Voltage Control of DC Microgrids: Robustness for Unknown ZIP-Loads

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***Abstract*—In this letter we propose a new passivity-based control technique for Buck converter based DC microgrids comprising ZIP-loads, i.e., loads with the parallel combination of constant impedance (Z), current (I) and power (P). More precisely, we propose a novel passifying input and a storage function based on the mixed potential function introduced by Brayton and Moser, relaxing restrictive (sufficient) conditions on Z, P and the voltage reference, which are usually assumed to be satisfied in the literature. Consequently, we develop a new passivity-based controller that is robust with respect to uncertain ZIP-loads.**

***Index Terms*—Decentralized control, power systems, uncertain systems.**

I. Introduction

The main control objective in (islanded) DC microgrids is voltage regulation, guaranteeing that the electrical signals converge to desired values for a proper functioning of the connected loads [2]. In the last decades, several controllers based on different techniques have been proposed in the literature, e.g., plug-and-play [3], sliding mode [4], barrier function [5] and PID passivity-based [6] controllers. However, these works consider only constant impedance (Z) and/or current (I) loads. In order to address the voltage destabilizing effect of the negative incremental impedance introduced by constant power (P) loads, several approaches have been recently proposed in [7]–[15] to stabilize DC microgrids including ZIP-loads. However, restrictive conditions on the system parameters are assumed to be satisfied (see [7, Th. 1], [8, Sec. V.B], [9, Sec. 5], [10, Th. 2], and [11, Proposition 3, Th. 1]). The adaptive controller in [12] requires the knowledge of some system parameters for the observer design, while the adaptive negative impedance strategy in [13] does not establish stability guarantees. A data-driven controller is proposed in [14], where the errors of the output voltage and input current w.r.t. a desired steady-state are required. However, knowledge of the steady-state value of the input current usually requires load information. An H-infinity controller is designed in [15], where however the stability depends on the feasibility of an optimization problem, which might thus imply restrictive conditions on the system parameters.

In this letter, based on passivity [16] and inspired by the theory developed in [17]–[19], we design and analyze a decentralized robust control scheme for DC microgrids comprising Buck converters supplying *unknown* ZIP-loads.

The main contributions of this letter are listed below: *i) Robustness*: The proposed control scheme is decentralized, scalable and robust w.r.t. the uncertainty affecting the ZIP-loads, the power lines impedances and the filters inductances and capacitances. *ii) Passivity*: We establish a passivity property with the output port-variable equal to the first time derivative of the voltage. Then, we adopt the *output-shaping* technique introduced in [20] to shape the closed-loop storage function such that it has a minimum at the desired operating point. *iii) Less restrictive conditions*: Differently from the results in the literature, where restrictive (sufficient) conditions on the system parameters are assumed to be satisfied, the considered DC microgrid in closed loop with the proposed controller is passive for positive voltages.

Note however that the proposed controller requires an estimate of the first time derivative of the voltage (as conventional PID controllers).

Notation: The set of real numbers is denoted by $\mathbb{R}$. For a vector $x \in \mathbb{R}^n$ and a symmetric and positive semidefinite matrix $M \in \mathbb{R}^{n\times n}$, let $\|x\|_M := (x^\top M x)^{1/2}$. Given a function $f : \mathbb{R}^n \to \mathbb{R}$, $\nabla_x f(x) \in \mathbb{R}^n$ denotes the partial derivatives of $f(x)$ w.r.t. x. $\mathcal{I}_n$ ($\mathcal{O}_n$) represents the identity (zero) matrix of order n, while '$\mathbb{1}$' denotes the ones vector of appropriate dimension. Let $v \in \mathbb{R}^n$, then $[v] := \mathrm{diag}\{v_1, \dots, v_n\}$, and $\ln v := [\ln v_1, \dots, \ln v_n]^\top$. We omit the time dependence if it is clear from the context.

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II. Brayton-Moser Framework

In the early 1960s, Brayton and Moser (BM) developed an unified framework to describe a class of RLC circuits by a gradient structure [17], [18]. This property has been widely used for controlling RLC circuits (see, e.g., [19], [20]).

A. Preliminaries

Consider the class of topologically complete RLC circuits with σ inductors, ρ capacitors and $k \leq \sigma$ voltage sources $u : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^k$ connected in series with the inductors. This class of systems can be represented as in [17], [18], i.e.,

$$-L\dot{I} = \nabla_I \mathcal{P}(I, V) + Bu, \quad C\dot{V} = \nabla_V \mathcal{P}(I, V), \tag{1}$$

where $L \in \mathbb{R}^{\sigma \times \sigma}$ and $C \in \mathbb{R}^{\rho \times \rho}$ are positive definite symmetric matrices with the inductances and capacitances as entries, respectively. The signals $I : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^{\sigma}$ and $V : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^{\rho}$ denote the currents through the σ inductors and the voltages across the ρ capacitors, respectively. The matrix $B \in \mathbb{R}^{\sigma \times k}$ is the input matrix and $\mathcal{P} : \mathbb{R}^{\sigma} \times \mathbb{R}^{\rho} \rightarrow \mathbb{R}$ represents the so-called *mixed potential* function, i.e.,

