# Decentralized Temperature and Storage Volume Control in Multiproducer District Heating

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# Decentralized Temperature and Storage Volume Control in Multiproducer District Heating

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***Abstract*—Modern district heating technologies have a great potential to make the energy sector more flexible and sustainable due to their capabilities to use energy sources of varied nature and to efficiently store energy for subsequent use. Central control tasks within these systems for the efficient and safe distribution of heat refer to the stabilization of overall system temperatures and the regulation of storage units state of charge. These are challenging goals when the networked and nonlinear nature of district heating system models is taken into consideration. In this letter, for district heating systems with multiple, distributed heat producers, we propose a decentralized control scheme to provably meet said tasks stably.**

***Index Terms*—Stability of nonlinear systems, Lyapunov methods, energy systems.**

## I. Introduction

District heating systems (DHSs) distribute heat from heating stations, conventionally a single combined heat and power plant, towards clusters of consumers within a neighborhood or a small city using a network of underground insulated pipelines [2]. The newest generation DHSs have the potential to improve the sustainability of a fossil fuel-dependent heating sector by increasing the share of distributed generations units based on renewables (e.g., geothermal or solar thermal) as well as units based on residual heat from some industrial processes. Such units can be flexibly incorporated with the support of heat storage devices (typically water tanks) [2] (see also [3], [4]).

Heat distribution in DHSs, in the face of varying weather conditions and heat demand profiles, is supported by a control system in charge of regulating the supply (return) temperature of heat producers (consumers) and the state of charge of storage units [5], [6]. This is done by adjusting producers' power injections and the overall system's flow rates so that the heat supply matches the demand and the system temperatures stay within prescribed domains. Optimal, predictive (and centralized) control of DHSs has been addressed for systems without ([7], [8]) and with ([9]) heat storage units. Supply temperature control and storage volume regulation is performed in [10] on a dynamic, nonlinear system model of a DH system comprising a single heating station with an adjacent storage tank. The control design is based on the internal model principle and offers closed-loop stability guarantees and robustness against uncertainty of some parameters. For a similar system model, which does not consider storage units, but does consider transport dynamics of the distribution network (codified as a delay), a control design based on Lyapunov-Krasovksii theory to achieve (supply and return) temperature regulation is reported in [11]. Recently, the control of supply temperatures of heating stations was investigated in [12] using passivity-based design within the context of electro-thermal microgrids consisting of distributed generation units. Optimal, open-loop end-user temperature control is investigated in [13] via numerical simulations of a DH system with multiple *prosumers*. Focused on the hydraulic dynamics, i.e., neglecting the thermal behavior, the works [14]–[16] address pressure regulation on single producer DH systems and [17], [18] consider flow and volume control in the multiproducer setting.

In Section II, we describe the setup of the considered DH system, which features multiple, distributed producers with heat storage capabilities. We then develop a (modular) dynamic model that describes the behavior of the overall system temperatures, including heat exchangers and storage tanks, and the supply and return layers of the distribution network. Our main contribution is in Section III, where we design novel decentralized controllers to regulate supply temperature of heat producers, the temperature and volume of the storage tanks and the return temperature of heat consumers. Stability analysis of the overall closed-loop system concludes

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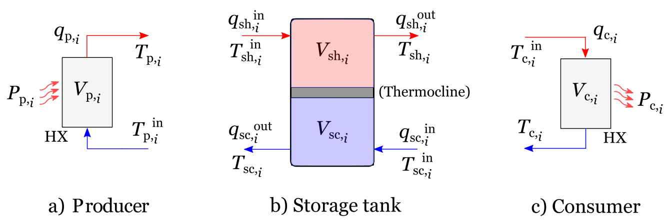


Fig. 1. Topologies of producers, consumers and storage tanks [10], [11]: $T_{\chi,i}$, $V_{\chi,i}$, $q_{\chi,i}$ and $P_{\chi,i}$ stand for temperature, volume, flow rate and thermal power associated with the $i$th device $\chi \in \{\text{p, sh, sc, st, c}\}$.

the section. Note that the simultaneous treatment of the mentioned tasks, using decentralized controllers and with the considered system setup is not addressed in the above cited works.

## II. System Model

### A. Setup and Main Modeling Assumptions

We consider a DH system with $n_p$ distributed heat producers and $n_c$ consumers interconnected through a distribution network (DN) that has a supply (hot) layer and return (cold) layer. The specific composition of producers and consumers is shown in Fig. 1. Note that each producer continuously drains water from the DN's return layer, heats it through a heat exchanger (HX) and injects the heated stream into the DN's supply layer. A converse operation follows for consumers.

