sum_{j(Y)} n_{ij} d$.

We study two variants. The first variant assumes that $(a - b) < (d - c)$, so that choosing $Y$ not only can lead to the efficient payoff $d$, but also is less risky, because the expected payoff from playing $Y$ is relatively high if you do not know what others will do. The second variant assumes $(a - b) > (d - c)$, so that choosing $Y$ is the risky choice because the expected payoff from playing $Y$ is relatively low if you do not know what others will do.

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In addition, we assume that ties are costly. We abstract from separate costs for the creation or the deletion of ties. Thus, the costs of existing ties have to be paid in every period of interaction, but ties can be deleted without any cost. We assume increasing marginal tie costs, and so the more ties one has, the more ‘effort’ is required for another tie. Increasing marginal tie costs can also be interpreted as diminishing marginal returns of relationships; an equivalent model assumes constant tie costs and benefits that decrease with an actor’s number of interactions. The total costs for an actor of having $t$ ties are $k(t) = \alpha t + \beta t^2$, where $\alpha > 0$ and $\beta \geq 0$. If $\beta = 0$, the tie costs are linear in the number of ties, and there are no increasing marginal costs of having more ties. Otherwise, there is a

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2 In terms of the two-person game, this would be the condition that $(Y, Y)$ is not only the payoff-dominant equilibrium, but also the risk-dominant equilibrium (in the sense of Harsanyi & Selten, 1988).
maximum number of network ties one can maintain, given the payoffs one can obtain related to relationships. In one period of interactions, the total payoff of an actor equals \( \sum_{i \neq j} n_{ij} p_{ij} - k(n_+,) \), where \( p_{ij} \) is the payoff \( i \) receives as a result of his own and \( j \)'s behavior and \( n_+ = \sum_{i \neq j} n_{ij} \) is the number of ties of actor \( i \).\(^3\) Finally, we define how the network is changed. Actors can add and sever ties. Because of their undirected nature, ties can be created only with the consent of both partners, but can be removed unilaterally. In other words, we assume a two-sided link-formation process (Jackson & Wolinsky, 1996). We also assume that actors have full information on the behavior of all actors in the network.

Stable States

In line with related works (Berninghaus & Vogt, 2006; Goyal & Vega-Redondo, 2005; Jackson & Watts, 2002), we search for stable networks first, that is, networks in which

- no actor wants to change behavior,
- no actor wants to sever a tie, and
- no pair of actors wants to add a tie given the actors’ behavior in the network.

This corresponds to pairwise stability (Jackson & Wolinsky, 1996) in networks in which only ties can be changed. The condition on behavior is added to guarantee stability in terms of the behavior chosen by the actors. In order to characterize stable networks, two definitions are helpful.

Definition A (sub)network is \( t \)-full if and only if no actor has more than \( t \) ties and either the addition of a tie causes that at least one of the actors has more than \( t \) ties or no ties can be added to the (sub)network.

Definition For \( Z = X, Y, \bar{t}_z \) is the maximum number of ties an actor wants to have if he chooses \( Z \) and all actors with whom he has a relation choose \( Z \) as well.

These definitions are useful because the number of relations an actor wants to maintain is determined by the tie costs and the payoffs an actor can obtain. More specifically, a new relation with someone with whom an actor \( i \) can earn \( p_{ij} \) is started only if the number of ties actor \( i \) has will be less than or equal to \( (p_{ij} - \alpha + \beta)/2\beta \) after adding this new relation. Otherwise, the marginal costs of adding this new tie are larger than its benefits. This implies that

\[
\bar{T}_X = \left\lfloor \frac{a - \alpha + \beta}{2\beta} \right\rfloor \quad \text{and} \\
\bar{T}_Y = \left\lfloor \frac{d - \alpha + \beta}{2\beta} \right\rfloor.
\]

Theorem If tie costs are equal to \( k(t) = \alpha t + \beta t^2 \) (\( \alpha > b \) and \( \beta > 0 \)), networks are stable if and only if one of these conditions holds:

1. All actors choose the same behavior \( Z \), where \( Z = X, Y \), and the network is \( \bar{t}_z \)-full.
2. The network consists of two subgroups of actors choosing \( X \) and \( Y \) and these subgroups are \( \bar{t}_X \)-full and \( \bar{t}_Y \)-full, respectively, and there are no ties between the two subgroups.