$$\mathcal{P}(I, V) = I^{\top}\Gamma V + f(I) - g(V), \tag{2}$$

where the matrix $\Gamma \in \mathbb{R}^{\sigma \times \rho}$ captures the instantaneous power transfer between the storage elements (i.e., inductors and capacitors). The resistive content $f : \mathbb{R}^{\sigma} \rightarrow \mathbb{R}$ and the resistive co-content $g : \mathbb{R}^{\rho} \rightarrow \mathbb{R}$ capture the power dissipated for instance in the resistors connected in series to the inductors and in parallel to the capacitors, respectively. Compactly, the BM equations (1) can be expressed as follows:

$$Q\dot{x} = \nabla_x \mathcal{P}(x) + \tilde{B}u, \tag{3}$$

where $x = (I^{\top}, V^{\top})^{\top}$, $Q = \operatorname{diag}\{-L, C\}$ and $\tilde{B} = (B^{\top}, \mathcal{O}_{k \times \rho})^{\top}$.

B. Generalized Gradient Structure

The mixed potential function (2) satisfies

$$\begin{aligned} \dot{\mathcal{P}}(x) &= \nabla_x^{\top}\mathcal{P}(x)\dot{x} = (Q\dot{x} - \tilde{B}u)^{\top}\dot{x} \\ &= \frac{1}{2}\|\dot{x}\|^2_{(Q+Q^{\top})} - \dot{x}^{\top}\tilde{B}u, \end{aligned} \tag{4}$$

along the solutions to (3). Therefore, (4) implies that (3) is passive with supply rate $-\dot{x}^{\top}\tilde{B}^{\top}u$, if $\mathcal{P}$ is positive semi-definite and Q is negative semi-definite. Unfortunately, the class of systems that satisfies this property is small and restricted to RL or RC circuits [21]. For the more general class of systems (3), the symmetric part of Q is indeed indefinite [17]. Then, in [17] BM observed that it is possible to generate a new pair $\{Q_A, \mathcal{P}_A\}$ that preserves the gradient structure (3). Differently from [19], we propose in the following proposition a *novel* family of BM descriptions.

Proposition 1 (A Novel Family of BM Descriptions): For $\lambda \in \mathbb{R}$, full rank matrix $D \in \mathbb{R}^{k \times k}$, constant symmetric matrix $M \in \mathbb{R}^{(\sigma+\rho)\times(\sigma+\rho)}$ and $Q_0 : \mathbb{R}^{\sigma+\rho} \rightarrow \mathbb{R}^{k \times (\sigma+\rho)}$, system (3) can be (re)written as follows:

$$Q_A(x)\dot{x} = \nabla_x \mathcal{P}_A(x) + \tilde{B}_A(x)\upsilon, \tag{5}$$

where

$$Q_A(x) := (\lambda \mathcal{I}_{\sigma+\rho} + \nabla_x^2 \mathcal{P}(x) M)(Q - \tilde{B}Q_0(x)) \tag{6a}$$

$$\mathcal{P}_A(x) := \lambda \mathcal{P}(x) + \frac{1}{2}\nabla_x^{\top}\mathcal{P}(x) M \nabla_x \mathcal{P}(x) \tag{6b}$$

$$\tilde{B}_A(x) := (\lambda \mathcal{I}_{\sigma+\rho} + \nabla_x^2 \mathcal{P}(x) M)\tilde{B}D \tag{6c}$$

$$\upsilon := D^{-1}(u - Q_0(x)\dot{x}), \tag{6d}$$

with λ and M such that $(\lambda \mathcal{I}_{\sigma+\rho} + \nabla_x^2 \mathcal{P}(x) M)$ has full rank.

Proof: To prove that the solution to (3) coincides with the solution to (5), one has to show that:

$$\begin{aligned} Q^{-1}(\nabla_x \mathcal{P}(x) + \tilde{B}u) = {}& (Q_A(x) + \tilde{B}_A(x) D^{-1} Q_0(x))^{-1} \\ & \cdot\ (\nabla_x \mathcal{P}_A(x) + \tilde{B}_A(x) D^{-1} u), \end{aligned}$$

which can be verified by directly substituting in the right-hand side the definitions in (6), where $\mathcal{P}_A$ in (6b) satisfies $\nabla_x \mathcal{P}_A(x) = (\lambda \mathcal{I}_{\sigma+\rho} + \nabla_x^2 \mathcal{P}(x) M)\nabla_x \mathcal{P}(x)$. ■

As a direct consequence of Proposition 1, and in analogy with [19], the following preliminary result is presented.

Proposition 2 [Passivity Property of (5)]: Assume that the pair $\{Q_A, \mathcal{P}_A\}$ in (6) satisfies $\mathcal{P}_A \geq 0$ and $Q_A + Q_A^{\top} \leq 0$, for all $x \in \mathbb{R}^{\sigma+\rho}$. Then, system (5) is passive w.r.t. the storage function $\mathcal{P}_A$ and supply rate $-\dot{x}^{\top}\tilde{B}_A \upsilon$.