Following [10], we consider a DH system with stratified storage tanks. Each tank stores a mixture of hot and cold water perfectly separated by a thermocline. The volume of hot water is on top and the cold one at the bottom. It is assumed that there is no heat or mass exchange between the mixtures. Moreover, each storage tank is considered to have two inlet/outlet pairs for hot and cold water, respectively. The topology of storage tanks is shown in Fig. 1. As a simplifying assumption, we consider that each producer is interfaced to the DN via a storage tank. Then, each producer drains water from the cold layer of a storage tank and injects it into the tank's hot layer. Using the remaining inlet/outlet pair, the tank can fill in its cold layer with water taken from the return layer of the DN. At the same time, the storage tank injects water from its hot layer into the DN's supply layer. We note that our results can be slightly adjusted to consider producers directly connected to the DN. However, standalone storage tanks, i.e., storage tanks with no immediate access to a heat producer are not considered in this letter. Additional modeling assumptions, some of which are fairly standard in related literature (see, e.g., [10], [11], [13]) are the following (see [19] for more details).

*Assumption 1:* (i) the density $\rho > 0$ and specific heat $c_{\text{s.h.}} > 0$ of water are spatially uniform and constant in time (for ease of notation we take $\rho = c_{\text{s.h.}} = 1$); (ii) the flow through any pipe is (spatially) one-dimensional. (iii) each device (pipe, storage tank, junction) is completely filled with water at all times; (iv) the internal energy of any water stream portion depends linearly on its temperature; the overall DH system is leak-free and lossless.

### B. Dynamics of Producers and Storage Tanks

Consider the notation used in Fig. 1. Then, the temperature dynamics of the producers and storage tanks, as well as the volume dynamics of the storage tanks can be modeled (see [1] for more details) as follows [10] (see also [11], [19]):

$$V_{p,i}\dot{T}_{p,i} = q_{p,i}\big(T_{sc,i} - T_{p,i}\big) + P_{p,i}, \tag{1a}$$
$$V_{sh,i}\dot{T}_{sh,i} = q_{p,i}\big(T_{p,i} - T_{sh,i}\big), \tag{1b}$$
$$V_{sc,i}\dot{T}_{sc,i} = q_{st,i}\Big(T_{sc,i}^{in} - T_{sc,i}\Big), \tag{1c}$$
$$\dot{V}_{sh,i} = q_{p,i} - q_{st,i}, \tag{1d}$$
$$\dot{V}_{sc,i} = q_{st,i} - q_{p,i}. \tag{1e}$$

We assume that $q_{p,i}$ and $P_{p,i}$ are control variables [10], [11] and $T_{sc,i}^{in}$ is an external input. Later, when we introduce the temperature dynamics of the return layer of the DN, $T_{sc,i}^{in}$ will be related with the temperature of a given junction in the return layer of the DN. We also identify each $q_{st,i}$ as an independent control variable, with the exception of one (see Remark 1 for details about this).

### C. Consumer Temperature Dynamics

Analogously to producers, the heat balance at each consumer's heat exchanger is given by [10], [11]:

$$V_{c,i}\dot{T}_{c,i} = q_{c,i}\Big(T_{c,i}^{in} - T_{c,i}\Big) - P_{c,i}. \tag{2}$$

The flow rate $q_{c,i}$ is an independent control variable, while $T_{c,i}^{in}$ and $P_{c,i} \geq 0$ are external inputs. The power load $P_{c,i}$ will be treated as an *unknown* constant disturbance. Later we will equate $T_{c,i}^{in}$ with the temperature of a certain junction in the supply layer of the DN.

### D. Distribution Network's Temperature Dynamics

Following [14], [20], [21], we represent the supply and return layers of the DN as connected graphs with no self-loops. For the supply layer we introduce $\mathcal{G}_s = (\mathcal{N}_s, \mathcal{E}_s)$, where the set of edges $\mathcal{E}_s$ represents all distribution pipes, and the set of nodes $\mathcal{N}_s$ denotes pipe junctions. An analogous description follows for the return layer of the DN, for which we use the notation $\mathcal{G}_r = (\mathcal{N}_r, \mathcal{E}_r)$. The focus of this letter is on DNs in which the supply and return layers are symmetric. Then, we assume that $\mathcal{G}_s$ and $\mathcal{G}_r$ are isomorphic and the bijection between $\mathcal{N}_s$ and $\mathcal{N}_r$ is referred to as $\gamma_{dn}$. We refer the reader to [22] for a discussion of prospective DH systems with non-symmetric DNs.[1]

We identify with the $i$th storage tank a unique node $k \in \mathcal{N}_s$ such that there is a stream with rate $q_{st,i}$ from the tank's hot layer into $k$, and at the same time there is a stream (with the same rate) from $\gamma_{dn}(k) \in \mathcal{N}_r$ towards the tank's cold layer. Analogously, to the $i$th consumer we associate a unique node $k \in \mathcal{N}_s$ such that the there is a stream with rate $q_{c,i}$ from $k$ towards the consumer's heat exchanger and finally reaching the node $\gamma_{dn}(k) \in \mathcal{N}_r$.

