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Finite-Time Output Parameter Estimation for a Class of Nonlinear Systems

Gianmario Rinaldi, Juan E. Machado, Michele Cucuzzella, *Member, IEEE*, Prathyush P. Menon, Antonella Ferrara, *Fellow, IEEE*, and Jacquelien M. A. Scherpen, *Fellow, IEEE*

***Abstract*—In this letter, a novel scheme is proposed to identify in finite time the value of an unknown output parameter for a class of nonlinear dynamical systems. Inspired by the Super-Twisting Sliding Mode Algorithm (STA), the identification problem is solved in an innovative way, consisting of two steps. An STA observer is firstly designed to track in finite time the system output. Exploiting the observer behaviour during the sliding motion, a second scheme, still inspired by the STA, is used to estimate the unknown value of the output parameter, thus solving the identification problem in finite time. The numerical simulations based on district heating systems validate the effectiveness of the proposed identification method.**

***Index Terms*—Observers for nonlinear systems, variable-structure/sliding-mode control, control applications.**

I. INTRODUCTION

THE IDENTIFICATION of model parameters and dynamical systems is a strategy to build mathematical representations to mimic, simulate and better understand real phenomena of interest [1]. Classically, parameter identification for nonlinear systems has been solved by using Extended Kalman Filtering techniques and Least-Square methods, which typically aim to asymptotically minimize (also via a solution of an optimization problem) the identification error [1], [2]. More recently, the so called Generalised Parameter Estimation-based Observer (GPEBO) method has been proposed in [3], where the task of state observation is framed (and solved) as an online parameter estimation problem. Sliding Mode (SM) methodologies have been proposed to stabilise in finite time dynamical systems, guaranteeing also a level of robustness against input disturbances and uncertainties [4]–[6]. SM techniques have also been successfully applied to identify not only unknown parameters, but also systems properties (such as the relative degree) [7]. For example, in [8], the identification of bounded constant coefficients has been executed via a two-steps approach: a Super-Twisting Sliding Mode Algorithm (STA) differentiator was proposed to estimate in finite time the state vector derivatives, and the coefficients were asymptotically identified via a least-squares estimation algorithm. Conventional least-squares estimation principles were also adopted, along with STA control laws, for the stabilisation of fuel cells systems [9]. The use of an higher order SM differentiator was also employed in [10], to asymptotically estimate model parameters with application to tank and automotive models. SM observers were also created for the identification of electrical drives. For example, in [11] a terminal SM observer was designed to estimate the inertia, the damping coefficient and the unknown load torque inputs. In [12], a conventional first order SM observer was made use to asymptotically determine the rotor resistance in induction motors. The importance of model parameter identification utilising SM principles has also been pointed out for single link robot arms [13] and for the Duffing oscillator [14].

Main Contribution: In this letter, motivated by the requirement to estimate a class of output parameters for district heating systems, we propose a novel strategy based on the STA [15] to estimate *in finite time* the output parameter for a class of Single Input Single Output (SISO) nonlinear systems. Differently from the existing literature, we develop a cascade estimation scheme including: *i*) an STA observer to force in finite time the equivalence of the output and its estimate; *ii*) an STA scheme that, by exploiting the STA observer behavior during the sliding motion, nullifies in finite time the so-called equivalent injection term [4], [15], thus estimating the true value of the unknown parameter. To the best of our knowledge, the proposed solution is innovative and exhibits relevant novelties when compared with the existing solutions proposed in the literature (see, e.g., [8], [9], [11]). In particular, the solution provided in this letter is easy to implement, and whilst classical solutions display an asymptotic convergence of the

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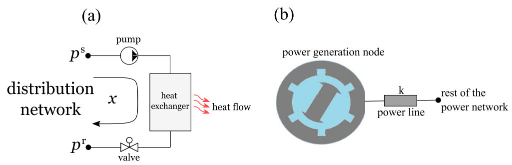

Fig. 1. **(a)**: Topology of an end-user (or heating substation) of a district heating system. **(b)**: Topology of a power system generation node.

estimate to the real value, our proposal is characterised by finite time convergence. In this letter, we provide a detailed simulation section to demonstrate the effectiveness of the proposed solution, also in presence of measurement noise.