Proof: Similarly to (4), the function $\mathcal{P}_A$ in (6b) satisfies $\dot{\mathcal{P}}_A(x) = \frac{1}{2}\|\dot{x}\|^2_{(Q_A(x)+Q_A^{\top}(x))} - \dot{x}^{\top}\tilde{B}_A(x)\upsilon(x, u) \leq -\dot{x}^{\top}\tilde{B}_A(x)\upsilon$, along the solutions to (5). ■

III. DC Power Network

In this letter we consider a Buck converter based DC microgrid, which is represented by a connected and undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where the nodes, $\mathcal{V} = \{1, \ldots, n\}$, represent Distributed Generation Units (DGUs) and the edges, $\mathcal{E} = \{1, \ldots, m\}$, represent the power lines. The network topology is described by its corresponding incidence matrix $\mathcal{B} \in \mathbb{R}^{n \times m}$. Then, the equations describing the average[1] dynamic behavior of the overall microgrid are given by (see, e.g., [4], [8], [10], [11], [22] and the references therein):

$$\begin{aligned} -L_s\dot{I}_s &= R_s I_s + V - u \\ -L_t\dot{I}_t &= R_t I_t + \mathcal{B}^{\top}V \\ C_s\dot{V} &= I_s + \mathcal{B}I_t - I_l(V), \end{aligned} \tag{7}$$

where $I_s : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^n$, $I_t : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^m$, $V : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^n$, $u : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^n$, represent the generated current, the current on the transmission lines, the load voltage (i.e., the voltage across the capacitor of an RLC filter in parallel to the load) and the voltage control input,[2] respectively; $C_s, L_s, R_s \in \mathbb{R}^{n \times n}$, $R_t, L_t \in \mathbb{R}^{m \times m}$ are positive definite diagonal matrices. Moreover, the term $I_l : \mathbb{R}^n \rightarrow \mathbb{R}^n$ represents the *unknown* current demand. In this letter, we consider a general nonlinear load model including the parallel combination of constant impedance (Z), current (I) and power (P) components, i.e.,

$$I_l(V) := Z_l^{*-1}V + I_l^* + [V]^{-1}P_l^*, \tag{8}$$

[1] The average model is accurate under the reasonable assumption that the Pulse Width Modulation (PWM) frequency is sufficiently high (see [6]).

[2] Note that in practical applications, u_i in (7) corresponds to $\delta_i V_{\mathrm{DC}_i}$, where δ_i is the duty cycle of the Buck converter i and V_{DC_i} is the DC voltage source provided by a generic energy source at node i.

with $P_l^*,\ I_l^* \in \mathbb{R}^n$, and $Z_l^* \in \mathbb{R}^{n\times n}$.

Then, (7) can be written in the BM structure (3) using $\sigma = n+m$, $\rho = n$, $k = n$, $I = [I_s^\top, I_t^\top]^\top$, $x = [I_s^\top, I_t^\top, V^\top]^\top$, $L = \operatorname{diag}\{L_s, L_t\}$, $C = C_s$, and

$$Q = \operatorname{diag}\{-L_s, -L_t, C_s\} \tag{9a}$$

$$\Gamma = [\mathcal{I}_n\ \ \mathcal{B}]^\top \tag{9b}$$

$$f(I) = \frac{1}{2}\|I_t\|_{R_t}^2 + \frac{1}{2}\|I_s\|_{R_s}^2 \tag{9c}$$

$$g(V) = \frac{1}{2}\|V\|_{Z_l^{*-1}}^2 + P_l^{*\top}\ln V + I_l^{*\top}V \tag{9d}$$

$$\tilde{B} = [-\mathcal{I}_n\ \ \mathcal{O}_{n\times m}\ \ \mathcal{O}_n]^\top. \tag{9e}$$

A. Problem Formulation

The main control objective in DC microgrids is to regulate the voltage across the loads to a desired reference value, i.e.,

Objective 1 (Voltage Regulation):

$$\lim_{t\to\infty} V_i(t) = V_i^*,\ \forall i \in \mathcal{V}, \tag{10}$$

where $V_i^* \in \mathbb{R}_{>0}$ is the desired voltage value of node $i \in \mathcal{V}$.

Because the value of Z_l^*, I_l^* and P_l^* are generally not known, it is desired to design a controller that does not require load information to achieve Objective 1. In Section V, we present a passivity-based robust controller that requires (locally) the following system information.

Assumption 1 (Available information): R_{si}, L_{si} and $\pi_i \in \mathbb{R}_{\geq 0}$, satisfying $P_{li}^* \leq \pi_i$, are available at node $i \in \mathcal{V}$.

For the sake of convenience, let $\Pi := \operatorname{diag}\{\pi_1, \dots, \pi_n\}$. Then, Assumption 1 implies $[P_l^*] \leq \Pi$.

Remark 1 (Plausibility of Assumption 1): Note that R_{si} and L_{si} are the filter components and, thus, design parameters whose values are generally known (see also Subsection V-A for a robustness analysis). We note also that the power absorbed by any load in practice cannot be infinite and, as a consequence, Π always exists and can generally be determined by data analysis and engineering understanding.

IV. Passivity Properties

We now briefly present and discuss some of the existing passivity properties for DC networks including ZIP-loads. Then, we establish a novel passivity property for system (7).