For ease of presentation, let us introduce further notation using $\mathcal{G}_s$ as reference. We fix an arbitrary reference orientation

[1] W.l.o.g., we assume that any two pipes $(i, j) \in \mathcal{E}_s$ and $(\gamma_{dn}(i), \gamma_{dn}(j)) \in \mathcal{E}_r$ have the same length and diameter.

to every edge of $\mathcal{G}_s$. Then, for any $i \in \mathcal{E}_s$ with end nodes $j, k \in \mathcal{N}_s$, $j \neq k$, we say that $j$ is the head and $k$ is the tail of $i$, or viceversa, that $j$ is the tail and $k$ is the head of $i$. Moreover, following [8], [21], [23], we introduce the functions $\mathcal{N}_s^-, \mathcal{N}_s^+ : \mathcal{E}_s \to \mathcal{N}_s$ and $\mathcal{E}_s^-, \mathcal{E}_s^+ : \mathcal{N}_s \to \mathcal{E}_s$ defined as follows. For any $i \in \mathcal{E}_s$, $\mathcal{N}_s^-(i)$ and $\mathcal{N}_s^+(i)$ respectively denote the tail and head of $i$; also, for any $j \in \mathcal{N}_s$, $\mathcal{E}_s^-(j)$ and $\mathcal{E}_s^+(j)$ denote sets of edges with $j$ as tail node and $j$ as head node, respectively. To streamline the subsequent definition of the DN's temperature dynamics, we assume that the reference orientation of any edge $i \in \mathcal{E}_s$ matches the direction of the stream through it. That is, if $j, k \in \mathcal{N}_s$, $j \neq k$, are the tail and head of any $i \in \mathcal{E}_s$, respectively, then the stream through $i$, henceforth denoted by $q_{s,i}$, is assumed to flow from $j$ to $k$ and we consider that $q_{s,i} \geq 0$.[2]

Considering the above definitions and assumptions, we write the temperature dynamics of each $i \in \mathcal{E}_\chi$, $j \in \mathcal{N}_\chi$, $\chi \in \{s, r\}$ as follows (see [1] for more details):

$$V_{\chi,i}\dot{T}_{\chi,i} = q_{\chi,i}\big(T_{\chi,j} - T_{\chi,i}\big)\,|_{j=\mathcal{N}_\chi^-(i)}, \tag{3a}$$

$$V_{\chi,j}\dot{T}_{\chi,j} = \sum_{k\in\mathcal{E}_\chi^+(j)} q_{\chi,k}\big(T_{\chi,k} - T_{\chi,j}\big) + \Phi_{\chi,j}, \tag{3b}$$

$$\Phi_{\chi,j} = \begin{cases} \sum_{k=1}^{n_p} \alpha_{k,j} q_{st,k}\big(T_{sh,k} - T_{s,j}\big), & \chi = s, \\ \sum_{k=1}^{n_c} \beta_{k,j} q_{c,k}\big(T_{c,k} - T_{r,j}\big), & \chi = r, \end{cases} \tag{3c}$$

where $V_{\chi,i}$, $T_{\chi,i}$ and $q_{\chi,i}$ respectively stand for volume, temperature and flow rate of the respective elements in $\mathcal{G}_\chi$. Moreover, $\alpha_{k,j} = 1$ if $j \in \mathcal{N}_s$ receives a stream from the $k$th storage tank (and $\alpha_{k,j} = 0$ otherwise). Analogously, $\beta_{k,j} = 1$ if from $j \in \mathcal{N}_s$ a stream is directed towards the $k$th consumer.