Notation: For a given scalar signal, x, $\hat{x}$ denotes its estimate, whilst $\text{sign}(x)$ denotes the sign function. For a scalar function $f(x)$, $\nabla f(x)$ denotes its gradient, $\nabla_{x_i} f(x)$ represents the i-th component of the gradient of $f(x)$, whilst $\text{lnd}f(x) := \frac{\nabla f(x)}{f(x)}$ is the logarithmic derivative of the function $f(x)$. For a vector or a matrix x, the expression x^T denotes its transpose.

Structure of This Letter: The rest of the present letter is organised as follows. Section II introduces practical examples that justify the importance of estimating output parameters of dynamical nonlinear systems. In this section, we describe also the considered class of nonlinear systems, and a set of assumptions are imposed to undertake the stability analysis. Section III presents the design of the estimation scheme along with its stability analysis. Section IV describes a set of numerical simulations to validate the effectiveness of the proposed method, whilst Section V concludes this letter.

II. Motivational Examples and Problem Formulation

This section presents to the reader two distinct motivational examples to further support the importance of the identification of the output parameters for dynamical systems. The two chosen examples are the hydraulic model of end-users in district heating systems, and the single-area power system model, and they are depicted in Fig. 1. The problem formulation is also provided in this section.

A. District Heating

The district heating is a system for delivering heat from distributed heating stations towards clusters of consumers within a neighbourhood (or even a small city) using a network of underground pipelines [16]. Such a network has a supply layer (hot stream) and a return layer (cold stream): a heat consumer can continuously drain water from the supply layer, drain heat through a heat exchanger, and inject a colder stream into the distribution network's return layer; an analogous description follows for heat production stations. A schematic diagram of a consumer is shown in Fig. 1. A fundamental control task of a consumer is to regulate its stream's flow rate to ensure that heat extraction meets the demand and to maintain the temperature of the stream entering the return layer within bounds specified by the network operator [17]. This is a challenging task due to the nonlinear and uncertain nature of detailed hydraulic models for pipes, valves and heat exchangers (c.f., [18]–[21]) To make the above discussion precise, we introduce the following hydraulic model for a heat consumer [22] (c.f., [18]):

$$J\dot{x} = -k|x|x - k_2|x|x + u + \Delta p \tag{1a}$$

$$y = k|x|x \tag{1b}$$

where $x \in \mathbb{R}$ represents the flow rate of the stream through the consumer. The term $J\dot{x}$ denotes the stream's inertia and $k|x|x$ models the pressure drop due to friction in the heat exchanger. The term $k_2|x|x$ is the pressure drop through the control valve and u is the pressure difference produced by the hydraulic (control) pump between its terminals, which here we assume to be a control input. Also, $\Delta p = p_s - p_r$ denotes the pressure across the supply and return connection nodes. By selecting the output y we assume that the measurement of the pressure across the heat exchanger is available for control purposes. Considering (1a), the regulation of the flow rate x towards a setpoint $x^\star$ can be done through the regulation of y towards some constant value [18]. The uncertainty about the true value of k highlights the challenges for achieving accurate flow regulation indirectly through pressure control. Then, a scheme to simultaneously estimate k and x is desirable. Moreover, accurate estimations of x will also lead to more precise heat meter readings, implying better billing that would benefit both the district heating operator and the consumer [21]. Even though flow rate estimation through pressure measurements has been explored in the literature before (see, e.g., [23]), the use of techniques based on sliding modes has not been used, to the best of our knowledge, in this specific application.

B. Power System

A single-area power system can be described by the following equation [24]

$$\dot{x} = \Delta\omega(x, k, u, d) \tag{2a}$$

$$y = k \sin(x) \tag{2b}$$

where $x \in \mathbb{R}$ is the voltage phase angle, measured in (rad), $\Delta\omega(x, k, u, d) \in \mathbb{R}$ is the frequency deviation measured in (Hz) or (p.u.), and $y \in \mathbb{R}$ is the output measurement, which is the electrical active power exchange (measured in (p.u.) or (MW)) by the area with the rest of the grid. The positive parameter k is the reciprocal of the line inductance interconnecting the area with the rest of the system [25], and it is often unknown or uncertain [26], [27]. The knowledge of the transmission line parameter is required not only for solving the nonlinear power flow equations [24], but also to design and implement optimal control methods (see, e.g., [6], [28]).