A. Existing Passivity Properties

Motivated by the well-known port-Hamiltonian representation of RLC circuits [23], we first introduce the total energy stored in the considered DC network (7), i.e.,

$$S(I_s, I_t, V) = \frac{1}{2}\|I_s\|_{L_s}^2 + \frac{1}{2}\|I_t\|_{L_t}^2 + \frac{1}{2}\|V\|_{C_s}^2. \tag{11}$$

The storage function (11) satisfies

$$\begin{aligned}\dot{S} &= I_s^\top(-R_sI_s - V + u) + I_t^\top(-R_tI_t - \mathcal{B}^\top V)\\ &\quad +\ V^\top(I_s + \mathcal{B}I_t - Z_l^{*-1}V - I_l^* - [V]^{-1}P_l^*)\\ &= -\|I_t\|_{R_t}^2 - \|I_s\|_{R_s}^2 - \|V\|_{Z_l^{*-1}}^2 - 1^\top P_l^*\\ &\quad -\ V^\top I_l^* + u^\top I_s,\end{aligned} \tag{12}$$

along the solutions to (7). Now, let $\mathcal{X} := \{(I_s, I_t, V) \in \mathbb{R}^{2n+m} | V_i > 0,\ \forall i \in \mathcal{V}\}$. Then, the following result holds.

Proposition 3 (Stored Energy): System (7) is passive w.r.t. (11) and supply rate $u^\top I_s$, for every $(I_s, I_t, V) \in \mathcal{X}$.

Note that because of the notorious *dissipation obstacle*,[3] the passivity property in Proposition 3 could be not useful for solving a stabilization problem at non trivial operating points. As an alternative to (11), inspired by the BM theory [17], [18], the following result follows from [19].

Proposition 4 (Generalized Mixed Potential): Assume that $(\lambda, M) \in \mathbb{R} \times \mathbb{R}^{(2n+m)\times(2n+m)}$ satisfies $\mathcal{P}_A \geq 0$ and $\dot{\mathcal{P}}_A \leq u^\top \dot{I}_s$, for every $(I_s, I_t, V) \in \mathcal{X}$, where $\mathcal{P}_A$ is given by (6b), with $\mathcal{P}$ in (2) and Γ, f, g in (9b)–(9d). Then, (7) is passive w.r.t. $\mathcal{P}_A$ and supply rate $u^\top \dot{I}_s$, for every $(I_s, I_t, V) \in \mathcal{X}$.

We notice that finding the pair (λ, M) for the considered network (7) including ZIP-loads (8) may require a nontrivial endeavour. Alternatively, in [7], [9], [10], the authors propose the Bregman distance associated to the total energy (11) as storage function, i.e.,

$$\begin{aligned}S_{\mathrm{B}}(I_s, I_t, V) &= \frac{1}{2}\|I_s - \bar{I}_s\|_{L_s}^2 + \frac{1}{2}\|I_t - \bar{I}_t\|_{L_t}^2\\ &\quad +\ \frac{1}{2}\|V - V^*\|_{C_s}^2.\end{aligned} \tag{13}$$

Following similar steps as in (12), (13) satisfies

$$\begin{aligned}\dot{S}_{\mathrm{B}} &= -\|I_t - \bar{I}_t\|_{R_t}^2 - \|I_s - \bar{I}_s\|_{R_s}^2 - \|V - V^*\|_{G_{\mathrm{B}}(V)}^2\\ &\quad +\ (u - u^*)^\top(I_s - \bar{I}_s),\end{aligned} \tag{14}$$

where $G_{\mathrm{B}}(V) := Z_l^{*-1} - [P_l^*][V]^{-1}[V^*]^{-1}$. Let $\mathcal{X}_{\mathrm{B}} := \{(I_s, I_t, V) \in \mathcal{X} | G_{\mathrm{B}}(V) \geq 0\}$, then, the following result holds.

Proposition 5 (Bregman Distance): System (7) is (shifted) passive w.r.t. the storage function (13) and supply rate $(u - u^*)^\top(I_s - \bar{I}_s)$, for every $(I_s, I_t, V) \in \mathcal{X}_{\mathrm{B}}$.

Remark 2 (Restrictive Conditions): Note that $\mathcal{X}_{\mathrm{B}}$ is empty for any $Z_{lj}^{*-1} = 0, j \in \mathcal{V}$. Moreover, V_i^* must satisfy the inequality $V_i^* \geq \sqrt{Z_{li}^* P_{li}^*}$, for all $i \in \mathcal{V}$. However, in practice, the loads are generally unknown and the voltage reference cannot be chosen arbitrarily large. Thus, $\mathcal{X}_{\mathrm{B}}$ could be often empty or not contain the steady-state solution corresponding to $V_i^*, i \in \mathcal{V}$. Furthermore, the trajectories $(I_s(t), I_t(t), V(t))$ must evolve in $\mathcal{X}_{\mathrm{B}}$ for all $t \geq 0$.