Equation (3a) represents the heat balance at any pipe $i \in \mathcal{E}_\chi$, in which we have used the boundary conditions $T_{s,i}^{in} = T_{s,j}\cdot|_{j=\mathcal{N}_s^-(i)}$ and $T_{s,i}^{out} = T_{s,i}$ [8], [21], meaning that the stream entering any pipe will have the temperature of the node from which the stream sources from and that the temperature of the stream at the outlet of any pipe will have the same temperature as the spatially-averaged temperature of the pipe's control volume (upwind scheme). Equation (3b) models the heat balance at each node $j \in \mathcal{N}_\chi$. The term in the left-hand side of (3b) is the rate of change of the thermal energy stored at $j$ and in the right-hand side we have the sum of the thermal energies of the streams that target or source from $j$. The interaction with storage tanks and consumers is represented by the term $\Phi_{\chi,j}$. Further details appear in [19].

*Remark 1:* **(I)** Following [19] (see also [20]), we take as independent variables each $q_{p,i}$, $q_{c,j}$ and $q_{s,k}$ ($q_{r,k}$) associated with any edge of $\mathcal{G}_s$ ($\mathcal{G}_r$) being a *chord*. The same can be done for each $q_{st,i}$, except for one, say for the $m$th tank. Such a constraint stems from the need to meet Kirchhoff's current laws and has the implication that $q_{st,m} = \sum_{\forall i} q_{c,i} - \sum_{\forall j\neq m} q_{st,j}$ (see [1]). **(II)** Having defined the temperature dynamics of the DN, we can define $T_{sc,i}^{in}$ and $T_{c,i}^{in}$ in (1) and (2), respectively, as follows:

$$T_{sc,i}^{in} = \alpha_{i,j} T_{r,k}|_{k=\gamma_{dn}(j)}, \;\; T_{c,i}^{in} = \beta_{i,j} T_{s,k}|_{k=\gamma_{dn}^{-1}(j)}. \tag{4}$$

Therefore, the overall temperature dynamics of the DH system are given by (1), (2), (3) and (4).

[2] Since flow reversals may occur, or the flow through an edge may simply not match the edge's reference orientation, $q_s$-dependent functions analogous to $\mathcal{N}_s^-$, $\mathcal{N}_s^+$, $\mathcal{E}_s^-$ and $\mathcal{E}_s^+$ can be defined to identify the source and target node of the stream through any edge (see, e.g., [21])

## III. Control Design and Stability Analysis

Control design is conducted in this section to meet the objectives of regulating each producer supply temperature $T_{p,i}$, each consumer return temperature $T_{c,i}$ and each storage tank (hot layer) volume $V_{sh,i}$ towards constant setpoints, usually specified by the DH operator and possibly based on some optimization criteria.

In this development, both the consumer and producer controllers are fully decentralised, requiring only local measurements for implementation. The advantage of a decentralised architecture over a distributed one is twofold. Firstly, the control design is independent of the network topology, and since only local measurements are required for control implementation, no communications are required among the controllers. Secondly, as stability is verified at the individual nodes, producers and consumers can be added or removed from the network without impacting the overall stability.

### A. Control of Producer Temperatures

First, we consider the temperature regulation of the producer. The objective is to utilise the power inputs $P_{p,i}$ to regulate the temperature to a known constant value $T_{p,i}^\star$.

*Proposition 1:* Consider the $i$th producer's temperature dynamics (1a) in closed-loop with the control law

$$P_{p,i} = -q_{p,i}\big(T_{sc,i} - T_{p,i}\big) - k_{p,i}\Big(T_{p,i} - T_{p,i}^\star\Big), \tag{5}$$

where $k_{p,i} > 0$ is a tuning parameter. The resulting closed-loop dynamics are given by

$$V_{p,i}\dot{T}_{p,i} = -k_{p,i}\Big(T_{p,i} - T_{p,i}^\star\Big) \tag{6}$$

and the producer temperature converges monotonically to $T_{p,i}^\star$ at an exponential rate.

*Proof:* The verification of the closed-loop dynamics (6) follows by direct substitution of (5) into (1a). The temperature dynamics (6) have the solution $T_{p,i}(t) = T_{p,i}^\star + [T_{p,i}(0) - T_{p,i}^\star]e^{-k_{p,i}V_{p,i}^{-1}t}$, verifying the stability properties. ■

### B. Storage Tank Control (Hot Layer)

Next we consider regulating both the volume and temperature of each storage tank's hot layer. The objective is to regulate the volume $V_{sh,i}$ to a known constant value $V_{sh,i}^\star$ via control of the producer's flow rate $q_{p,i}$. As the temperature of the $i$th producer is regulated to the (specified) constant value $T_{p,i}^\star$, it is not surprising that the temperature of the tank's hot layer converges to the same value.