Proof As tie costs are always larger than \( b \), a tie between actors with different behavior is never sustainable, because one of the actors wants to sever the tie or change behavior. It is easily checked that the definition of \( \bar{t}_z \)-full implies that no pair of actors wants to add a tie and no actor wants to sever a tie. Because

\(^3\) Although our cost function is provided in a very explicit form, the only crucial aspect is the number of ties an actor wants to have depending on whether he obtains mostly \( a \) or \( d \) in his relations. The possible number of ties is varied in the simulation over the complete relevant range.
actors are not connected with actors who behave differently, they do not want to change behavior. All other networks are unstable, because in (sub)networks that are neither $\tilde{t}_Y$-full nor $\tilde{t}_X$-full, some actors want to remove or add ties. This completes the proof.

The theorem is a reformulation of the corresponding theorem in Berninghaus & Vogt (2006) for costs that are non-linear in the number of ties and for the conditions we want to consider. We extend the results of Berninghaus & Vogt, not only by allowing for non-linear costs but also by studying the likelihood of the emergence of different structures depending on initial conditions; see the simulation section below. Jackson & Watts (2002) study coordination and endogenous formation of networks. They analyze which networks are stochastically stable in a specific dynamic context, showing analytically that homogeneous networks emerge in which all the actors coordinate on one behavior (cf. Young, 1998). Which behavior is chosen depends on the tie costs. In their analysis, conflict situations are excluded as possible long-term outcomes, because they are less stable than non-conflict situations. In contrast with their study, we do not include ‘trembles’, but analyze how the likelihood of emerging structures in a deterministic dynamic environment depends on initial conditions. In a deterministic environment, networks with polarized behavior can be stable. This makes it possible to address the likelihood of conflict. Goyal & Vega-Redondo (2005) analyze a similar model but assume a one-sided link-formation cost. More importantly, they characterize only stable states without analyzing the likelihood that certain stable networks emerge.

The condition in the theorem that (sub)networks should be $\tilde{t}_X$-full or $\tilde{t}_Y$-full suggests that the payoff and cost structure does not allow much variation in network structure. This is true in the sense that the density (proportion of ties present in the network) of the emerging network hardly varies with the cost function, the payoffs, and the distribution of behavior in the emerging network. Some variation is still possible. Let us consider as an example a nine-actor network in which everybody wants to have two ties. If behavior is homogeneous, both three closed triads and one circle of nine actors are stable networks. Another type of variation is related to the possibility that one actor may still have fewer ties than he wants to have but all other actors in his group have their maximum number of ties (compare Jackson & Watts, 2002: 182, for more details on variations in network structures). The simulation results presented below indeed confirm that the density of the emerging network is almost perfectly determined by the emerging distribution of behavior, the payoffs in the game, and the tie costs. Other network measures turn out to be hard to predict from the initial network structure. Therefore, the analyses below focus on predicting how the emerging distribution of behavior depends on initial conditions.

Simulation Design

To analyze the effects of the model parameters on the emergence of stable networks by means of computer simulation, we systematically vary the initial conditions of the dynamic process and relate the outcomes of the process to these conditions. The conditions include the initial network, the initial distribution of behavior, the payoff structure of the coordination game, the tie costs, and the adaptation rules in the dynamic process. Network size ranges from 2 to 50 actors. For networks of 2 to 8 actors, we include all 13,597 possible non-isomorphic networks. For networks of 9 to 50, the number of possible networks becomes extremely large, and we take a sample stratified on the size (number of actors) and density of the network. In other words, we draw a set of random
networks, while density and size are fixed such that for each size there are about the same number of networks. Also, for each density within each size, a similar number of networks are drawn. Extreme densities for which the number of non-isomorphic networks is small are excluded. This results in a set of 95,729 networks. The probability of each actor initially choosing 'Y' equals 0.25, 0.50, or 0.75. This results in a wide range of distributions of actual initial behavior. For reasons mentioned above, we vary only payoff $c$ (see Figure 1) such that $RISK$ takes the values 0.467 ($c = 4$) and 0.538 ($c = 8$), fixing the other payoffs at $b = 0$, $a = 14$, and $d = 20$. With regard to the tie costs, we vary both the linear and quadratic components such that all $0 \leq i_X \leq i_Y \leq n - 1$ are possible. Linear cost $\alpha$ is chosen as an integer number from 1 through 16 (excluding $c$, and 14 to avoid equalities with payoffs), each with an equal probability. For $\alpha < 14$, $\beta$ is chosen such that all values of $\bar{i}_X (0 < i_X \leq n - 1)$ are equally likely by choosing $\beta = (14 - \alpha)/(1 + r(n - 1))$ where $r \sim U[0,2]$. If $\alpha > 14$, we have $\bar{i}_X = 0$. In these situations, $\beta$ is chosen such that all values of $\bar{i}_Y (0 < i_Y \leq n - 1)$ are equally likely by choosing $\beta = (20 - \alpha)/(1 + r(n - 1))$ and again $r \sim U[0,2]$.