More recently, for the sake of robustness, in [20], [24], [25], the authors have proposed the following Krasovskii's Lyapunov function as storage function:

$$S_{\mathrm{K}}(I_s, I_t, V, u) = \frac{1}{2}\|\dot{I}_s\|_{L_s}^2 + \frac{1}{2}\|\dot{I}_t\|_{L_t}^2 + \frac{1}{2}\|\dot{V}\|_{C_s}^2. \tag{15}$$

Following similar steps as in (12), (15) satisfies

$$\dot{S}_{\mathrm{K}} = -\|\dot{I}_t\|_{R_t}^2 - \|\dot{I}_s\|_{R_s}^2 - \|\dot{V}\|_{G_{\mathrm{K}}(V)}^2 + \dot{u}^\top \dot{I}_s, \tag{16}$$

where $G_{\mathrm{K}}(V) := Z_l^{*-1} - [P_l^*][V]^{-2}$. Let us define the set $\mathcal{X}_{\mathrm{K}} := \{(I_s, I_t, V, u) \in \mathcal{X} \times \mathbb{R}_{>0}^n | G_{\mathrm{K}}(V) \geq 0\}$, then, the following result holds (see Remark 2 also for Proposition 6).

[3] For system with non-zero supply rate at the desired operating point, the controller has to provide unbounded energy to stabilize the system. In the literature, this is usually referred to as *dissipation obstacle* or *pervasive dissipation* [16].

Proposition 6 (Krasovskii): System (7) is passive w.r.t. (15) and supply rate $\dot{u}^\top \dot{I}_s$, for every $(I_s, I_t, V, u) \in \mathcal{X}_\mathrm{K}$.

We propose now a *novel* passifying input and a storage function based on the generalized mixed potential function (6b), leading to a passivity property for every type of load (even loads consisting of only the P component), for every positive voltage reference and for every $(I_s, I_t, V) \in \mathcal{X}$.

First, we notice that the passive output of the existing passivity properties for general RLC circuits is the current or its time derivative. However, from the dissipation inequalities (14) and (16), we can observe that in order to counteract the effects of the P-loads, it would be desired that the passive output is (function of) the voltage or its time derivative, allowing for damping injection. In the following theorem, based on Proposition 1, we establish a new passivity property with output port-variable equal to the first time derivative of the voltage. This property is essential in Section V to design the proposed passivity-based control achieving Objective 1.

Theorem 1 (Novel Passivity Property): Let Assumption 1 hold. Given system (7), define the mapping $u_\mathrm{PBC} : \mathcal{X} \to \mathbb{R}^n$ as follows:

$$u_\mathrm{PBC} := R_s I_s + Q_0(x)\dot{x}, \tag{17}$$

where $Q_0 : \mathcal{X} \to \mathbb{R}^{n\times(2n+m)}$. Consider the following mixed potential function:

$$\mathcal{P}(I, V) = \frac{1}{2}\|I_t\|^2_{R_t} + I^\top \Gamma V - g(V), \tag{18}$$

where Γ and $g(V)$ are given by (9b) and (9d), respectively. Then, the following statements hold:

(i) Let $\upsilon \in \mathbb{R}^n$. Consider the following input:

$$u = u_\mathrm{PBC} + L_s \upsilon. \tag{19}$$

The closed-loop system (7), (19) is described by the generalized gradient structure (5) with input υ and mixed potential function (18).

(ii) Let $Q_0 = \begin{bmatrix}\mathcal{O}_n & \mathcal{O}_{n\times m} & -L_s\Pi[V]^{-2}\end{bmatrix}$. The closed-loop system (7), (19) is passive w.r.t. $\mathcal{P}_A$ in (6b) and supply rate $\upsilon^\top \dot{V}$, for every $(I_s, I_t, V) \in \mathcal{X}$.

Proof: Part *(i)*. Along the solutions to (7), $\mathcal{P}_A$ in (6b) (with $\mathcal{P}$ given by (18)) satisfies

$$\begin{aligned}\nabla_x \mathcal{P}_A &= (\lambda \mathcal{I}_{2n+m} + \nabla_x^2 \mathcal{P} M)\nabla_x \mathcal{P}\\ &= (\lambda \mathcal{I}_{2n+m} + \nabla_x^2 \mathcal{P} M)(Q\dot{x} - \tilde{B}(-R_s I_s + u))\\ &= (\lambda \mathcal{I}_{2n+m} + \nabla_x^2 \mathcal{P} M)(Q\dot{x} - \tilde{B}(Q_0\dot{x} + L_s\upsilon))\\ &= (\lambda \mathcal{I}_{2n+m} + \nabla_x^2 \mathcal{P} M)((Q - \tilde{B}Q_0)\dot{x} - \tilde{B}L_s\upsilon)\\ &= Q_A\dot{x} - \tilde{B}_A\upsilon,\end{aligned} \tag{20}$$

where in (6c) and (6d) we use $D = L_s$. Part *(ii)*. Let $\lambda = 0$ and $M = \mathrm{diag}\{L_s^{-1}, L_t^{-1}, C_s^{-1}\}$. Then, $\mathcal{P}_A$ in (6b) can be expressed as follows:

$$\begin{aligned}\mathcal{P}_A(I_s, I_t, V) = \;& \frac{1}{2}\|V\|^2_{L_s^{-1}} + \frac{1}{2}\|R_t I_t + \mathcal{B}^\top V\|^2_{L_t^{-1}}\\ &+ \frac{1}{2}\|I_s + \mathcal{B}I_t - I_l(V)\|^2_{C_s^{-1}}.\end{aligned} \tag{21}$$

