Multi-loop Hysteresis and Recursive Remnant Control

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DOI:
10.33612/diss.215199709

Publication date:
2022

Chapter 6

The recursive remnant control algorithm

Make everything as simple as possible, but not simpler.

—Albert Einstein

In this chapter, we study the control of hysteresis-based actuator systems where its remnant behavior (e.g., the remaining memory when the actuation signal is set to zero) must follow a desired reference point. For high-precision mechatronic systems, a number of novel actuator systems have been proposed that exploit the remnant output value of hysteretic systems. For instance, in [30, 45], a piezoelectric actuator with two stable configurations is developed. A commercial piezoelectric actuator, so-called PIRest, is developed and presented in [60]. We formulate firstly the remnant control problem using an input of a predefined form with modulating amplitude factors. Using this input, we present recursive algorithms for the output regulation of the hysteresis remnant behavior of two hysteresis operators: the Preisach hysteresis operator and the Duhem hysteresis operator. Under some mild conditions, we prove that our proposed algorithms guarantee that the output remnant converges to the desired value. Simulation results show the efficacy of our proposed algorithms.

6.1 The remnant control problem of the Preisach hysteresis operator

To introduce our formulation of the *remnant* control problem for the Preisach operator, let us start considering an input u defined on a time interval $[0,\tau]$ with $\tau > 0$ such that $u(0) = u(\tau) = 0$, and an initial interface $L_0 \in \mathscr{I}$ satisfying $(0,0) \in L_0$. When such input is applied to a Preisach operator in the form $\mathscr{P}(u, L_0)$, the final output value $y(\tau)$ may be different from the initial output value $y(0)$ due to the switching of some relays in the Preisach domain P which occurs as a result of the variations of u within the interval $[0,\tau]$. Let $L_\tau \in \mathscr{I}$ be the final interface which describes the state of relays in the Preisach operator at time instance $t = \tau$. It is clear that $(0,0) \in L_\tau$ (because $(u(\tau), u(\tau)) = (0,0)$). Consequently, when the input of the Preisach operator is restricted to satisfy $u(0) =$

$u(\tau) = 0$, the initial and final interfaces are contained in a subset of $\mathscr{I}$ defined by

$$\mathscr{I}_\varsigma := \{L \in \mathscr{I} \,|\, (0,0) \in L\}.$$

Note that the restriction $u(0) = u(\tau) = 0$ also compels relays whose (α, β) are in certain subdomains of P to have fixed initial and final states regardless the behavior of u within the interval $[0, \tau]$. Consider a subdomain of the Preisach plane defined by

$$P_\varsigma := \{(\alpha, \beta) \in P \mid \alpha \geq 0, \beta \leq 0\}.$$

We have that every interface in $\mathscr{I}_\varsigma$ lies entirely in P_ς. Consequently, relays whose (α, β) are not in the subdomain P_ς are restrained to the state -1 (resp. $+1$) at both time instances $t = 0$ and $t = \tau$ if they have $\beta > 0$ (resp. $\alpha < 0$). In other words, the set of relays $\mathscr{R}_{\alpha,\beta}$ which have different initial and final state due to the variation of the signal u in $(0, \tau)$ belongs to P_ς.

The *remnant* of the Preisach operator refers to the instantaneous value of the output $y(t)$ when the input value satisfies $u(t) = 0$ for some t. Roughly speaking, our remnant control problem corresponds to designing a feedforward control input u whose values at initial and terminal time are zero, and the corresponding output of the Preisach operator has the desired remnant value $\varsigma^* \in \mathbb{R}$ at the terminal time. To solve this problem, we propose a recursive algorithm based on an input of the form

$$u_\varsigma(t) := \sum_{k=0}^{\infty} w_k \rho_k(t) \tag{6.1}$$

where $k \in \mathbb{Z}_+$, $w_k \in \mathbb{R}$ and ρ_k is defined by

$$\rho_k(t) := \begin{cases} \frac{2}{\tau}(t - k\tau) & \text{if } k\tau \leq t \leq \left(k + \frac{1}{2}\right)\tau, \\ \frac{2}{\tau}\left(-t + (k+1)\tau\right) & \text{if } \left(k + \frac{1}{2}\right)\tau < t \leq (k+1)\tau, \\ 0 & \text{otherwise}, \end{cases} \tag{6.2}$$

with $\tau > 0$. The function ρ_k corresponds to a triangular pulse of unit amplitude and time length τ, which starts at $t = k\tau$ and finishes at $t = (k+1)\tau$ and whose peak value occurs at $t = \left(k + \frac{1}{2}\right)\tau$. Therefore, the input u_ς is a train of triangular pulses whose amplitudes are modulated by the factors w_k.