The dynamic model assumes discrete time. All actors simultaneously choose behavior in each period. We distinguish three methods for how actors change behavior and ties between periods. All methods assume actors to be myopic, that is, optimizing under the restriction that the behavior and network of the previous period persists. It is also assumed that all actors know the behavior of all other actors. The three methods are different in the relative adjustment rates of behavior and ties. Since these adjustment rates can be expected to affect outcomes (Skyrms & Pemantle, 2000), we compare three approaches:

1. 'Actor-based': Actors decide themselves which type of change they prefer. After every period, a random actor is allowed to change either behavior or one network tie, whatever is most beneficial. We assume the actor will choose the alternative with the highest payoff to him, given the network and behavior of other actors in the preceding period. If multiple tie changes yield the same maximal benefit, one is selected at random.

2. 'Alternating': After every period, the actors in a random dyad decide whether they want to add or remove a tie between them. In addition, one random actor decides whether or not he wants to change behavior. This mechanism is similar to the one applied by Jackson & Watts (2002).

3. 'Fast network': Now the network is allowed to change relatively fast compared to behavior. In comparison to the 'alternating' version, not one but $[n/2]$ dyads are considered to change their tie after every period, and, next, one random actor is allowed to change behavior.

In any of the three methods, actors are informed about previous changes before they have to decide. If actors are indifferent between a change and the existing situation, we assume that actors do not change. We also postulate that actors do not change ties if

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4 Using this procedure, we obtain satisfactory amounts of variation in the variables that we want to use as independent variables to explain polarization and efficiency. Selectivity on the independent variables should, in principle, not affect the estimations of the regression models we use below. Nevertheless, we did some checks to ensure that the sampling procedure does not affect the substantive conclusions, and this does not seem to be the case.

5 A natural extension of the model is to relax this assumption and assume that actors know only behavior of actors they are connected to, or that are within a given distance in the network. First analyses of such extensions (to be reported elsewhere), however, suggest that although assuming limited information in itself has interesting effects, the conclusions as reached in the current study are not undermined.
both actors are indifferent between having and not having a tie. One additional assumption is needed here to handle actors who are not connected to anyone at all. These actors cannot adapt their behavior to any connected actor; moreover, in some cases, they are not able to connect to anyone else, regardless of the behavior they choose, because the other actors already have the optimal number of ties. In such cases, the unconnected actor changes behavior only if that might create opportunities to become connected in the next period. Otherwise, the actor does not change behavior. Clearly, other rules for changing behavior and relations can be conceived. We have chosen some of the most straightforward options on who might change what and when. However, further investigation into how the dynamics depend on these options is called for only if outcomes differ dramatically between them. As we will see below, this is not the case.  

For each of the 13,597 + 95,729 = 109,326 networks and each of the three versions of the dynamics, we varied the initial conditions in the following way. The simulation was done four times with different values of the quadratic cost component for each version of the dynamic process and for 13,597 networks of size up to 8. We distinguished only two levels of the quadratic cost component for the 95,729 larger networks. One random choice was made to select the other conditions. Then, each of the initial conditions was repeated four times. These repetitions with the same set of initial conditions enable us to distinguish between stochastic variation in the outcomes that is related to variations in initial conditions and randomness caused by the dynamic process. At each repetition, we let the process continue until it converges. This leads to 13,597 × 48 + 95,729 × 24 = 2,950,152 simulation runs.