Furthermore, $\tilde{B}_A = [\mathcal{O}_{n\times(n+m)} \;\; -\mathcal{I}_n]^\top$ and Q_A in (6a) can be expressed as follows:

$$Q_A = \begin{bmatrix}\mathcal{O}_n & \mathcal{O}_{n\times m} & \mathcal{I}_n\\ \mathcal{O}_{m\times n} & -R_t & \mathcal{B}^\top\\ -\mathcal{I}_n & -\mathcal{B} & -G_\Pi(V)\end{bmatrix},$$

where, for every $(I_s, I_t, V) \in \mathcal{X}$, $G_\Pi := Z_l^{*-1} + (\Pi - [P_l^*])[V]^{-2} \geq 0$ by virtue of Assumption 1. Therefore, $Q_A + Q_A^\top \leq 0$, and following Proposition 2, (21) satisfies

$$\begin{aligned}\dot{\mathcal{P}}_A &= \frac{1}{2}\dot{x}\left(Q_A + Q_A^\top\right)\dot{x} - \upsilon^\top \tilde{B}_A^\top \dot{x}\\ &= -\|\dot{I}_t\|^2_{R_t} - \|\dot{V}\|^2_{G_\Pi(V)} + \upsilon^\top \dot{V},\end{aligned} \tag{22}$$

implying that the closed-loop system (7), (19) is passive with port-variables υ and $\dot{V}$. ■

Note that the input u_PBC in (17) can be written as

$$u_\mathrm{PBC} = R_s I_s - L_s \Pi [V]^{-2}\dot{V}. \tag{23}$$

Moreover, we notice that, differently from $G_\mathrm{B}(V)$ and $G_\mathrm{K}(V)$, $G_\Pi(V)$ is positive for every $V > 0$.

V. Robust Passivity-Based Control

In this section, we use the new passivity property established in Theorem 1 to design a controller that stabilizes the closed-loop system and achieves Objective 1 despite the uncertainty affecting the load components. Specifically, we shape the closed-loop storage function such that it has a minimum at the desired operating point. To do so, we adopt the *output-shaping* technique introduced in [20], where the integrated passive output is used to shape the closed-loop storage function.

Theorem 2 (Closed-Loop Stability): Let Assumption 1 hold. Given system (7), define the mapping $u_\mathrm{Stab} : \mathcal{X} \to \mathbb{R}^n$ as follows:

$$u_\mathrm{Stab} := -L_s K_1 (V - V^*) - L_s K_2 \dot{V} + V^*, \tag{24}$$

$K_1 \geq 0, K_2 > 0 \in \mathbb{R}^{n\times n}$, $V^* \in \mathbb{R}^n_{>0}$. Then, the following statements hold:

(i) Let $\mu \in \mathbb{R}^n$. Consider the following input:

$$u = L_s\mu + u_\mathrm{PBC} + u_\mathrm{Stab}, \tag{25}$$

with u_PBC and u_Stab given by (23) and (24), respectively. The closed-loop system (7), (25) is passive w.r.t. the supply rate $\mu^\top \dot{V}$ and storage function $S_d = \mathcal{P}_A + S_a$, with $\mathcal{P}_A$ in (21) and S_a defined as follows:

$$S_a(V) := \frac{1}{2}\|V - V^*\|^2_{K_1} - V^\top L_s^{-1} V^* + \frac{1}{2}\|V^*\|^2_{L_s^{-1}}, \tag{26}$$

for every $(I_s, I_t, V) \in \mathcal{X}$.

(ii) Consider the closed-loop system (7), (25) with μ equal to zero. Then, the equilibrium $(\bar{I}_s, \bar{I}_t, V^*) \in \mathcal{X}$ is asymptotically stable.

Proof: Part *(i)*. Consider $\mathcal{P}_A$ in (21), and $S_a : \mathbb{R}^n_{>0} \to \mathbb{R}$ in (26), then we choose the desired closed-loop storage function $S_d : \mathcal{X} \to \mathbb{R}$ as $S_d = \mathcal{P}_A + S_a$, i.e.,

$$S_d = \frac{1}{2}\|V - V^*\|^2_{L_s^{-1}+K_1} + \frac{1}{2}\|R_t I_t + \mathcal{B}^\top V\|^2_{L_t^{-1}} + \frac{1}{2}\|I_s + \mathcal{B}I_t - I_l(V)\|^2_{C_s^{-1}}. \quad (27)$$