Assume that u_ς is applied as input to the Preisach operator and let $I_k \in \mathscr{I}_\varsigma$ be the interface that describes the state of the relays at time instance $t = k\tau$ (i.e. $I_k = L_{(k\tau)}$). We can compute the remnant by a function $\varsigma : \mathbb{R} \times \mathscr{I}_\varsigma \to \mathbb{R}$ defined by

$$\begin{aligned}\varsigma(w_k, I_k) &:= \left[\mathscr{P}(u_\varsigma, I_0)\right]((k+1)\tau) \\ &= \left[\mathscr{P}(w_k \rho_k, I_k)\right]((k+1)\tau) = \left[\mathscr{P}(w_k v_0, I_k)\right](\tau)\end{aligned} \tag{6.3}$$

In other words, the function ς gives the remnant after the application of the k-th triangular pulse of u_ς to a Preisach operator whose relays have initial states described by the interface I_0, or equivalently, the remnant after the application of a single triangular pulse with amplitude w_k to a Preisach operator whose relays have initial states described by the interface I_k. In this way, we formulate the remnant control problem as finding the sequence of values w_k that yields $\varsigma(w_k, I_k) \to \varsigma^*$ as $k \to \infty$.

6.2 The properties of the remnant rate of the Preisach hysteresis operator

We analyze in this section the behavior of the remnant when the triangular pulses of the input u_ς defined in (6.1) are applied to the Preisach operator. For this, we consider the difference of remnant between two consecutive triangular pulses of u_ς, which is defined by

$$\Delta_k \varsigma := \varsigma(w_{k+1}, I_{k+1}) - \varsigma(w_k, I_k). \tag{6.4}$$

Let us introduce the auxiliary functions

$$\begin{aligned}\ell_\beta^M(\alpha, L) &:= \max\left\{\beta \mid (\alpha, \beta) \in L\right\}, \\ \ell_\beta^m(\alpha, L) &:= \min\left\{\beta \mid (\alpha, \beta) \in L\right\}, \\ \ell_\alpha^M(\beta, L) &:= \max\left\{\alpha \mid (\alpha, \beta) \in L\right\}, \\ \ell_\alpha^m(\beta, L) &:= \min\left\{\alpha \mid (\alpha, \beta) \in L\right\},\end{aligned} \tag{6.5}$$

which are used in the following proposition to re-parameterize the coordinates (α, β) of the interface.

Proposition 6.1. *Consider the remnant difference $\Delta_k \varsigma$ defined in* (6.4). *For every $k \in \mathbb{Z}_+$,* *we have that*

$$\Delta_k \varsigma = \begin{cases} 2\int\limits_{M_{k+1}}^{w_{k+1}} \int\limits_{\ell_\beta^M(\alpha,I_{k+1})}^{0} \mu(\alpha,\beta)\,\mathrm{d}\beta\mathrm{d}\alpha, & \text{if } w_{k+1} > M_{k+1}, \\ -2\int\limits_{w_{k+1}}^{m_{k+1}} \int\limits_{0}^{\ell_\alpha^m(\beta,I_{k+1})} \mu(\alpha,\beta)\,\mathrm{d}\alpha\mathrm{d}\beta, & \text{if } w_{k+1} < m_{k+1}, \\ 0, & \text{otherwise}, \end{cases} \tag{6.6}$$

with

$$M_{k+1} = \ell_\alpha^M(0,I_{k+1}) \quad \text{and} \quad m_{k+1} = \ell_\beta^m(0,I_{k+1}).$$

Proof. Consider the case when $w_{k+1} > M_{k+1}$ and let P_{k+1}^+ and P_{k+1}^- be the subdomains of the Preisach domain P that are below and above the interface I_{k+1}, respectively (see Fig. 6.1a). Using these domains, the remnant of the Preisach operator at time instance $t = (k+1)\tau$ can be expressed by

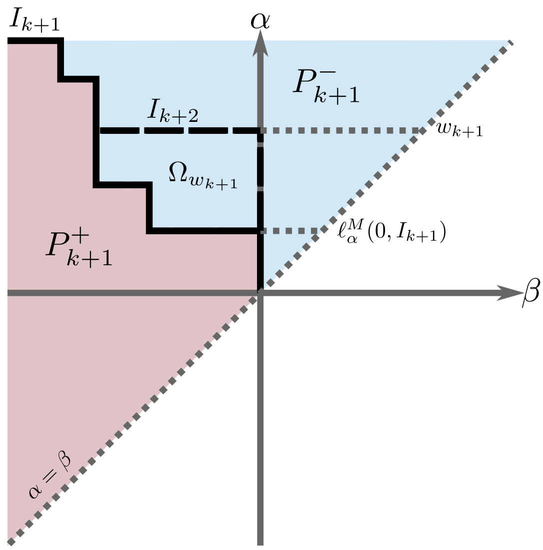
$$\varsigma(w_k,I_k) = \iint\limits_{P_{k+1}^+} \mu(\alpha,\beta)\,\mathrm{d}\alpha\mathrm{d}\beta - \iint\limits_{P_{k+1}^-} \mu(\alpha,\beta)\,\mathrm{d}\alpha\mathrm{d}\beta.$$