To analyze the effects of the initial network and to evaluate the emerging networks, we need to define and compute some key network characteristics:

- **Size** represents the number of actors.
- **Density** is the number of existing ties divided by size × (size − 1)/2, size − 1 the possible number of ties in the network (Wasserman & Faust, 1994: 101).
- **Degree** is the number of ties of an actor divided by size − 1. This ‘relative’ definition differs slightly from the more common ‘absolute’ definition (Wasserman & Faust, 1994: 100), but facilitates comparison across network size.
- **Centralization** is measured through the standard deviation of the degrees as defined above. This measure is derived directly from the variance in degrees as proposed by Snijders (1981; see also Wasserman & Faust, 1994: 101). Other centralization measures (Freeman, 1979) lead to similar results in the analyses that we present below.
- **Distance** is a dyadic measure, the minimum path length between two actors (Wasserman & Faust, 1994: 110).
- **Segmentation** is the proportion of dyads at distance larger or equal to 3 of all dyads at distance larger or equal to 2 (Baerveldt & Snijders, 1994). In accordance with Baerveldt & Snijders, we count disconnected dyads as having a distance larger than 3. In the complete network, there is clearly no segmentation, which implies that this measure is equal to 0.
- **Segregation** measures the extent to which ties are limited to actors with the same behavior, rather than between actors with different behavior. It is defined as the expected number of between-group ties minus the observed number of between-group ties, divided by the expected number of between-group ties.
of between-group ties, assuming random matching (Freeman, 1978). This is the only measure that combines behavior and network structure. Because there are no ties between actors with the same behavior in the emerging networks, this measure always equals 1 for the emerging network.

- **Number of components** is the number of connected subgraphs (including isolates) that are not connected to the rest of the network. If the number of ties actors can have is low, owing to the high tie costs, groups of actors that behave in the same way might still fall apart into different components.

Size, density, and centralization are included because these represent the most basic network measures: the number of actors, the number of ties in the network, and the variation of ties between actors. The measures for segregation, segmentation, and components are included because they are particularly relevant for group-formation processes in general and polarization in particular. While polarization indicates the extent to which actors are divided into subgroups due to their behavior, segmentation and components indicate the extent to which actors are divided into subgroups due to the network structure. Segregation is a measure of the extent to which behavioral and structural groupings coincide. The other measures are described only to define the network measures that we need in the analyses below. We also record the distribution of behavior in the initial and the emerging networks, the number of tie changes until convergence, and the number of behavior changes until convergence.

Some parameter values among the initial conditions are less interesting to analyze and are difficult to compare with the large set of cases. These are the following subsets:

- Cases for which $\alpha > 14$. If $\alpha > 14$, actors who choose $X$ cannot maintain any ties.

As a result, in over 80% of the cases, all actors choose $Y$. Actors will stick to $X$ only if no one chooses $Y$ or all $Y$-choosing actors do not want to add ties.

- Cases that start in a situation in which everyone chooses $X$ or everyone chooses $Y$. Behavior does not change in these cases, and only the network ties are adapted until the network is $\bar{t}_X$-full or $\bar{t}_Y$-full.

Excluding these cases, 1,544,100 cases remain for the analyses. Summary statistics for the initial conditions are shown in Table I.

**Simulation Results**

In this section, we explain properties of the stable networks in terms of polarization and efficiency by the model parameters, using statistical regression analysis. Table II presents summary statistics for the stable states. One result is that the relative number of behavioral changes (number of changes per actor) is considerably lower than the relative number of network changes (number of changes per dyad). This can be understood by recognizing that changing one’s behavior has much more impact than changing one tie: in the case of a behavioral change, the payoffs resulting from all interactions are affected, while changing a tie affects only one relation. Therefore, an actor mostly does not change behavior more than once. Usually actors need to adapt multiple relationships to optimize their situation.

The type of network dynamics does not have a large influence on the stable networks. In both ‘alternating’ and ‘fast network’ dynamics, there are somewhat more changes in behavior and ties than in ‘actor-based’ dynamics. Surprisingly, the number of tie changes is only marginally larger in ‘fast network’ dynamics than in ‘alternating’ dynamics, although there are many more opportunities to change ties in the former. Because of the limited differences between
the types of dynamics, we provide only joint summary statistics in Table II.