Moreover, along the closed-loop dynamics (7), (25), S_a satisfies $\dot{S}_a = \dot{V}^\top(K_1(V - V^*) - L_s^{-1}V^*)$, and S_d satisfies

$$\begin{aligned}\dot{S}_d &= -\|\dot{I}_t\|^2_{R_t} - \|\dot{V}\|^2_{G_\Pi(V)} + \upsilon^\top\dot{V} + \dot{S}_a \\ &= -\|\dot{I}_t\|^2_{R_t} - \|\dot{V}\|^2_{G_\Pi(V)} + \mu^\top\dot{V} \\ &\quad + \dot{V}^\top(-K_1(V - V^*) - K_2\dot{V} + L_s^{-1}V^*) \\ &\quad + \dot{V}^\top(K_1(V - V^*) - L_s^{-1}V^*) \\ &= -\|\dot{I}_t\|^2_{R_t} - \|\dot{V}\|^2_{G_\Pi(V)+K_2} + \mu^\top\dot{V}, \end{aligned} \quad (28)$$

where we used $\dot{\mathcal{P}}_A$ given by (22), $\upsilon = \mu + L_s^{-1}u_{\text{Stab}}$ and u_{Stab} given by (24). Part *(ii)*. System (7) in closed-loop with $u = u_{\text{PBC}} + u_{\text{Stab}}$ becomes

$$\begin{aligned}\dot{I}_s + \left(\Pi[V]^{-2} + K_2\right)\dot{V} &= -\left(K_1 + L_s^{-1}\right)(V - V^*) \\ -L_t\dot{I}_t &= R_t I_t + \mathcal{B}^\top V \\ C_s\dot{V} &= I_s + \mathcal{B}I_t - I_l(V). \end{aligned} \quad (29)$$

Now, we observe that S_d in (27) is positive and attains a minimum at the (unique) equilibrium point $(\overline{I}_s, \overline{I}_t, V^*) \in \mathcal{X}$ of the closed-loop system (29). Then, we use S_d in (27) as a candidate Lyapunov function. Therefore, (28) implies that there exists a forward invariant set Υ and by Lasalle's invariance principle the solutions that start in Υ approach to the largest invariant set contained in

$$\Upsilon \cap \left\{(I_s, I_t, V) \in \mathcal{X} : \dot{V} = 0,\ \dot{I}_t = 0\right\}. \quad (30)$$

On this invariant set, by differentiating the third line of (29), it follows that $\dot{I}_s = 0$. Moreover, from (29), it also follows that $V = V^*$, $I_s = \overline{I}_s$ and $I_t = \overline{I}_t$. ■

Remark 3 (Control law): The control law (25) with $\mu = 0$ can be written compactly as follows:

$$u = \underbrace{R_s I_s + V^*}_{(a)} - \underbrace{L_s K_1(V - V^*)}_{(b)} - \underbrace{L_s(\Pi[V]^{-2} + K_2)\dot{V}}_{(c)}. \quad (31)$$

In (31), the term *(a)* represents a feedforward control action. The term *(b)* represents an integral action on the passive output (or, equivalently, a proportional action on the voltage error), with gain $-L_sK_1$. Finally, the term *(c)* represents a proportional action on the passive output (or, equivalently, a derivative action on the voltage error), with adaptive gain $-L_s(\Pi[V]^{-2} + K_2)$.

A. Robustness With Respect to Parameter Uncertainty

We observe from (31) that the proposed controller does not require any information about the filters capacitances, the lines impedances and load parameters, except for an upperbound of only the P component. Moreover, it is robust w.r.t. uncertainty affecting the filter inductance. Let L_s denote the (unknown) actual value of the inductance and $\hat{L}_s$ its (known) nominal value. Then, (31) can be rewritten as follows $u = R_sI_s + V^* - L_s\hat{K}_1(V - V^*) - L_s(\hat{\Pi}[V]^{-2} + \hat{K}_2)\dot{V}$, with $\hat{K}_1 = {L_s}^{-1}\hat{L}_sK_1$, $\hat{K}_2 = {L_s}^{-1}\hat{L}_sK_2$ and $\hat{\Pi} = {L_s}^{-1}\hat{L}_s\Pi$. Thus, the uncertainty affecting L_s modifies only the control gains. If we assume to know the maximum deviation of L_s from its nominal value $\hat{L}_s$, it is then always possible to select Π such that $\hat{\Pi} \geq [P_l^*]$. Since $\hat{K}_1 \geq 0$ and $\hat{K}_2 > 0$, stability is then preserved according to Theorem 2. The only system parameter required is R_s. However, we notice that, in presence of uncertainty affecting R_s, the steady-state voltage of the closed-loop system satisfies $\overline{V} = V^* - (\mathcal{I}_n + L_sK_1)^{-1}(R_s - \hat{R}_s)\overline{I}_s$, where $\hat{R}_s$ denotes the nominal value of the filter resistance. Then, it is evident that a sufficiently large value of K_1 implies that $\overline{V}$ is sufficiently close to V^*. Furthermore, the value of R_s is very small in practice and often neglected.

B. Robustness With Respect to Noisy Voltage Measurement

We observe from (31) that the proposed controller requires information of the voltage derivative. It is indeed well known that the derivative action may considerably improve the power converter performances [6], yet, the voltage derivative might not be perfectly known in practice. Thus, we now investigate the robustness properties of the proposed controller w.r.t. some classes of estimation errors of the voltage derivative, leading to useful tuning rules for the controller gains.