Note that the value $M_{k+1} = \ell_\alpha^M(0,I_{k+1})$ is the α-coordinate of the vertex in the interface I_{k+1} which corresponds to the last maximum of the input applied to the Preisach operator at time instance $t = (k+1)\tau$ (i.e. the last maximum of the truncated input $\{u_\varsigma(t)\,|\,0 \leq t \leq (k+1)\tau\}$). Therefore, since $w_{k+1} > M_{k+1}$, at the time instance $t = (k+2)\tau$ when the $(k+1)$-th triangular pulse finishes, there is a region $\Omega_{w_{k+1}} \subset P_{k+1}^-$ of relays whose states have switched from -1 to $+1$. This region is given by

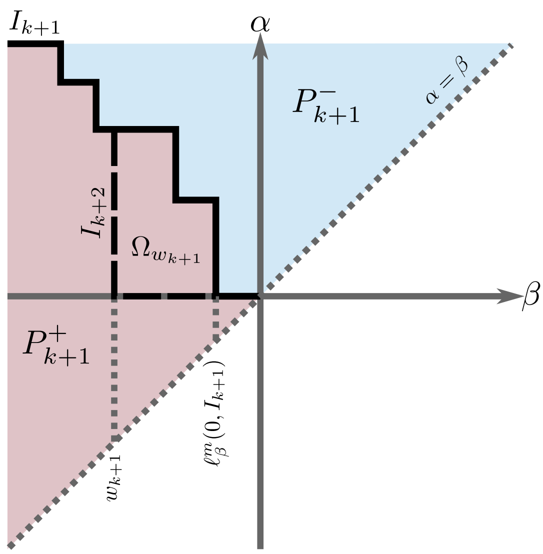
$$\Omega_{w_{k+1}} = \{(\alpha,\beta)\,|\,M_{k+1} \leq \alpha \leq w_{k+1},\ \ell_\beta^M(\alpha,I_{k+1}) \leq \beta \leq 0\}.$$

Consequently, it can be checked that the remnant of the Preisach operator at time instance $t = (k+2)\tau$ is given by

$$\begin{aligned} \varsigma(w_{k+1},I_{k+1}) = & \iint\limits_{P_{k+1}^+} \mu(\alpha,\beta)\,\mathrm{d}\alpha\mathrm{d}\beta - \iint\limits_{P_{k+1}^-} \mu(\alpha,\beta)\,\mathrm{d}\alpha\mathrm{d}\beta \\ & + 2\iint\limits_{\Omega_{w_{k+1}}} \mu(\alpha,\beta)\,\mathrm{d}\alpha\mathrm{d}\beta, \end{aligned}$$



(a) when $w_{k+1} > M_{k+1}$



(b) when $w_{k+1} < M_{k+1}$

Figure 6.1: Partition of the Preisach plane P used in Proposition 6.1 to compute $\Delta_k \varsigma := \varsigma(w_{k+1}, I_{k+1}) - \varsigma(w_k, I_k)$.

and subtracting both values of the remnant we have

$$\varsigma(w_{k+1},I_{k+1}) - \varsigma(w_k,I_k) = 2 \iint\limits_{\Omega_{w_{k+1}}} \mu(\alpha,\beta)\, \mathrm{d}\alpha \mathrm{d}\beta,$$

and from the definition of the region $\Omega_{w_{k+1}}$, the integral limits can be parameterized as follows

$$\Delta_k \varsigma = 2 \int\limits_{M_{k+1}}^{w_{k+1}} \int\limits_{\ell_\beta^M(\alpha,I_{k+1})}^{0} \mu(\alpha,\beta)\, \mathrm{d}\beta \mathrm{d}\alpha.$$

Consider now the case when $w_{k+1} < m_{k+1}$ and again let P_{k+1}^+ and P_{k+1}^- be the subdomains of the Preisach domain P that are below and above the interface I_{k+1}, respectively (see Fig. 6.1b). As in the previous case, the remnant of the Preisach operator at time instance $t = (k+1)\tau$ is given by

$$\varsigma(w_k,I_k) = \iint\limits_{P_{k+1}^+} \mu(\alpha,\beta)\, \mathrm{d}\alpha \mathrm{d}\beta - \iint\limits_{P_{k+1}^-} \mu(\alpha,\beta)\, \mathrm{d}\alpha \mathrm{d}\beta.$$

Observe that in this case the value $m_{k+1} = \ell_\beta^m(0,I_{k+1})$ is the β-coordinate of the vertex in the interface I_{k+1} which corresponds to the last minimum of the input applied to the Preisach operator at time instance $t = (k+1)\tau$ (i.e. the last minimum of the truncated input $\{u_\varsigma(t)\,|\,0 \le t \le (k+1)\tau\}$). Since in this case $w_{k+1} < m_{k+1}$, at the time instance $t = (k+2)\tau$ when the $(k+1)$-th triangular pulse finishes, the region $\Omega_{w_{k+1}} \subset P_{k+1}^+$ of relays whose states have switched from $+1$ to -1 is given by

$$\Omega_{w_{k+1}} = \{(\alpha,\beta)\,|\,0 \le \alpha \le \ell_\alpha^m(\beta,I_{k+1}),\, w_{k+1} \le \beta \le m_{k+1}\},$$

and the remnant of the Preisach operator at time instance $t = (k+2)\tau$ is given by

$$\begin{aligned}\varsigma(w_{k+1},I_{k+1}) = &\iint\limits_{P_{k+1}^+} \mu(\alpha,\beta)\, \mathrm{d}\alpha \mathrm{d}\beta - \iint\limits_{P_{k+1}^-} \mu(\alpha,\beta)\, \mathrm{d}\alpha \mathrm{d}\beta \\ &-2 \iint\limits_{\Omega_{w_{k+1}}} \mu(\alpha,\beta)\, \mathrm{d}\alpha \mathrm{d}\beta.\end{aligned}$$