Polarization is our first important dependent variable. Recall that polarization is defined as $4p(Y)(1-p(Y))$, where $p(Y)$ is the proportion of actors choosing $Y$. In more than 60% of the stable states, behavior is homogeneous and, thus, without polarization. Efficiency, as expressed by the proportion of actors choosing the $Y$-behavior, is a little above 50%, which is only slightly higher than the average efficiency in initial networks (see Table I). The standard deviation of efficiency in stable states is rather large, reflecting the large proportion of homogeneous stable states. These percentages are not easily interpreted, because they depend to a large extent on our choice of initial conditions. Therefore, we focus below on how polarization and efficiency change with initial conditions.

Predicting Stable States I: Polarization

To examine how polarization depends on the parameters of the simulation, we use linear regression models, with polarization as the dependent variable and the initial conditions (the network structure, the initial distribution of behavior, the tie costs, and the dynamics version) as independent variables. It is important to realize that standard errors reflect uncertainties due to the randomness of the dynamic process and misspecification of the model, not ‘sampling’. In addition, with more than a million cases, even very small effects are significant. Therefore, we restrict ourselves to comparisons of relative sizes of standard errors between variables to provide information on the relative stability of the effects, and we do not report significance levels. Standard errors are adapted for clustering within initial conditions (Rogers, 1993).

Polarization in stable states has an extremely skewed distribution, with over 60% of the stable networks showing homogeneous behavior where the value of polarization is 0. Our statistical model has to take this unusual distribution into account. We decide to model polarization with two separate analyses. First, we estimate a model predicting whether stable states are heterogeneous or not, using logistic regression. Then, we apply a linear regression model to predict the extent of polarization in the cases with polarization. The tie costs are included in the analysis as the difference in the number of ties an actor can have while choosing $Y$ as compared to the number of ties he can have while choosing $X$ divided by size. The effect of tie costs is small, and only the quadratic cost component turns out to be relevant for predicting behavior in the analysis. This component can be adequately summarized by $(\bar{t}_Y - \bar{t}_X)/\text{size}$. We add dummies for the different types of dynamics.

Because the outcomes of the analyses strongly depend on initial polarization, we
ran separate analyses for different categories of initial polarization. These analyses suggested adding interactions of initial polarization with density and segregation to our model.\textsuperscript{7} To facilitate the interpretation of the main effects, density is centered at the mean before taking the interaction. Initial polarization and segregation already have a mean equal to 0.

In Table III, we report the results separately for high and low RISK, although the models do not differ substantially between RISK values, to facilitate comparison with the results on efficiency reported below. The initial distribution of behavior, operationalized as the polarization at the start of the process, has a large positive effect on polarization in stable networks. There is a slightly higher probability of persisting polarization for the ‘alternating’ version of the dynamics compared to the ‘actor-based’, which is the reference category. The probability of persisting polarization is highest for the ‘fast network’ version, which is understandable, since many network changes result in a network falling apart in groups with different behaviors before actors have an opportunity to adapt behavior. Substantively, this leads to the plausible interpretation that in societies in which relations are more volatile while people tend to stick more strongly to their behaviors or opinions, it is more likely that persistent disagreement arises that may induce conflicts. The difference between the number of ties one wants to have choosing Y compared to choosing X has a negative effect on the probability of any polarization as well as the extent of polarization. The probability of persistent polarization and the extent of polarization decrease for larger networks.

Considering the network effects, density has a large negative effect on the probability of any polarization as well as on the extent of polarization (given positive polarization). Segregation has a small positive effect on the probability of any emerging polarization for average initial polarization and a small negative effect on the extent of emerging polarization. Segmentation increases the likelihood of persistent polarization as well as the extent of polarization. Centralization promotes the probability of homogeneity, although if there is some polarization, polarization will be larger if initial centralization is higher.