C. Problem Formulation

The class of SISO systems in (1a)-(1b) and (2a)-(2b) can be written in a general form as

$$\dot{x} = g(x, u, k), \tag{3a}$$

$$y = kf(x), \tag{3b}$$

where $x := [x_1, \ldots, x_n]^T \in \mathcal{X} \subset \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ is the control input, $y \in \mathbb{R}$ is the output measurement, and $k \in \mathbb{R}$ is an unknown scalar parameter to be estimated. The function $f(x) \in \mathbb{R}$ and the vector values function $g(x, u, k) := [g_1(\cdot), \ldots, g_n(\cdot)]^T \in \mathbb{R}^n$ are assumed to

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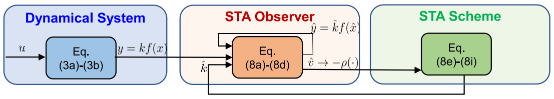

Fig. 2. A schematic of the approach adopted in this letter.

be known and sufficiently smooth. The two main objectives are:

Objective 1 (Parameter Estimation): Estimate **in finite time** the unknown parameter k in (3b).

Objective 2 (State Estimation): Estimate **in finite time** the state vector $x \in \mathbb{R}^n$ associated with the dynamics in (3a).

Following the principles of the SM theory [4], Objectives 1 and 2 are achieved in two stages:

i) An STA observer is designed to force the condition of equivalence of output (y) and its estimate ($\hat{y}$):
$$\hat{k}f(\hat{x}) = kf(x) \tag{4a}$$
in finite time, where $\hat{k}$ is an estimate of k, and $\hat{x}$ is an estimate of x;

ii) Another STA scheme regulates the estimate of the unknown parameter, i.e., $\hat{k}$, such that
$$\hat{k} = k \quad \text{and} \quad \hat{x} = x \tag{4b}$$

A schematic of the approach adopted in this letter is provided in Fig. 2.

Assumption 1: Let the following assumptions hold:

(A1) The unknown parameter k to be estimated is constant or piecewise constant, i.e., $\dot{k} = 0$, bounded, and satisfies [29]
$$k_m < k < k_M, \tag{5}$$
where k_m and k_M are known positive constants.

(A2) There exists a control input u which ensures that the SISO system in (3a)-(3b) is both input-to state and input-to-output stable [30]. It follows that
$$x \in \mathcal{X}, \quad \mathcal{X} := \{x \in \mathbb{R}^n | x_m < \|x\| < x_M\}, \tag{6}$$
where x_m and x_M are known real values and $\mathcal{X} \subset \mathbb{R}^n$ contains the origin. The function $g(x, u, k)$ in (3a) and its first time derivative remain bounded
$$\|g(x, u, k)\| \leq \Delta_g, \quad \|\dot{g}(x, u, k)\| \leq \Delta_{dg}, \tag{7}$$
where Δ_g and Δ_{dg} are known positive constants.

(A3) The function $f(x)$ and its gradient $\nabla f(x)$ are known, injective, and bounded in the domain $\mathcal{X}$. Since $f(x)$ is injective, $\nabla f(x) \neq 0 \ \forall x \in \mathcal{X}$ [31].

