Modelling the hydrodynamics of swimming fish, from individuals to infinite schools

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APPENDIX A : RAY / MOVING LINE INTERSECTION

Let the ray be parametrically expressed in two dimensions as

$$\mathbf{r}(t) = \mathbf{r}_0 + t\,d\mathbf{r}. \tag{A.1}$$

The particle is at $\mathbf{r}_0$ at the beginning of the current time step, which we identify with $t = 0$, and t is continuous time.

Let $\mathbf{p}(t)$ and $\mathbf{q}(t)$ similarly be the position of the endpoints of the moving line over time, as follows :

$$\begin{aligned} \mathbf{p}(t) &= \mathbf{p}_0 + t\,d\mathbf{p} & \text{(A.2)} \\ \mathbf{q}(t) &= \mathbf{q}_0 + t\,d\mathbf{q}. & \text{(A.3)} \end{aligned}$$

Any point on the edge can be expressed as $\mathbf{E}(s,t) = s\mathbf{q}(t) + (1-s)\mathbf{p}(t)$, where s is the coordinate along the edge. The movements of particle and edge intersect if at any time t' the equality $\mathbf{E}(s,t') = \mathbf{r}(t')$ holds, in other words

$$s\left(\mathbf{q}_0 + t'\,d\mathbf{q}\right) + (1-s)\left(\mathbf{p}_0 + t'\,d\mathbf{p}\right) = \mathbf{r}_0 + t'\,d\mathbf{r}. \tag{A.4}$$

First we solve for t', focussing on the x component of Eq. A.4

$$t' = -\frac{s\,(q_{0,x} - p_{0,x}) + (p_{0,x} - r_{0,x})}{s\,(dq_x - dp_x) + (dp_x - dr_x)}. \tag{A.5}$$

Then we solve for t', focussing on the y component, with a result similar to the one above but with all subscripts x replaced by y. Equating the two expressions for t', we arrive at a quadratic equation in s:

$$as^2 + bs + c = 0, \tag{A.6}$$

where the coeffients a, b and c can be expressed using the binary perpendicular dot product ($\perp$), which greatly simplifies the coefficients of the quadratic equation and allows the solution to be calculated efficiently (note that $\perp$ is basically the z component of the cross product of vectors in the x-y plane):

$$\perp(\mathbf{A}, \mathbf{B}) \equiv A_x B_y - A_y B_x \tag{A.7}$$
$$a = \perp(\mathbf{q}_0 - \mathbf{p}_0, d\mathbf{q} - d\mathbf{p}) \tag{A.8}$$
$$b = \perp(\mathbf{q}_0 - \mathbf{p}_0, d\mathbf{p} - d\mathbf{r}) + \perp(\mathbf{p}_0 - \mathbf{r}_0, d\mathbf{q} - d\mathbf{p}) \tag{A.9}$$
$$c = \perp(\mathbf{p}_0 - \mathbf{r}_0, d\mathbf{p} - d\mathbf{r}) \tag{A.10}$$

Solving the quadratic equation yields two values for s, which, when inserted in Eq. A.5 give two corresponding values for t'. If any of the s lie in the interval $[0, 1]$ and the corresponding t' lies in the interval $[0, \Delta t]$ a collision has occurred. If there are two solutions within this interval, the one with smallest t' occurred first and is picked for further processing.