Therefore, subtracting again both values of the remnant we have

$$\varsigma(w_{k+1},I_{k+1}) - \varsigma(w_k,I_k) = -2 \iint\limits_{\Omega_{w_{k+1}}} \mu(\alpha,\beta)\, \mathrm{d}\alpha \mathrm{d}\beta,$$

and parameterizing the limits of the integral over the region $\Omega_{w_{k+1}}$ we have

$$\Delta_k \varsigma = -2 \int_{w_{k+1}}^{m_{k+1}} \int_{0}^{\ell_\alpha^m(\beta, I_{k+1})} \mu(\alpha,\beta)\, \mathrm{d}\alpha \mathrm{d}\beta.$$

Finally, when $0 \leq w_{k+1} < M_{k+1}$ or $m_{k+1} < w_{k+1} \leq 0$, then $I_{k+1} = I_{k+2}$ and at both time instances $t = (k+1)\tau$ and $t = (k+2)\tau$ all relays in the Preisach domain P are in the same state which immediately implies $\varsigma(w_{k+1}, I_{k+1}) - \varsigma(w_k, I_k) = 0$. □

Based on the explicit expression of $\Delta_k \varsigma$ given by (6.6) in Proposition 6.1 and assuming that μ is compactly supported in a subset $P_\mu \subset P$, we can find sector bounds for $\Delta_k \varsigma$ as a function of $\Delta_k w = w_{k+1} - w_k$. In other words, we find that the rate of the remnant difference respect to the difference between two consecutive amplitudes w_{k+1} and w_k is bounded.

Proposition 6.2. *Let μ have a compact support $P_\mu \subset P$ whose intersection with P_ς is not empty (i.e. $P_\mu \cap P_\varsigma \neq \emptyset$), and consider $\Delta_k \varsigma$ as given by* (6.6). *Then there exist constants $\Gamma_{1_+} \leq \Gamma_{2_+}$ and $\Gamma_{1_-} \leq \Gamma_{2_-}$ such that*

$$\begin{aligned} \Gamma_{1_+} \Delta_k w \leq \Delta_k \varsigma \leq \Gamma_{2_+} \Delta_k w, \quad \textit{if}\ \Delta_k w > 0, \\ \Gamma_{1_-} \Delta_k w \leq \Delta_k \varsigma \leq \Gamma_{2_-} \Delta_k w, \quad \textit{if}\ \Delta_k w < 0, \end{aligned}$$

with $\Delta_k w = w_{k+1} - w_k$.

Proof. Following analysis from Proposition 6.1, assume that $w_{k+1} > M_{k+1} = \ell_\alpha^M(0, I_{k+1})$. Then by taking the maximum and minimum of the inner integral in the first case of (6.6), we define

$$\Gamma_{1_+} := 2 \min_{(\alpha,\beta_1) \in P_\mu \cap P_\varsigma} \int_{\beta_1}^{0} \mu(\alpha,\beta)\, \mathrm{d}\beta, \tag{6.7}$$

$$\Gamma_{2_+} := 2 \max_{(\alpha,\beta_1) \in P_\mu \cap P_\varsigma} \int_{\beta_1}^{0} \mu(\alpha,\beta)\, \mathrm{d}\beta. \tag{6.8}$$

Note that since $\beta_1 \leq 0$ for every $(\alpha, \beta_1) \in P_\varsigma$, then either one of the values (6.8) or (6.7) is zero (i.e. $\Gamma_{1_+} = 0$ or $\Gamma_{2_+} = 0$), or they have opposite signs (i.e. $\Gamma_{1_+} < 0 < \Gamma_{2_+}$).

Consequently, we find that

$$\Delta_k\varsigma \geq \int\limits_{M_{k+1}}^{w_{k+1}} \Gamma_{1_+} \mathrm{d}\alpha = \Gamma_{1_+}(w_{k+1} - M_{k+1}),$$

$$\Delta_k\varsigma \leq \int\limits_{M_{k+1}}^{w_{k+1}} \Gamma_{2_+} \mathrm{d}\alpha = \Gamma_{2_+}(w_{k+1} - M_{k+1}).$$

Moreover, since $M_{k+1} = \ell_\alpha^M(0, I_{k+1})$ is the α-coordinate of the vertex in the interface I_{k+1} corresponding to the last maximum of the truncated input $\{u_\varsigma(t) \mid 0 \leq t \leq (k+1)\tau\}$, then we have that $w_k \leq M_{k+1}$, which leads us to

$$\Gamma_{1_+}(w_{k+1} - w_k) \leq \Delta_k\varsigma \leq \Gamma_{2_+}(w_{k+1} - w_k).$$