III. Estimation Scheme Design

To achieve the Objectives 1 and 2, in the first stage a **STA observer** as follows is proposed to attain the equivalence of the output y in (3b) and its estimate $\hat{y}$ as given in (4a):
$$\dot{\hat{x}}_i = g_i(\hat{x}, u, \hat{k}) - \frac{\alpha_1 |e_{1y}|^{\frac{1}{2}} \text{sign}\big(e_{1y}\big) + \xi f(\hat{x}) - \hat{v}}{n\hat{k}\nabla_{x_i} f(\hat{x}_i)}, \tag{8a}$$
$$\dot{\hat{v}} = -\alpha_2 \ \text{sign}\big(e_{1y}\big), \tag{8b}$$
$$\hat{y} = \hat{k}f(\hat{x}), \tag{8c}$$
$$e_{1y} := \hat{y} - y, \tag{8d}$$
the term $\hat{x}_i$ is the estimate of the i-th state x_i of x, n is the positive integer representing the dimension of the state vector x, $\alpha_1, \alpha_2 \in \mathbb{R}$ are positive design scalar constants. In (8a), $\hat{v} \in \mathbb{R}$ is an auxiliary observer scalar variable, $\hat{k}$ is the estimate of unknown scalar parameter k, and $e_{1y} \in \mathbb{R}$ is the output estimate error, which is considered as the sliding variable of the STA structure to be nullified in finite time [15]. To regulate the convergence of the parameter as well as state estimation, an additional **STA scheme** is proposed as follows:
$$\sigma_k = -\hat{v}, \tag{8e}$$
$$\hat{k} = \int_0^t \varphi d\tau, \qquad \hat{k}(0) = k_0, \tag{8f}$$
$$\varphi = \begin{cases} 0, & \text{if} \quad |e_{1y}| > 0 \\ \xi, & \text{else} \end{cases} \tag{8g}$$
where σ_k is the sliding variable of the STA scheme, $\hat{k}(0)$ is the initial condition of $\hat{k}$ satisfying (5) in **(A1)**. When $|e_{1y}| \leq 0$, the relation $\varphi = \dot{\hat{k}} \equiv \xi$ is determined, like a classical STA control scheme as in [4] depending on the sliding variable σ_k, as follows:
$$\xi = -\beta_1 |\sigma_k|^{\frac{1}{2}} \text{sign}(\sigma_k) + w, \tag{8h}$$
$$\dot{w} = -\beta_2 \text{sign}(\sigma_k), \quad w(0) = w_0, \tag{8i}$$
where $\beta_1 \in \mathbb{R}$ and $\beta_2 \in \mathbb{R}$ are positive scalar constants to be designed, $w \in \mathbb{R}$ is an auxiliary scalar variable, and w_0 is its initial condition. Note that $\hat{k}$ is started being regulated after the enforcement of the sliding motion $e_{1y} = 0$, which takes place in finite time.

Proposition 1: Under Assumptions 1, the proposed observer framework in (8a)-(8d) satisfies $e_{1y} = \dot{e}_{1y} = 0$ for the system in (3a)-(3b), which ensure $\hat{k}f(\hat{x}) = kf(x)$ and $\hat{y} = y$ are reached in finite time.

Proof: Given the system in (3a)-(3b) and the STA observer in (8a)-(8d), the first time derivative of the output observation error e_{1y} is
$$\dot{e}_{1y} = \dot{\hat{y}} - \dot{y} = e_{2y} - \alpha_1 |e_{1y}|^{\frac{1}{2}} \text{sign}\big(e_{1y}\big), \tag{9}$$
where
$$e_{2y} = \rho(x, \hat{x}, u, k, \hat{k}) + \hat{v}, \tag{10}$$
and the function
$$\rho(\cdot) := \hat{k}\nabla f(\hat{x})^T g(\hat{x}, u, \hat{k}) - k\nabla f(x)^T g(x, u, k) \ . \tag{11}$$
Note that in the derivation of the compact expression for e_{2y}, the vector representation $\dot{\hat{x}} = [\dot{\hat{x}}_1, \ldots, \dot{\hat{x}}_n]^T$ has been made used from (8a). Furthermore, by defining the auxiliary vector $\nabla^{-1} f(x) := [1/\nabla_{x_1} f(x), \ldots, 1/\nabla_{x_n} f(x)]^T$ the basic vectorial identity
$$\nabla f(x)^T \nabla^{-1} f(x) = n \tag{12}$$
is utilised. The presence of the state vector dimension n in (8a) is also required for the straightforward algebraic simplifications. The STA canonical form [15] is therefore obtained
$$\dot{e}_{1y} = e_{2y} - \alpha_1 |e_{1y}|^{\frac{1}{2}} \text{sign}\big(e_{1y}\big), \tag{13a}$$