The outgoing flow from each tank, $q_{st,i}$, could be controlled locally via a valve (or pump, see [17]). Each of these flows are treated as independent inputs with the exception for one node. As noted in Remark 1, there exists an index $m$ such that

$q_{\text{st},m} = \sum_{\forall i} q_{\text{c},i} - \sum_{\forall j \neq m} q_{\text{st},j}$, which could potentially lead to negative $q_{\text{st},m}$ in some scenarios. To avoid such a situation, we make the following assumption.

*Assumption 2:* The flows $q_{\text{st},i}$ are non-negative at all times.

In subsequent design, each $q_{\text{c},i}$ will be chosen to be non-negative, ensuring that $\sum_{\forall i} q_{\text{c},i} \geq 0$. Note also that this assumption can be satisfied in practice by placing a check valve at the hot water outlet of each tank.

*Proposition 2:* Consider the volume dynamics of the $i$th tank's hot layer (1d) in closed-loop with the continuous control law

$$q_{\text{p},i} = \begin{cases} -\kappa_{\text{p},i}\left(V_{\text{sh},i} - V^{\star}_{\text{sh},i}\right) + q_{\text{st},i}, & V_{\text{sh},i} \leq V^{\star}_{\text{sh},i}, \\ q_{\text{st},i} e^{-\left(V_{\text{sh},i} - V^{\star}_{\text{sh},i}\right)}, & V_{\text{sh},i} > V^{\star}_{\text{sh},i}, \end{cases} \tag{7}$$

where $\kappa_{\text{p},i} > 0$ is a tuning parameter that adjusts the rate of convergence of $V_{\text{sh},i}$ towards $V^{\star}_{\text{sh},i}$. The resulting closed-loop dynamics are described by

$$\dot{V}_{\text{sh},i} = \begin{cases} -\kappa_{\text{p},i}\left(V_{\text{sh},i} - V^{\star}_{\text{sh},i}\right), & V_{\text{sh},i} \leq V^{\star}_{\text{sh},i}, \\ -\xi_{\text{sh},i}(V_{\text{sh},i}) q_{\text{st},i}, & V_{\text{sh},i} > V^{\star}_{\text{sh},i}, \end{cases} \tag{8}$$

where $\xi_{\text{sh},i}(V_{\text{sh},i}) = (1 - e^{-(V_{\text{sh},i} - V^{\star}_{\text{sh},i})})$ is non-negative for $V_{\text{sh},i} > V^{\star}_{\text{sh},i}$. The equilibrium point $\bar{V}_{\text{sh},i} = V^{\star}_{\text{sh},i}$ is Lyapunov stable and, in the case that $q_{\text{st},i} > 0$, asymptotically stable.

*Proof:* The verification of the closed-loop dynamics (8) follows by direct substitution of (7) into (1d). To verify stability, consider the Lyapunov function candidate $W_{V_{\text{sh},i}} = \frac{1}{2}(V_{\text{sh},i} - V^{\star}_{\text{sh},i})^2$. Its time derivative along solutions of (8) can be straightforwardly verified to satisfy

$$\dot{W}_{V_{\text{sh},i}} = \begin{cases} -\kappa_{\text{p},i}\left(V_{\text{sh},i} - V^{\star}_{\text{sh},i}\right)^2, & V_{\text{sh},i} \leq V^{\star}_{\text{sh},i}, \\ -\xi_{\text{sh},i}(V_{\text{sh},i}) q_{\text{st},i}\left(V_{\text{sh},i} - V^{\star}_{\text{sh},i}\right), & V_{\text{sh},i} > V^{\star}_{\text{sh},i}. \end{cases} \tag{9}$$

By Assumption 2, $q_{\text{st},i}$ is non-negative for all the time, implying that $W_{V_{\text{sh},i}}$ is non-increasing along solutions of (8), which verifies Lyapunov stability. Note that if $q_{\text{st},i}(t) > 0$, $W_{V_{\text{sh},i}}$ is *strictly* decreasing, which implies asymptotic stability and convergence of $V_{\text{sh},i}$ to $V^{\star}_{\text{sh},i}$. ■

Next we show that the temperature of the $i$th storage tank's hot layer converges to the $i$th producer's (desired) outlet temperature $T^{\star}_{\text{p},i}$.

*Proposition 3:* Consider the temperature $T_{\text{sh},i}$ of the $i$th storage tank and assume that the flow rate $q_{\text{p},i}$ satisfies (7). Then, the temperature of the storage tank satisfies the following: **(I)** If the $i$th producer and storage tank have initial conditions satisfying $|T_{\text{p},i}(0) - T^{\star}_{\text{p},i}|, |T_{\text{sh},i}(0) - T^{\star}_{\text{p},i}| \leq \phi_i$, for some $\phi_i > 0$, then the storage tank temperature satisfies $|T_{\text{sh},i}(t) - T^{\star}_{\text{p},i}| \leq \phi_i$ all the time. **(II)** If the $i$th producer's flow rate $q_{\text{p},i}$ is strictly positive, the temperature $T_{\text{sh},i}$ converges to the $i$th producer's set-point $T^{\star}_{\text{p},i}$.