First, let assume that the estimation error can be expressed as $\delta\dot{V}$, with $\delta \geq \delta_{\min} > -1$. Then, replacing $\dot{V}$ in (31) by $\dot{V} + \delta\dot{V}$, it is straightforward to show that Theorems 1 and 2 hold if $\pi_i \geq P^*_{li}/(1 + \delta_{\min})$, $i \in \mathcal{V}$. Secondly, we consider a more general estimation error and replace $\dot{V}$ in (31) by $\dot{V} + \beta, \beta \in \mathbb{R}$. Analogously to Theorem 2, it can be shown that (27) satisfies $\dot{S}_d = -\|\dot{I}_t\|^2_{R_t} - \|\dot{V}\|^2_{G_\Pi(V)+K_2} - \dot{V}^\top(\Pi[V]^{-2} + K_2)\beta + \mu^\top\dot{V}$, implying that the closed-loop system (7), (25) is passive w.r.t. the storage function S_d and supply rate $\mu^\top\dot{V}$, for every $(I_s, I_t, V) \in \mathcal{X}$ and every β sufficiently small. Then, for $\mu = 0$, $\dot{S}_d \leq 0$ and, as a consequence, we can conclude that the voltage error is bounded and, consequently, also the generated and line currents. Moreover, from the steady-state analysis of the closed-loop system (7), (25), i.e., $(\Pi[\overline{V}]^{-2} + K_2)\beta = -(K_1 + L_s^{-1})(\overline{V} - V^*)$ we observe that independently of β, the voltage error can be made arbitrarily small by choosing a sufficiently large K_1.

Now, in order to make β sufficiently small, we suggest to use a powerful and robust Filtering-Differentiator (FD) that is suitable for the considered application. Let assume that the measured voltage at each node $i \in \mathcal{V}$, $V_i(t), t \geq 0$, can be expressed as $V_i(t) = V_{0i}(t) + \eta_i(t)$, where V_{0i} is the basic voltage signal and η_i is a Lebesgue-measurable noise. Assume also that $\dot{V}_{0i}$ has the Lipschitz constant $\mathcal{L}_i > 0$, and $|\eta_i| \leq \epsilon_i$, where $\epsilon_i \geq 0$ is unknown. Then, according to [26, Th. 2], the filtering differentiator [26, eq. 9] provides in finite time the estimation of the voltage derivative with an error whose upperbound depends on $\mathcal{L}_i, \epsilon_i$ and the choice of the differentiator's parameters. The finite time convergence property of the FD allows to use it within the proposed scheme, providing sufficient time for the FD to converge.

VI. EXAMPLE

In this section, we show through MATLAB/Simulink (Simscape Electrical Toolbox) the performance of the proposed controller on a DC microgrid comprising 4 nodes in a ring topology as in [4, Figure 2]. The control gains in (31)

TABLE I
NETWORK PARAMETERS

Node		1	2	3	4
R_{si}	(Ω)	0.25	0.20	0.15	0.10
L_{si}	(mH)	1.8	2.0	3.0	2.2
C_{si}	(mF)	2.2	1.9	2.5	1.7
V_i^*	(V)	379.50	379.75	380.00	380.25
P_{li}^*	(kW)	10	2	6	10
Z_{li}^{*-1}	(S)	0.08	0.04	0.05	0.07
I_{li}^*	(A)	10	15	10	15
ΔP_{li}^*	(kW)	2	2	-4	-2

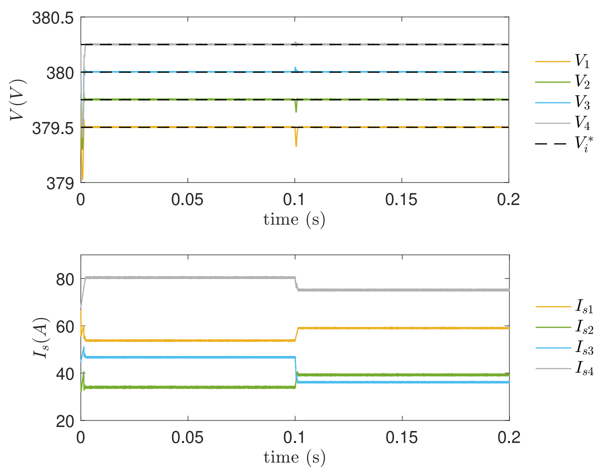

Fig. 1. Time evolution of the voltages together with the corresponding references (top); and generated currents (bottom).

are $K_{1i} = 1 \times 10^6$, $K_{2i} = 25$, $\pi_i = 25 \times 10^3$ for $i = 1, \ldots, 4$, the parameters of each node are reported in Table I, and those of the lines are chosen as in [4, Table III]. At the time instant $t = 0.1$ s, the value of the P-load has a variation equal to ΔP_{li}^* (see Table I). Figure 1 shows the microgrid voltages and generated currents, respectively. We can observe that the voltages converge to the corresponding references (see Objective 1), independently from the load parameters.

VII. Conclusion and Future Work

In this letter we have addressed the notorious instability issue related to the presence of unknown constant power loads in DC microgrids. Inspired by the theory developed by Brayton and Moser, under a very mild assumption, we have designed a passivity-based voltage controller that is robust with respect to the uncertainty affecting the load parameters. A deeper analysis in presence of a larger class of noises and input saturation is left to future works as well as the extension to other types of converters, e.g., the Boost converter.

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