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$$\dot{e}_{2y} = \dot{\rho}(x, \hat{x}, u, k, \hat{k}) - \alpha_2 \text{sign}\big(e_{1y}\big). \tag{13b}$$

Under Assumptions 1, the function $\rho(\cdot)$ and its derivative are in the form of linear combination of bounded contributions, and therefore:

$$|\dot{\rho}(x, \hat{x}, u, k, \hat{k})| \leq \Delta_{d\rho}, \tag{14}$$

where $\Delta_{d\rho}$ is a known positive constant. It is possible to choose the design constants α_1 and α_2 such that the following set of conditions [4], [15]

$$\alpha_1 = 1.5\sqrt{\Delta_{d\rho}}, \tag{15a}$$
$$\alpha_2 = 1.1\Delta_{d\rho}, \tag{15b}$$

are satisfied. It is guaranteed that the trajectories of the error system (13a)-(13b) converge to the origin in finite time T_y which can be upper-bounded as

$$T_y \leq T_{y0}, \tag{16}$$

where T_{y0} is a known positive constant that can be determined following the methodology illustrated in [15]. ■

Remark 1: During the sliding of the system (13a)-(13b), as $e_{2y} = \dot{e}_{2y} = 0$, the algebraic conditions

$$\rho(x, \hat{x}, u, k, \hat{k}) + \hat{v} = 0, \tag{17a}$$
$$\dot{\rho}(x, \hat{x}, u, k, \hat{k}) - \alpha_2 \text{sign}\big(e_{1y}\big) = 0, \tag{17b}$$

are verified. It follows from (17a) that an estimate $\hat{\rho}(\cdot)$ of $\rho(x, \hat{x}, u, k, \hat{k})$ can be extracted online from the auxiliary variable $\hat{v}$ as

$$\hat{\rho}(\cdot) = -\hat{v}. \tag{18}$$

The estimate $\hat{\rho}(\cdot)$ is made use of to obtain a suitable sliding variable to regulate $\hat{k}$, as it will be clarified in the rest of the manuscript.

Remark 2: Not that the unknown function estimation methods based on SM principles, often require the use of a low-pass filter to eliminate the high-frequency components and obtain good quality unknown function [4], [32]. The filtering process only ensures that the estimate of the unknown function **asymptotically** converges to its true value. However, such a low-pass filtering is not required for the reconstruction of $\rho(\cdot)$ in (18). This is guaranteed by the **continuous** signal $\hat{v}$, as the order of the STA scheme in (8a)-(8d) is increased by one with respect to the order of the system in (3a)-(3b). Therefore, following the proposed approach, the unknown function $\rho(x, \hat{x}, u, k, \hat{k})$ can be estimated in finite time.

A. Estimation of the Parameter k

During the sliding behaviours of the STA system (13a)-(13b), an online estimation of $\rho(\cdot)$ is made use of to design the regulation scheme for $\hat{k}$. It is immediate to verify that $\rho(\cdot) = 0$ when $\hat{k} = k$. Let the error parameter estimation variable be defined as

$$e_k := \hat{k} - k, \tag{19}$$

and consider the following lemma:

Lemma 1 (Sufficient Conditions): The function $\rho(x, \hat{x}, u, k, \hat{k})$ defined as in (11) verifies the system of inequalities

$$\rho(x, \hat{x}, u, k, \hat{k}) \begin{cases} > 0, & \text{if} \quad e_k > 0 \\ = 0, & \text{if} \quad e_k = 0 \\ < 0, & \text{if} \quad e_k < 0 \end{cases} \tag{20}$$

if one of the following two cases are met:

- *Case (a):* $f(x)$ is monotonically increasing in the domain $x \in \mathcal{X}$, and ($\mathbf{a_{1.1}}$) OR ($\mathbf{a_{1.2}}$), AND ($\mathbf{a_2}$) conditions are met, which are:
  - ($\mathbf{a_{1.1}}$) Each component of $\nabla f(x)$ is monotonically decreasing;
  - ($\mathbf{a_{1.2}}$) Each component of $\nabla f(x)$ is monotonically increasing, but the logarithmic derivative $\text{ln}df(x)$ is monotonically decreasing.
  - ($\mathbf{a_2}$) Each component of the function $g(x, u, k)$ is monotonically increasing whenever k increases and simultaneously x decreases.
- *Case (b):* $f(x)$ is monotonically decreasing in the domain $x \in \mathcal{X}$, and ($\mathbf{b_{1.1}}$) OR ($\mathbf{b_{1.2}}$), AND ($\mathbf{b_2}$) conditions are met, which are:
  - ($\mathbf{b_{1.1}}$) Each component of $\nabla f(x)$ is monotonically increasing;
  - ($\mathbf{b_{1.2}}$) Each component of $\nabla f(x)$ is monotonically decreasing, but the logarithmic derivative $\text{ln}df(x) := \nabla f(x)/f(x)$ is monotonically increasing.
  - ($\mathbf{b_2}$) Each component of the function $g(x, u, k)$ is monotonically increasing whenever both k and x simultaneously increase.

Proof: The lemma can be proven following straightforward algebraic manipulations and making use of the definition of $\rho(x, \hat{x}, u, k, \hat{k})$ in (11). It is always possible to represent $\hat{k}$ as $\hat{k} = \gamma k$, where $\gamma > 0$ is a (time-varying) parameter. The STA sliding motion condition $e_{1y} = 0$ can be written as:

$$\gamma k f(\hat{x}) = kf(x) \rightarrow \frac{f(x)}{f(\hat{x})} = \gamma. \tag{21}$$

Consider now the situation when $\gamma > 1$, which means that $e_k > 0$ by using (19). Under **Case** (**a**), it is easy to show that $x > \hat{x}$, whilst under **Case** (**b**) it follows that $x < \hat{x}$. Given the conditions on **Case** (**a**) or **Case** (**b**) on $f(x)$ and on $g(x, u, k)$, stated in Lemma 1, the set of inequalities

$$\gamma k \nabla f(\hat{x}) > k \nabla f(x), \tag{22a}$$
$$g(\hat{x}, u, \gamma k) > g(x, u, k), \tag{22b}$$

hold. In particular, note that the inequality (22a) is ensured thanks to the conditions ($\mathbf{a_{1.1}}$) **or** ($\mathbf{a_{1.2}}$) or to the ($\mathbf{b_{1.1}}$) **or** ($\mathbf{b_{1.2}}$) ones. The inequality (22b) is ensured instead thanks to the conditions ($\mathbf{a_2}$) or ($\mathbf{b_2}$). Under (22a)-(22b), it is immediate that $\rho(\cdot) > 0$ whenever $e_k > 0$. Analogous (complementary) considerations can also be made for the case when $\hat{k} = \gamma k$, $0 < \gamma < 1$, which yield

$$\gamma k \nabla f(\hat{x}) < k \nabla f(x), \tag{23a}$$
$$g(\hat{x}, u, \gamma k) < g(x, u, k). \tag{23b}$$

Under (23a)-(23b), it follows that $\rho(\cdot) < 0$ whenever $e_k < 0$, and the Lemma is proven. ■