Analogously, for the case $w_{k+1} < m_{k+1} = \ell_\beta^m(0, I_{k+1})$, we take the maximum and minimum of the inner integral in the second case of (6.6) and define

$$\Gamma_{1_-} := 2 \max_{(\alpha_1, \beta) \in P_\mu \cap P_\varsigma} \int\limits_0^{\alpha_1} \mu(\alpha, \beta)\, \mathrm{d}\alpha, \tag{6.9}$$

$$\Gamma_{2_-} := 2 \min_{(\alpha_1, \beta) \in P_\mu \cap P_\varsigma} \int\limits_0^{\alpha_1} \mu(\alpha, \beta)\, \mathrm{d}\alpha. \tag{6.10}$$

Similarly to the previous case, observe that since $\alpha_1 \leq 0$ for every $(\alpha_1, \beta) \in P_\varsigma$, then either one of the values (6.10) or (6.9) is zero (i.e. $\Gamma_{1_-} = 0$ or $\Gamma_{2_-} = 0$), or they have opposite signs (i.e. $\Gamma_{2_-} < 0 < \Gamma_{1_-}$). Therefore, in this case we have that

$$\Delta_k\varsigma \geq -\int\limits_{w_{k+1}}^{m_{k+1}} \Gamma_{1_-} \mathrm{d}\beta = -\Gamma_{1_-}(m_{k+1} - w_{k+1}),$$

$$\Delta_k\varsigma \leq -\int\limits_{w_{k+1}}^{m_{k+1}} \Gamma_{2_-} \mathrm{d}\beta = -\Gamma_{2_-}(m_{k+1} - w_{k+1}).$$

Furthermore, in this case $m_{k+1} = \ell_\beta^m(0, I_{k+1})$ is the β-coordinate of the vertex in the interface I_{k+1} corresponding to the last minimum of the truncated input $\{u_\varsigma(t) \mid 0 \leq t \leq (k+1)\tau\}$. Thus $w_k \geq m_{k+1}$ and we can obtain

$$\Gamma_{1_-}(w_{k+1} - w_k) \leq \Delta_k\varsigma \leq \Gamma_{2_-}(w_{k+1} - w_k).$$

Finally, when $m_{k+1} \leq w_{k+1} \leq M_{k+1}$ we have $\Delta_k \varsigma = 0$ and both inequalities hold with the same values defined in (6.7)-(6.10). □

Proposition 6.2 proves the existence of general sector bounds for $\Delta_k \varsigma$ as a function of $\Delta_k w$ disregarding the sign of μ. In the next proposition, we show that when μ is positive in a compact subset of P_ς, then under mild assumptions over the initial interface I_0 and the magnitude of every factor w_k, we have that $\Delta_k \varsigma$ is monotonic respect to $\Delta_k w$.

Proposition 6.3. *Assume that there exists a non-empty subdomain $Q \subseteq P_\mu \cap P_\varsigma$ of the form*

$$Q = \{(\alpha, \beta) \in P_\mu \cap P_\varsigma \mid 0 \leq \alpha \leq \alpha_2, \beta_2 \leq \beta \leq 0\}, \tag{6.11}$$

with $\alpha_2 > 0$ and $\beta_2 < 0$, such that $\mu(\alpha, \beta) \geq 0$ for every $(\alpha, \beta) \in Q$. Moreover, let the initial interface $I_0 \in \mathscr{I}_\varsigma$ be such that for every $(\alpha, \beta) \in I_0$ we have $\alpha \geq \alpha_2$ whenever $\beta \leq \beta_2$ and $\beta \leq \beta_2$ whenever $\alpha \geq \alpha_2$, and assume that $w_k \in [\beta_2, \alpha_2]$ for every $k \in \mathbb{Z}_+$. Then

$$0 \leq \frac{\Delta_k \varsigma}{\Delta_k w} \leq \max\left\{\Gamma^Q_{2_+}, \Gamma^Q_{1_-}\right\}, \qquad \textit{when } \Delta_k w \neq 0, \tag{6.12}$$

with

$$\Gamma^Q_{2_+} = 2 \max_{(\alpha,\beta_1)\in Q} \int_{\beta_1}^{0} \mu(\alpha, \beta)\, \mathrm{d}\beta, \tag{6.13}$$

$$\Gamma^Q_{1_-} = 2 \max_{(\alpha_1,\beta)\in Q} \int_{0}^{\alpha_1} \mu(\alpha, \beta)\, \mathrm{d}\alpha. \tag{6.14}$$