As a result of the interaction of polarization and density, the effect of density is stronger for higher initial polarization. This implies that density really helps to solve the polarization problem, and if some disagreement persists, at most a small minority will stick to the ‘deviant’ behavior, and will be excluded by the majority. Although the main effect of segregation is small, the interaction effect with initial polarization is substantial. It shows that segregation enhances polarization if the

\begin{table}
\centering
\caption{Summary Statistics of Stable States ($N=1,544,100$)}
\begin{tabular}{lllll}
\hline
Variable & Mean & SD & Min & Max \\
\hline
Behavior changes per actor & 0.250 & 0.161 & 0 & 4.897 \\
Tie changes per dyad & 0.492 & 0.191 & 0 & 2.333 \\
Proportion actors playing Y & 0.518 & 0.423 & 0 & 1 \\
Polarization of behavior & 0.271 & 0.387 & 0 & 1 \\
Density & 0.591 & 0.295 & 0.02 & 1 \\
Segmentation & 0.379 & 0.430 & 0 & 1 \\
Centralization & 0.158 & 0.185 & 0 & 0.873 \\
Number of components & 1.664 & 1.712 & 1 & 25 \\
\hline
\end{tabular}
\end{table}

\textsuperscript{7} Similar series of analysis were done (e.g. for separate values of network size), but these did not provide strong evidence for the necessity to add interaction effects.
initial network is rather polarized. Thus, if differences in behavior are aligned with initial group boundaries, the situation most likely worsens as a result of network dynamics.

**Predicting Stable States II: Efficiency**

Efficiency has a somewhat peculiar distribution. In more than 60% of the stable states, behavior is homogeneous (efficiency is 0 or 1), while the remaining cases are more or less evenly distributed between the two extremes, such that the total distribution is U-shaped. An appropriate way to analyze such a dependent variable is logistic regression for grouped data, which predicts the number of successes (i.e. the number of actors choosing \( Y \)), given the number of actors in the network.

As the sizes of some network effects depend strongly on \( RISK \), we conduct the analyses separately for low \( RISK \) and high \( RISK \) (Table IV).

The coefficients refer to the log-odds of the predicted proportion. Clearly, the initial proportion \( Y \) determines, to a large extent, the emerging distribution of behavior. If the group starts with a majority of actors choosing \( Y \), there is a large probability that an even larger majority or the whole group will ultimately choose \( Y \). The other effects have to be interpreted in terms of the extent to which they affect this baseline dynamic. The ‘alternating’ and ‘fast network’ dynamics have higher rates of efficiency as compared to ‘actor-based’ dynamics for low \( RISK \), while

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### Table III. Logistic and Linear Regression of Behavioral Polarization on Simulation Parameters with Standard Errors Adapted for Clustering within Initial Condition

<table>
<thead>
<tr>
<th>Variable in initial condition</th>
<th>( RISK = 0.467, \ c = 4 )</th>
<th>( RISK = 0.538, \ c = 8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Probability of any polarization</td>
<td>Extent of polarization</td>
</tr>
<tr>
<td>Polarization</td>
<td>5.739 (0.025)</td>
<td>0.820 (0.005)</td>
</tr>
<tr>
<td>Version ‘alternating’</td>
<td>0.094 (0.008)</td>
<td>0.023 (0.001)</td>
</tr>
<tr>
<td>Version ‘fast network’</td>
<td>0.601 (0.009)</td>
<td>0.023 (0.001)</td>
</tr>
<tr>
<td>( \bar{Y} - \bar{x} )/size</td>
<td>-0.383 (0.020)</td>
<td>-0.013 (0.002)</td>
</tr>
<tr>
<td>Size</td>
<td>-0.016 (0.000)</td>
<td>-0.005 (0.000)</td>
</tr>
<tr>
<td>Density</td>
<td>-4.166 (0.051)</td>
<td>-0.003 (0.007)</td>
</tr>
<tr>
<td>Centralization</td>
<td>-1.150 (0.041)</td>
<td>0.182 (0.005)</td>
</tr>
<tr>
<td>Segregation</td>
<td>0.301 (0.028)</td>
<td>-0.083 (0.004)</td>
</tr>
<tr>
<td>Segmentation</td>
<td>0.636 (0.024)</td>
<td>0.047 (0.003)</td>
</tr>
<tr>
<td>Number of components</td>
<td>0.134 (0.003)</td>
<td>0.004 (0.000)</td>
</tr>
<tr>
<td>Polarization × Density</td>
<td>-0.402 (0.113)</td>
<td>-0.516 (0.014)</td>
</tr>
<tr>
<td>Polarization × Segregation</td>
<td>3.468 (0.091)</td>
<td>0.315 (0.012)</td>
</tr>
<tr>
<td>Constant</td>
<td>-3.096 (0.033)</td>
<td>0.037 (0.005)</td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>-360,165</td>
<td>0.283 0.394</td>
</tr>
<tr>
<td>(Pseudo) ( R^2 )</td>
<td>0.283 0.394</td>
<td>0.305 0.425</td>
</tr>
<tr>
<td>Number of observations</td>
<td>767,946 277,578</td>
<td>776,154 282,468</td>
</tr>
</tbody>
</table>
this is the other way around for high $RISK$. This can be interpreted in the following way. Changes in behavior mostly have the largest impact. Thus, if actors behave differently from most actors they interact with, these actors start changing behavior and thereafter they optimize their ties. As we know from models with static networks, adaptation of behavior leads to attraction to the risk-dominant equilibrium. In the ‘alternating’ and ‘fast network’ versions of the dynamics, changing ties often has to be done before behavioral changes. This apparently decreases the attraction to the risk-dominant equilibrium. Except for the situation in which no ties were possible between actors choosing $X$, which are excluded from the analysis, tie costs have only small effects.