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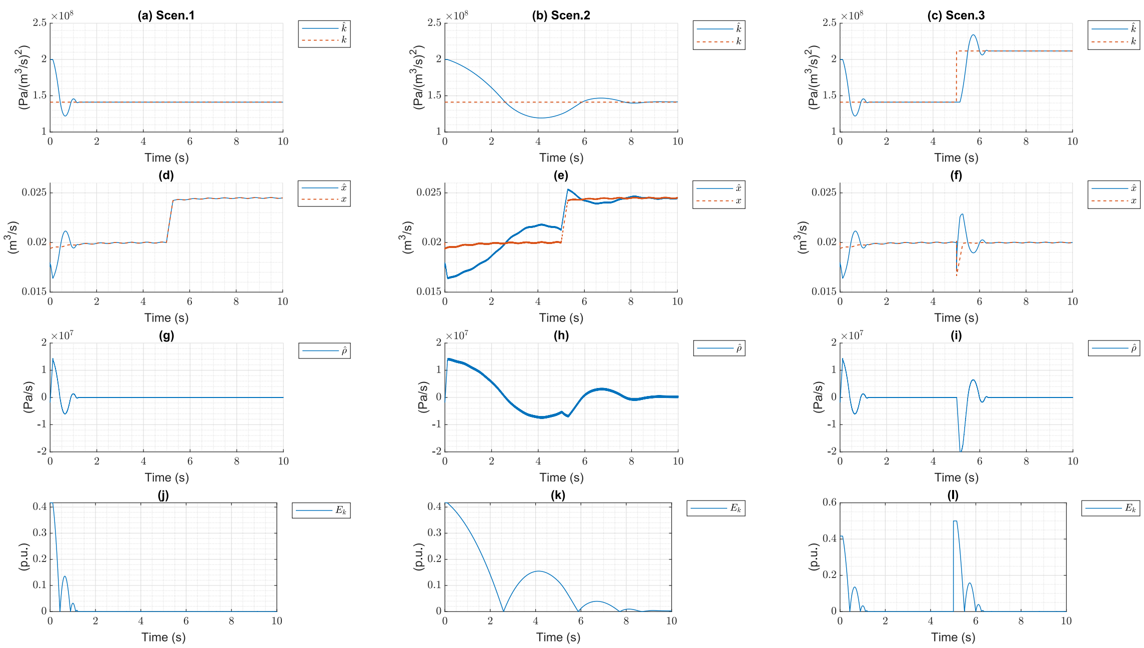

Fig. 3. **(Left:)** Scenario 1, **(Centre:)** Scenario 2, **(Right:)** Scenario 3. Time histories of: the output parameter k and its estimate $\hat{k}$ **((a)-(b)-(c))**; the state variable x and its estimate $\hat{x}$ (**(d)-(e)-(f)**); the estimate of the function $\rho(\cdot)$ (**(g)-(h)-(i)**); the error metric E_k (**(j)-(k)-(l)**).

Remark 3: Note that the set of conditions formulated in Lemma 1 constitutes ***sufficient*** conditions for the stability analysis of the STA scheme regulating $\hat{k}$. As long as the set of inequalities in (20) is verified, the scheme proposed in this letter is applicable and its stability analysis guaranteed.

Proposition 2: Provided that the function $\rho(\cdot)$ satisfies the condition (20), and under the SM enforced by Proposition 1, the condition $e_k = 0$ is reached in finite time.

Proof: Under (20), the sliding variable σ_k to be nullified is chosen as

$$\sigma_k := \hat{\rho}(\cdot) = \vartheta e_k, \tag{24}$$

where ϑ is an unknown positive bounded time-varying parameter, which is equal to zero only if $e_k = 0$. Consider the STA structure regulating $\hat{k}$ as in (8e)- (8i). As per (20), it is ensured that $\text{sign}(\sigma_k) = \text{sign}(e_k)$. The error system

$$\dot{e}_k = -\tilde{\beta}_1 |e_k|^{\frac{1}{2}} \text{sign}(e_k) + w, \tag{25a}$$
$$\dot{w} = -\beta_2 \text{sign}(e_k), \tag{25b}$$

exhibits a STA structure, where $\tilde{\beta}_1 := \beta_1 |\vartheta|^{\frac{1}{2}}$. The system (25a)-(25b) converges to the origin in finite time provided that $\beta_1 > 0$ and $\beta_2 > 0$ [4], [15]. The condition $\hat{k} = k$ holds in finite time. Since $f(x)$ is injective, and (4a) ensured, $\hat{x} = x$ is satisfied in finite time. ■