Proof. Note from the assumptions of the initial interface I_0 that none of its points lies in the subdomains $\{(\alpha, \beta) \mid \alpha > \alpha_2, \beta_2 < \beta \leq 0\}$ and $\{(\alpha, \beta) \mid \beta < \beta_2, 0 \leq \alpha < \alpha_2\}$. Moreover, since w_k is restricted to the interval $[\beta_2, \alpha_2]$ for every $k \in \mathbb{Z}_+$, then only the relays with $(\alpha, \beta) \in Q$ can be affected by the input u_ς defined in (6.1). Therefore, to find the sector bounds of $\Delta_k \varsigma$ as a function of $\Delta_k w$, it is enough to modify (6.7)-(6.10) to take the maximum and minimum over Q. Thus when $\mu(\alpha, \beta) \geq 0$, for every $(\alpha, \beta) \in Q$, we have that

$$\Gamma_{1_+}^{Q} = 2 \min_{(\alpha,\beta_1)\in Q} \int_{\beta_1}^{0} \mu(\alpha,\beta)\,\mathrm{d}\beta = 0,$$

$$\Gamma_{2_-}^{Q} = 2 \min_{(\alpha_1,\beta)\in Q} \int_{0}^{\alpha_1} \mu(\alpha,\beta)\,\mathrm{d}\alpha = 0,$$

and it follows that

$$\begin{aligned} 0 \le \Delta_k\varsigma &\le \Gamma_{2_+}^{Q}\Delta_k w, && \text{if } \Delta_k w > 0,\\ \Gamma_{1_-}^{Q}\Delta_k w \le \Delta_k\varsigma &\le 0, && \text{if } \Delta_k w < 0, \end{aligned}$$

which combined yield (6.12). □

We remark from Proposition 6.3 that in case the initial interface L_0 of a Preisach operator is unknown or does not satisfy the stated assumptions, it is possible to apply a single triangular pulse with amplitude either $w = \beta_2$ or $w = \alpha_2$ and to consider the new obtained interface, which will satisfy the assumptions, as the initial interface. Furthermore, when μ is negative in the set Q, an inequality to prove the monotonicity of $\Delta_k\varsigma$ respect to $\Delta_k w$ can be also obtained. However, in that case we would obtain values $\Gamma_{2_-}^{Q} \le 0$ and $\Gamma_{1_+}^{Q} \le 0$ such that $\min\left\{\Gamma_{2_-}^{Q}, \Gamma_{1_+}^{Q}\right\} \le \frac{\Delta_k\varsigma}{\Delta_k w} \le 0$.

6.3 The recursive algorithm for the remnant control of the Preisach hysteresis operator

In this section, we present the recursive control algorithm to compute w_{k+1} as a function of w_k and the error of the remnant after the k-th triangular pulse of u_ς. Our algorithm works for the case considered in Proposition 6.3 when there exists a compact subset $Q \subset P_\mu \cap P_\varsigma$ where μ is positive. The algorithm can easily be adapted to the case when μ is negative in a compact subset of $Q \subset P_\mu \cap P_\varsigma$. Before introducing the algorithm, we present the next lemma which provides a way to compute the maximum and minimum remnant that can be obtained from a Preisach operator whose weighting function and initial interface satisfy the conditions of Proposition 6.3.

Lemma 6.4. *Let $Q \subseteq P_\mu \cap P_\varsigma$ and $I_0 \in \mathscr{I}_\varsigma$ be a non-empty subdomain and initial interface, respectively, that satisfy conditions stated in Proposition 6.3. Then the maximum* *and minimum values of* ς *with the initial interface* I_0 *are given by*

$$\varsigma_{\max} = \max_{w\in[\beta_2,\alpha_2]} \varsigma(w,I_0) = \varsigma(\alpha_2,I_0), \tag{6.15}$$

$$\varsigma_{\min} = \min_{w\in[\beta_2,\alpha_2]} \varsigma(w,I_0) = \varsigma(\beta_2,I_0), \tag{6.16}$$

where α_2 *and* β_2 *are the values used for the definition of* Q *in* (6.11).

Proof. Note that since only relays with $(\alpha,\beta)\in Q$ can be affected by the input u_ς when $w\in[\beta_2,\alpha_1]$, and μ is positive in Q, then the maximum (resp. minimum) remnant possible is obtained when all relays in Q are in $+1$ state (resp. -1 state). It follows that after the application of a triangular pulse with amplitude $w=\alpha_2$ (resp. $w=\beta_2$), all relays in Q are in $+1$ state (resp. -1 state). □

Proposition 6.5. *Let* $Q\subseteq P_\mu\cap P_\varsigma$ *and* I_0 *be a non-empty subdomain and initial interface, respectively, that satisfy conditions stated in Proposition 6.3, and assume that* $w_0\in[\beta_2,\alpha_2]$ *and* $\varsigma^*\in[\varsigma_{\min},\varsigma_{\max}]$. *Consider the following update rule for the amplitude of the triangular pulse*

$$w_{k+1} = w_k - \kappa\varepsilon_k, \tag{6.17}$$

where $\varepsilon_k = \varsigma(w_k,I_k)-\varsigma^*$ *and* $\kappa>0$ *is the adaptation gain. If* κ *satisfies*

$$0<\kappa<\frac{2}{\max\left\{\Gamma^Q_{2_+},\Gamma^Q_{1_-}\right\}}, \tag{6.18}$$

then $\varepsilon_k\to 0$ *as* $k\to\infty$.