*Proof:* All details appear in [1]. ■

*Remark 2:* Propositions 1 and 2 rely on the exact compensation of some system dynamics. It can be shown with extended analysis that the corresponding closed-loop systems are ISS with respect to control imperfection and thus subsequent results share similar properties. This addition analysis, however, is omitted for brevity.

## C. Temperature Stability of the Supply Layer

We now focus on the dynamic behaviour of the DN's hot-layer and verify that the temperature of each node and edge within the layer is bounded and remains above a threshold value required for the consumers to operate correctly. Before proceeding, we define $T^{\star}_{\text{c}_{\max}}$ to be the maximum temperature reference among all consumers. We similarly define $T^{\star}_{\text{p}_{\min}}$ to be the minimum temperature reference among all producers. It is assumed that $T^{\star}_{\text{p}_{\min}} > T^{\star}_{\text{c}_{\max}} + \epsilon$ for some $\epsilon > 0$.

*Proposition 4:* Consider the set of all temperatures within the distribution supply layer $T_{\text{s}}$ and assume that all temperatures have initial condition satisfying $T_{\text{s},i}(0) \geq T^{\star}_{\text{c}_{\max}} + \epsilon$, $i \in \mathcal{G}_{\text{s}}$. If we additionally assume that the initial conditions of each producer and storage tank satisfy

$$|T_{\text{p},i}(0) - T^{\star}_{\text{p},i}|, |T_{\text{sh},i}(0) - T^{\star}_{\text{p},i}| \leq T^{\star}_{\text{p}_{\min}} - T^{\star}_{\text{c}_{\max}} - \epsilon, \tag{10}$$

the temperature of the distribution layer is bounded and satisfies

$$T_{\text{s},i}(t) \geq T^{\star}_{\text{c}_{\max}} + \epsilon, \ \forall i \in \mathcal{G}_{\text{s}} \tag{11}$$

all the time.

*Proof:* By Proposition 3, the tank temperatures satisfy $T_{\text{sh},i}(t) \geq T^{\star}_{\text{c}_{\max}} + \epsilon$ all the time. To verify the behavior of the distribution layer, we consider the coldest temperature in the layer and show that it is lower bounded by $T^{\star}_{\text{c}_{\max}} + \epsilon$. The temperature dynamics of each edge and node in the distribution layer are described by (3a) and (3b) with $\chi = s$.

At an arbitrary time $t$, the coldest temperature could occur at either an edge $T_{\text{s},i}$, which would imply that it is colder than all node temperatures, i.e., $T_{\text{s},i} \leq T_{\text{s},j}$, $j \in \mathcal{N}_{\text{s}}$. Recalling (3a) and noting that by the node ordering convention $q_{\text{s},i} \geq 0$, we have that $V_{\text{s},i}\dot{T}_{\text{s},i} \geq 0$, ensuring that if the coldest temperature is within an edge, it is non-decreasing. Now, consider that the coldest temperature is within a node $T_{\text{s},j}$. As the node is the coldest within the network, it is colder than all edges, i.e., $T_{\text{s},j} \leq T_{\text{s},i}$, $i \in \mathcal{E}_{\text{s}}$. Recalling (3b) and noting that by the node ordering convention $q_{\text{s},i} \geq 0$, we have that

$$V_{\text{s},j}\dot{T}_{\text{s},j} \geq \sum_{k=1}^{n_{\text{p}}} \alpha_{k,j} q_{\text{st},k}\left(T_{\text{sh},k} - T_{\text{s},j}\right). \tag{12}$$

As each storage tank satisfies $T_{\text{sh},i}(t) \geq T^{\star}_{\text{c}_{\max}} + \epsilon$ all the time, $\dot{T}_{\text{s},j}$ is non-decreasing for $T_{\text{s},j} \leq T^{\star}_{\text{c}_{\max}} + \epsilon$. As this inequality holds all the time and all temperatures initially satisfy $T_{\text{s},j} \geq T^{\star}_{\text{c}_{\max}} + \epsilon$ it follows that all temperatures within the supply layer are lower bounded by $T^{\star}_{\text{c}_{\max}} + \epsilon$.