IV. Simulation Results

In this section, an assessment of the proposed scheme is undertaken by considering the hydraulic system represented in Fig. 1 and modelled by (2a)-(2b). Based on case studies reported in [33] and [34], and following some design guidelines in [35], we produce the following synthetic system parameters: $J = 1.88 \times 10^6$ $\text{Pa}/(\text{m}^3/s^2)$, $k_1 = 14.12 \times 10^7$ $\text{Pa}/(\text{m}^3/s)^2$ and $k_2 = 38.53 \times 10^5$ $\text{Pa}/(\text{m}^3/s)^2$. Even though in the networked case, the instantaneous value of Δp depends on the state of the overall hydraulic system, here we assume out of simplicity that Δp is an exogenous disturbance bounded within the interval [150, 300] kPa (c.f., [36, Ch. 5]) given by $\Delta p = 150 + 150 \sin(2\pi t)$. For the control input u, based on the results of [18], we assign the following PI control law around the output y: $u = c_P(y - y^\star) + z \quad \dot{z} = c_I(y - y^\star)$, which ensures the asymptotic convergence of y towards $y^\star$ for any $c_\text{P}, c_\text{I} > 0$. The PI control gains are here selected equal to $c_P = c_I = 50$. The output reference is set equal to $y^\star = 56.48$ (kPa) for $t < 5$ and $y = 84.79$ (kPa) for $t > 5$. The function $f(x) := k|x|x$, is injective and monotonically increasing along with its partial derivative $f_x(x) := 2kx, \forall x > 0$. The logarithmic derivative $\ln df(x) = 2/x, \quad x > 0$ is monotonically decreasing for $x > 0$. The function

$$g(x, k, u) = \frac{1}{J}(k(-|x|x) - k_2|x|x + u),$$

is monotonically increasing whenever k increases and simultaneously x decreases. Therefore, all the sufficient conditions of **Case (a)-($\mathbf{a_{1,2}}$), ($\mathbf{a_2}$)** presented in Lemma 1 are met.

The STA observer in (8a) is created, with the design parameters chosen as: $\alpha_1 = 7.77$ and $\alpha_2 = 75$. The initial condition for the estimate of x is $\hat{x}(0) = 0.02$ (m^3/s). The STA scheme (8g) is implemented adopting: $\beta_1 = 7.77$ $\beta_2 = 75$, $\hat{k}(0) = 20 \times 10^7$. The proposed scheme is implemented using the MATLAB-Simulink R2021b environment with a fixed integration (Euler) step size set as 0.1 milliseconds. To evaluate the performance of our estimation scheme, the following per-unit metric is proposed:

$$E_k := \frac{|\hat{k} - k|^{\frac{1}{2}}}{k_\text{base}} \qquad k_\text{base} := 10^7 \;\; (\text{Pa}/(\text{m}^3/\text{s})^2)$$

Three scenarios are considered:

- *Scenario 1:* The unknown parameter remains constant and no measurement noise affects the output y.
- *Scenario 2:* The unknown parameter k remains constant and a band-limited measurement noise affects the output

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y as $y = k|x|x + n$ where $n = A_n \sin(2\pi f_n t)$, with $A_n = 0.5$ (kPa) and $f_n = 100$ (Hz).

- *Scenario 3:* The unknown parameter varies in a piece-wise constant manner. In particular, k increases from 14.13×10^7 to the value 21.18×10^7 after the first 5 seconds of the simulation.

Fig. 3 presents the results of the three considered scenarios. During Scenario 1, the estimate of the parameter $\hat{k}$ reaches the true value k in less than 2 seconds. The state $\hat{x}$ converges to x in the same amount of time, whilst the function $\hat{\rho}$ is nullified. Also in presence of measurement noise (Scenario 2), the proposed scheme still works with acceptable accuracy, estimating correctly the value of k. A piece-wise constant behaviour of the parameter k is simulated in Scenario 3. After a transient during which the SM is lost, the proposed scheme re-tracks in finite time k.

V. Conclusion

In this letter, a novel scheme has been proposed to determine in finite time the output parameter for nonlinear SISO system. The parameter estimation problem has been solved in an original way by adopting a cascade architecture composed of an STA observer and by an STA scheme. The results presented in this letter will be extended to the networked case, where the unknown parameter k is a vector. It seems likely that by exploiting distributed/decentralised observers-based estimation schemes, the finite time identification of k could be possible even in the vector case.

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