Proof. The remnant error after the application of the $(k+1)$-th triangular pulse is given by

$$\begin{aligned}\varepsilon_{k+1} &= \varsigma(w_{k+1},I_{k+1})-\varsigma^*\\ &= \varsigma(w_k,I_k)-\varsigma^*+\varsigma(w_{k+1},I_{k+1})-\varsigma(w_k,I_k)\\ &= \varepsilon_k+\Delta_k\varsigma,\end{aligned}$$

where $\Delta_k\varsigma$ is explicitly given by (6.6) in Proposition 6.1. Introducing $\Delta_k w = -\kappa\varepsilon_k$, we obtain

$$\varepsilon_{k+1} = \left(\varepsilon_k+\frac{\Delta_k\varsigma}{\Delta_k w}\Delta_k w\right) = \left(1-\kappa\frac{\Delta_k\varsigma}{\Delta_k w}\right)\varepsilon_k$$

which by Proposition 6.3 is a contraction mapping if κ is chosen to satisfy (6.18). □

6.4 Numerical example: Preisach butterfly hysteresis operator remnant control

To illustrate the application of the algorithm introduced in Proposition 6.5, we performed a simulation controlling the remnant of a particular class of Preisach operators known as the Preisach butterfly operator. The main characteristic of this class of Preisach operators is that its weighting function has disjoint subdomains of positive and negative values with a particular distribution and we refer interested readers to [26] for the details. In this numerical example, we used real data of the relation between electric-field and strain of a piezoelectric material sample made of doped Lead Zirconate Tinate (PZT) that exhibits the butterfly hysteresis loop on the left of Fig. 6.2. The measurements were taken by a laser interferometer applying triangular periodic inputs of 1400V of amplitude at a constant low frequency of 1Hz, which is significantly lower than the resonance frequency of the system for obtaining the rate-independent hysteresis measurement as in [59], and we fitted a weighting function to obtain the Preisach butterfly operator. For the obtained weighting function, the subdomain Q was approximated by $Q = \{(\alpha,\beta) \in P \mid -850 \leq \beta \leq 0, 0 \leq \alpha \leq 1400\}$, which is indicated by a dashed line enclosing a region of the weighting function illustrated in Fig. 6.2. We found for this Q that $\Gamma_{2_+} \approx 6.83$, $\Gamma_{1_-} \approx 5.50$, $\varsigma_{\text{max}} \approx 433.83$, and $\varsigma_{\text{min}} =\approx -141.96$, and the initial interface considered was $I_0 = \{(\alpha,\beta) \in P \mid \alpha = 1400, -\infty < \beta \leq -800\} \cup \{(\alpha,\beta) \in P \mid 0 \leq \alpha \leq 1400, \beta = -800\} \cup \{(\alpha,\beta) \in P \mid \alpha = 0, -800 \leq \beta \leq 0\}$. For simulation purpose, we took $\kappa = 0.28$ and $\varsigma^* = 250$ and used an input u_ς whose triangular pulses length was $\tau = 1$. We truncated it to zero after 20 steps (i.e. $u(t) = 0$ for $t \geq 20$) once the output remnant $\varsigma(w_k, I_k)$ was sufficiently close to ς^*. It can be observed in the simulation results of Fig. 6.3 that the output value $y(t) \approx \varsigma^*$ is maintained for $t \geq 20$ when the input u_ς has been removed.

6.5 The remnant control problem of the Duhem hysteresis operator

In this section, we formulate the remnant control problem for the Duhem hysteresis operator. We base our formulation on the existence of periodic solutions to the equation of the Duhem model. For this, let us recall from Section 5.1 that when a simple periodic input $u_p \in AC(\mathbb{R}_+,\mathbb{R})$ with maximum and minimum $\upsilon_{\text{max}}, \upsilon_{\text{min}} \in \mathbb{R}$ is applied to the Duhem hysteresis operator, the solution converges to a periodic orbit that can be parameterized by a pair of functions with the input instantaneous value υ and the initial condition as

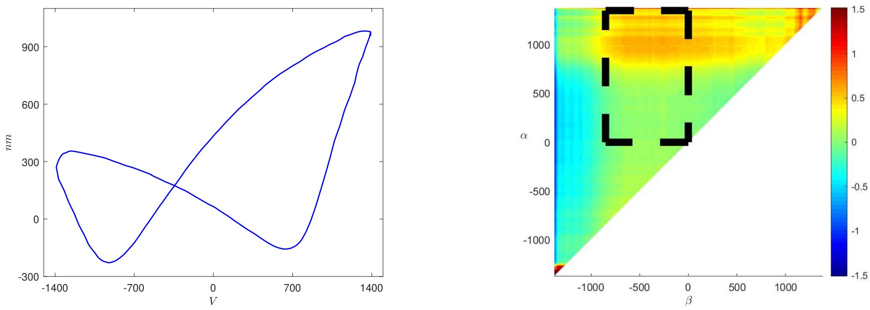


Figure 6.2: Experimental butterfly hysteresis loop exhibited in the relation between voltage (V) and strain (nm) of a piezoelectric material and the corresponding weighting function μ of the fitted Preisach butterfly operator with the region Q where μ is positive enclosed by the dashed line.

