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Modelling the hydrodynamics of swimming fish, from individuals to infinite schools

Reid, Daniel Alexander Peter

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APPENDIX A : RAY / MOVING LINE INTERSECTION

Let the ray be parametrically expressed in two dimensions as

$$\mathbf{r}(t) = \mathbf{r}_0 + t \mathbf{dr}. \quad (\text{A.1})$$

The particle is at \mathbf{r}_0 at the beginning of the current time step, which we identify with $t = 0$, and t is continuous time.

Let $\mathbf{p}(t)$ and $\mathbf{q}(t)$ similarly be the position of the endpoints of the moving line over time, as follows :

$$\mathbf{p}(t) = \mathbf{p}_0 + t \mathbf{dp} \quad (\text{A.2})$$

$$\mathbf{q}(t) = \mathbf{q}_0 + t \mathbf{dq}. \quad (\text{A.3})$$

Any point on the edge can be expressed as $\mathbf{E}(s, t) = s\mathbf{q}(t) + (1 - s)\mathbf{p}(t)$, where s is the coordinate along the edge. The movements of particle and edge intersect if at any time t' the equality $\mathbf{E}(s, t') = \mathbf{r}(t')$ holds, in other words

$$s (\mathbf{q}_0 + t' \mathbf{dq}) + (1 - s) (\mathbf{p}_0 + t' \mathbf{dp}) = \mathbf{r}_0 + t' \mathbf{dr}. \quad (\text{A.4})$$

First we solve for t' , focussing on the x component of Eq. A.4

$$t' = -\frac{s (q_{0,x} - p_{0,x}) + (p_{0,x} - r_{0,x})}{s (dq_x - dp_x) + (dp_x - dr_x)}. \quad (\text{A.5})$$

Then we solve for t' , focussing on the y component, with a result similar to the one above but with all subscripts x replaced by y . Equating the two expressions for t' , we arrive at a quadratic equation in s :

$$as^2 + bs + c = 0, \quad (\text{A.6})$$

where the coefficients a , b and c can be expressed using the binary perpendicular dot product (\perp), which greatly simplifies the coefficients of the quadratic equation and allows the solution to be calculated efficiently (note that \perp is basically the z component of the cross product of vectors in the x - y plane):

APPENDIX A : RAY / MOVING LINE INTERSECTION

$$\perp (\mathbf{A}, \mathbf{B}) \equiv A_x B_y - A_y B_x \quad (\text{A.7})$$

$$a = \perp (\mathbf{q}_0 - \mathbf{p}_0, d\mathbf{q} - d\mathbf{p}) \quad (\text{A.8})$$

$$b = \perp (\mathbf{q}_0 - \mathbf{p}_0, d\mathbf{p} - d\mathbf{r}) + \perp (\mathbf{p}_0 - \mathbf{r}_0, d\mathbf{q} - d\mathbf{p}) \quad (\text{A.9})$$

$$c = \perp (\mathbf{p}_0 - \mathbf{r}_0, d\mathbf{p} - d\mathbf{r}) \quad (\text{A.10})$$

Solving the quadratic equation yields two values for s , which, when inserted in Eq. A.5 give two corresponding values for t' . If any of the s lie in the interval $[0, 1]$ and the corresponding t' lies in the interval $[0, \Delta t]$ a collision has occurred. If there are two solutions within this interval, the one with smallest t' occurred first and is picked for further processing.