By a similar argument, we can conclude that the distribution layer is upper-bounded as a function of the layer temperature initial conditions $T_{\text{s},i}, T_{\text{s},j}$, storage tank initial conditions $T_{\text{sh},k}(0)$ and set-points $T^{\star}_{\text{p},i}$. ■

The assumptions in Proposition 4 require that the initial temperature of the supply layer is higher than $T^{\star}_{\text{c}_{\max}}$ and that the initial temperature of each producer and storage tank hot layer belongs to a boundary layer centered in $T^{\star}_{\text{p},i}$ (see (10)).

## D. Control of Consumer Temperatures

Now we propose a simple control law to ensure the regulation of the $i$th consumer's return temperature $T_{\text{c},i}$ to some specified value $T^\star_{c,i}$. It is assumed that the consumer can measure the temperature $T^{in}_{c,i}$ of the incoming stream from the DN's supply layer (see Fig. 1 and (4)). This value, however, does not need to be constant. Before proceeding with the control design, it is recalled that $P_{c,i} \geq 0$ is constant and unknown.

*Proposition 5:* Consider the $i$th consumer's temperature dynamics (2) in closed-loop with the control law

$$q_{c,i}(T^{in}_{c,i}, z_{c,i}) = \frac{1}{T^{in}_{c,i} - T^\star_{c,i}} z_{c,i}, \tag{13}$$

with initial condition $z_{c,i}(0) > 0$ where

$$z_{c,i} = x_{c,i} - V_{c,i}T_{c,i}, \tag{14a}$$

$$\dot{x}_{c,i} = q_{c,i}\left(T^{in}_{c,i} - T_{c,i}\right) - z_{c,i}. \tag{14b}$$

The resulting closed-loop dynamics are such that $T_{c,i} \to T^\star_{c,i}$ and $z_{c,i} \to P^\star_{c,i}$ exponentially (as $t \to \infty$). Also, the state $z_{c,i}$, and hence the input $q_{c,i}$, are strictly non-negative.

*Proof:* All details appear in [1]. ■

*Remark 3:* **(I)** Equations (7) and (13) define control laws for each $q_{p,i}$ and $q_{c,i}$, respectively. The remaining flow inputs, namely $q_{st,i}$ as well as $q_{s,i}$ ($q_{r,i}$) associated to chords of $\mathcal{G}_s$ ($\mathcal{G}_r$)—see Remark 1—can be fixed for simplicity, and to comply with Assumption 2, as positive constants. Nonetheless, the PI-like control law reported in [16, Sec. 4.1] can also be used, which is non-negative all the time. **(II)** In this letter, the consumer's power consumption $P_{c,i}$ is assumed to be constant. In practice, however, this quantity would be time-varying. If the dynamics of the power consumption are slow compared to the closed-loop network then changes in the load can be neglected. If the loads are periodic the loads can be dynamically compensated using a frequency estimator, similar to [24].

## E. Stability of the Cold Layer

In this subsection, we study the stability of the cold layer, consisting of the return layer temperature and cold storage tank temperatures and volumes. The analysis is analogous to the one performed to assess the stability of the DH system's hot layer in Section III-C.

*Proposition 6:* All edge and node temperatures within the return layer ($T_{r,i}$, $i \in \mathcal{G}_r$) are bounded all the time.

*Proof:* From Proposition 5, all consumer temperatures are bounded and converge to $T^\star_{c,i}$. The remainder of the proof follows analogously to the proof of Proposition 4. ■

*Proposition 7:* The volume of each cold-layer storage tank $V_{sc,i}$ is Lyapunov stable and asymptotically stable for $q_{st,i} > 0$. The temperature is bounded all the time.

*Proof:* Considering the reciprocal behaviour of the hot and cold-layer storage tanks in (1d), (1e) and Proposition 2, the volume of the cold storage tank $V_{sc,i}$ shares the same stability properties of the hot-layer storage $V_{sh,i}$. The cold storage tank temperature is described by (1c). Proposition 6 ensures that temperatures within the return layer are bounded, which ensures that the temperature of the cold storage tank is bounded as well. ■

## F. Overall System Stability

The final property to be verified is stability of the overall DH system. To achieve this, we invoke the propositions detailed through Section III-A–III-E and verify that the assumptions utilized throughout are satisfied by the closed-loop system.