Not only do the network effects vary greatly with $RISK$, but some also vary with the initial distribution of behavior. Therefore, we consider again interaction effects between the initial distribution of behavior and other variables. Especially the density and centralization effects depend considerably on initial behavior. In Table IV, interactions of the centered variables were included. In both models, strong and positive interaction effects exist between density and the initial distribution of behavior, such that the effect of density is negative for low initial proportions and positive for high initial proportions. For centralization and initial behavior, the interaction effect points in the opposite direction. The effects of density and centralization at their means are in different directions, and they also switch if one compares high and low $RISK$. At the means, density promotes efficiency for low $RISK$, but hampers efficiency for high $RISK$. In contrast, centralization hampers efficiency under low $RISK$, but promotes efficiency under high $RISK$. It is important to realize that the effects at the mean are relatively small compared to the interaction effects. Segregation has a negative effect on efficiency under low $RISK$, but a positive effect under high $RISK$. This result indicates that segregation, to some extent, is able to stabilize the more risky equilibrium. Segmentation has a negative

### Table IV. Logistic Regression for Grouped Data on the Proportion of Actors Choosing $Y$ with Standard Errors Adapted for Clustering within Initial Condition

<table>
<thead>
<tr>
<th>Variable in initial condition</th>
<th>$RISK = 0.467, ; c = 4$</th>
<th>$RISK = 0.538, ; c = 8$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coeff.</td>
<td>Std. err.</td>
</tr>
<tr>
<td>Proportion $Y$</td>
<td>16.728</td>
<td>(0.033)</td>
</tr>
<tr>
<td>Version ‘alternating’</td>
<td>0.129</td>
<td>(0.006)</td>
</tr>
<tr>
<td>Version ‘fast network’</td>
<td>0.107</td>
<td>(0.006)</td>
</tr>
<tr>
<td>$(\bar{y} - \bar{z})/size$</td>
<td>0.189</td>
<td>(0.014)</td>
</tr>
<tr>
<td>Size</td>
<td>-0.004</td>
<td>(0.000)</td>
</tr>
<tr>
<td>Density</td>
<td>1.479</td>
<td>(0.019)</td>
</tr>
<tr>
<td>Centralization</td>
<td>-0.413</td>
<td>(0.032)</td>
</tr>
<tr>
<td>Segregation</td>
<td>-0.281</td>
<td>(0.009)</td>
</tr>
<tr>
<td>Segmentation</td>
<td>-0.058</td>
<td>(0.013)</td>
</tr>
<tr>
<td>Number of components</td>
<td>-0.004</td>
<td>(0.000)</td>
</tr>
<tr>
<td>Prop. $Y \times$ Density</td>
<td>31.080</td>
<td>(0.120)</td>
</tr>
<tr>
<td>Prop. $Y \times$ Centralization</td>
<td>-23.000</td>
<td>(0.143)</td>
</tr>
<tr>
<td>Constant</td>
<td>-8.144</td>
<td>(0.026)</td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>-5,304,285</td>
<td></td>
</tr>
<tr>
<td>Number of observations</td>
<td>767,946</td>
<td></td>
</tr>
</tbody>
</table>
effect for low and high RISK. Network size has only a small negative effect on efficiency.

