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### Persistent holes in the Universe

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university of  
 groningen

# **Persistent Holes in the Universe**

## **A hierarchical topology of the cosmic mass distribution**

PhD Thesis

to obtain the degree of PhD at the  
 University of Groningen  
 on the authority of the  
 Rector Magnificus prof. E. Sterken  
 and in accordance with  
 the decision by the College of Deans.

This thesis will be defended in public on  
 Friday 18 December 2015 at 14.30 hours

by

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 in Darbhanga, India

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*Then was not non-existence nor existence  
There was no realm of air, no sky beyond it  
What covered in, and where? And what gave shelter?  
Death was not then, nor was there aught immortal  
No sign was there, the day's and night's divider*

*That One Thing, breathless, breathed by its own nature  
Darkness there was at first  
Concealed in darkness this all was indiscriminated chaos  
All that existed then was void and formless  
By the great power of Warmth was born that Unit*

*There were begetters, there were mighty forces  
Free action here and energy up yonder  
Who verily knows and who can here declare it  
Whence it was born  
Whence comes this creation?*

Cover Illustration: An artistic impression of the filamentary network that pervades the large scale Universe. Superposed is the nearby Universe depicting the stars in the night sky.

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