*Theorem 1:* Consider the DH model detailed in Section II in closed-loop with the decentralized control scheme described through Section III-A–III-E. Assuming that Assumption 2 holds, the producer and storage tank initial temperatures satisfy (10) and the supply layer initial temperatures satisfy (11), the closed-loop system satisfies the following properties: **(I)** The producer temperatures $T_{p,i}$ and hot storage temperatures $T_{sh,i}$ converge to the producer reference temperature $T^\star_{p,i}$. **(II)** The consumer temperatures $T_{c,i}$ converge their reference temperatures $T^\star_{c,i}$. **(III)** All network temperatures remain bounded. **(IV)** The hot-layer storage tank volumes $V_{sh,i}$ converge to the reference value $V^\star_{sh,i}$ and the cold-layer storage tank volumes $V_{sc,i}$ converge to a constant value.

*Proof:* Claim **(I)** follows from direct application of Propositions 1 and 3. The temperatures of the distribution layer satisfy the lower bound (11) by Proposition 4 which ensures that the inequality (11) is satisfied. Consequently, claim **(II)** is satisfied by Proposition 5. Claim **(III)** follows from direct application of Propositions 4, 6 and 7. Finally claim **(IV)** follows from application of Propositions 2 and 7. ■

# IV. Numerical Simulations

We illustrate the performance of the considered DH system model in closed-loop with the proposed controllers via numerical simulations. The configuration and parametrization are based on the case study reported in the arXiv version of [19], which corresponds to a DH system with three heat producers ($n_p = 3$), nine consumers ($n_c = 9$). Each tank is assumed to have a total capacity of 1000 $m^3$. Due to space limitations, we provide additional details about the setup and the values of the control gains in [1], and here we simply restrict ourselves to explain in general terms considered scenario.

For the results shown in Fig. 2, the system is initialized in the vicinity of a system's equilibrium in a context of low consumer demand (50% w.r.t. full demand) and with relatively small values for each $V^\star_{sh,i}$. Convergence of the signals on display is observed after a short transient. At $t = 6$h all storage tanks switch to a charging mode and attain their respective new setpoints at approximately $t = 9$h. At $t = 12$h the heat demand of consumers is simultaneously increased to higher values (75% w.r.t. full demand). Note that the consumers' temperatures return to their fixed setpoints (55° C) after a short transient. At $t = 18$h the tanks switch now to a discharging mode that ends at approximately $t = 21$h. The new, lower values for the entries of $V^\star_{sh}$ are maintained until the end of the simulation. Finally, it is observed that the producers' temperatures $T_{p,i}$ reach quickly (exponentially) their setpoints (85° C) and remain at this value during the whole simulation time. Additional simulations are reported in [1], where we analyze the system's behavior under input saturation.

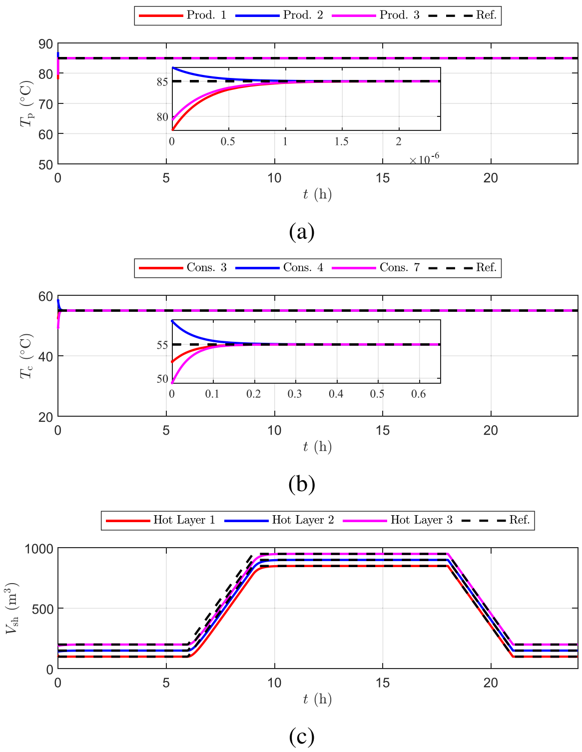


Fig. 2. From top to bottom: evolution of the producers' temperatures $T_{p,i}$, some of consumers' outlet temperatures $T_{c,i}$ and the volume of hot water in the storage tanks $V_{sh,i}$.

## V. Concluding Remarks

In this letter, we have addressed producer supply and consumer return temperature control in multiproducer DH systems through the design of novel decentralized controllers that also consider the regulation of the amount of hot water of multiple, distributed storage tanks. The design is complemented with a Lyapunov theory-based closed-loop stability analysis, from which convergence of the variables of interest is guaranteed. Extensions to this letter we are currently investigating include: time-varying heat demand profiles (c.f., [10]); more detailed consumer models (c.f., [13]); fair energy distribution [5]; and input saturation [16].

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