To sum up, the strongest effect of the initial network structure on efficiency is that density galvanizes initial behavioral tendencies. If we start at a rather high level of efficiency, higher density leads to the emergence of even more efficient networks. However, if initial behavior is inefficient, the situation is likely to worsen with dense networks. Other network effects have a modest size. In the low-risk situation, segregation prevents the efficient behavior for diffusion through the whole network and therefore has a negative effect on total efficiency. However, segregation helps to maintain efficient behavior in parts of the network in the high-risk situation. On average, density decreases efficiency in the high-risk situation, which is surprising because density increased efficiency in high-risk situations for static networks (see Buskens & Snijders, 2005). Centralization, in contrast to density, favors minority behavior in high-risk situations. An interpretation for this would be that if this minority has a central position in the network, they can induce the majority to change their behavior.

Conclusions and Discussion

We develop a model explaining opinion-formation or mass-mobilization processes as, for example, occurred during the 2004 elections in the Ukraine and the ‘Monday Demonstrations’ in Leipzig, 1989–91. The model formalizes the coevolution of coordination and networks to study under what conditions it is more or less likely that the emergence of stable states leads to inefficient situations or situations with considerable conflict potential. We assume that actors are organized in a specific network in which coordination problems emerge. Initially, all actors behave in a certain way (or have a certain opinion or attitude toward a given issue). Depending on their own behavior and the behavior of the actors with whom they have relations, actors change their behavior and their relationships. After the network has developed into a stable situation, we consider behavioral polarization and efficiency in the emerged stable situation.

It turns out that the initial network structure might affect the emerging distribution of behavior. The most important result with respect to polarization is that dense networks lead to more homogeneous behavior, while more segmented and segregated networks have the opposite effect. The latter effect becomes especially important if the initial behavior is already polarized. If polarized societies are indeed more prone to end up in conflicts, conflicts are less likely in dense cohesive societies, while conflicts are more likely in segregated and segmented societies, especially if the initial attitudes in sensitive issues are correlated with initial groups in social networks. The effect of centralization is multifold: centralization of the initial network increases the probability that all actors behave in the same way, but if this is not the case, centralization slightly promotes the extent of polarization.

The most salient finding on efficiency is that network density amplifies the effect of the initial distribution of behavior. The higher the density, the larger the effect of initial inefficient behavior on the inefficiency of the emerging network. In addition, if the initial behavior is equally distributed, density still increases the likelihood that the emerging behavior will be inefficient if the efficient behavior is risky. A similar effect is found for centralization, although smaller and in the opposite direction: centralization has a positive effect on efficiency if initial efficiency is low and a negative effect if initial efficiency is high. In addition, in larger, more segmented, and more segregated networks, behavior tends to become less efficient. These results are consistent with the fact that dictatorial states often survive for quite some
time without having to cope with major mass demonstrations (cf. Hardin, 1995). As soon as a status quo with no major opposition is reached, it is difficult to turn this situation around. The centralization result shows that the best opportunity to escape from such an inefficient situation should come from central people who can mobilize others to start a revolt.

Although this article provides an extension to existing models on network formation in coordination problems and provides more insight into the relation between initial conditions and the emerging networks, there are still a number of limitations. First, our model assumes extreme opinion-formation problems, in which people have a choice of only two opinions and, if they do not agree, a relationship would be very unattractive. In some of the examples mentioned, such as the ones where the choice is between standing up against the regime or remaining quiet, these assumptions are clearly more realistic than in less extreme situations. Second, while coordination problems represent the evolution of norms or opinions, many social interactions might lead to conflicts of a different character. Such situations can also be related to, say, trust, cooperation, or distribution problems (Heckathorn, 1996). Then, it becomes likely that actors differentiate behavior between their partners. People might trust some people and distrust others. Therefore, the study of the evolution of networks and the possible emergence of related social problems can be extended to other types of interactions in settings where behavior coevolves with networks. In addition, the theory developed has to be tested in experimental and real-world settings to corroborate the hypotheses developed in this article.

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