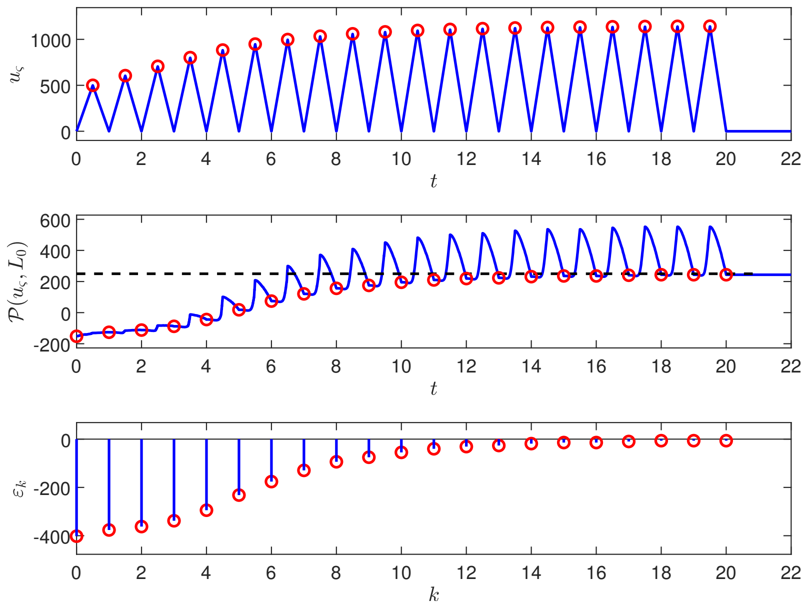


Figure 6.3: Simulation results for the first 20 steps of the algorithm controlling the remnant of a Preisach butterfly operator with an input u_ς whose triangular pulses length is $\tau = 1$. The upper plot shows the input $u_\varsigma(t)$ where the amplitude w_k of the k-th triangular pulse is marked in red. The middle plot corresponds to output $y(t)$ with the remnant $\varsigma(w_k, I_k)$ marked in red. The bottom plot shows the remnant error $\varepsilon_k = \varsigma(w_k, I_k) - \varsigma^*$.

independent variables in the form

$$\mathscr{Y}_{u_{p+}}(\upsilon,\gamma_{\min}) \qquad \text{and} \qquad \mathscr{Y}_{u_{p-}}(\upsilon,\gamma_{\max}),$$

where $u_{p+} \in AC([0,t_1],[\upsilon_{\min},\upsilon_{\max}])$ and $u_{p-} \in AC([t_1,T],[\upsilon_{\min},\upsilon_{\max}])$ are the monotonically increasing and monotonically decreasing sub-intervals of the input, respectively. Moreover, we have that $\gamma_{\max}$ and $\gamma_{\min}$ satisfy

$$\mathscr{Y}_{u_{p+}}(\upsilon_{\max},\gamma_{\min}) = \gamma_{\max} \qquad \text{and} \qquad \mathscr{Y}_{u_{p-}}(\upsilon_{\min},\gamma_{\max}) = \gamma_{\min}.$$

Roughly speaking, we formulate the remnant control problem of the Duhem hysteresis operator as finding a periodic input-output pair whose phase plot has two intersections with the axis $u = 0$ from which at least one of them is the desired remnant value ς^*. In other words, such that we have $(u(t_1),y(t_1)) = (0,y_1)$ and $(u(t_2),y(t_2)) = (0,\varsigma^*)$ for two time instances t_1 and t_2 within the periodic interval. We will present a solution to this remnant control problem fixing one of these intersection points to be given by one of the intersections of the axis $u = 0$ with the so-called major loop, which is parameterized by the pairs $(\upsilon,\mathscr{Y}_{u_{p+}}(\upsilon,\gamma_{\min}))$ and $(\upsilon,\mathscr{Y}_{u_{p-}}(\upsilon,\gamma_{\max}))$ with $\upsilon \in [\upsilon_{\min},\upsilon_{\max}]$, as described below.

We consider a Duhem hysteresis operator whose gradient functions satisfy conditions of Lemma 5.7 (i.e. the convergence to a periodic orbit is guaranteed when the input is periodic) and additionally are such that

$$f_1(\upsilon,\gamma) > 0 \qquad \text{and} \qquad f_2(\upsilon,\gamma) > 0, \tag{6.19}$$

for every $(\upsilon,\gamma) \in \mathbb{R}^2$. Since the input-output phase plot needs to cross the axis $u = 0$, we must have that $\upsilon_{\min} \leq 0 \leq \upsilon_{\max}$. Moreover, note that since the gradient functions are positive, then the minimum and maximum remnants attainable by a simple periodic input whose maximum and minimum are within the interval $[\upsilon_{\min},\upsilon_{\max}]$ are given by

$$\varsigma_{\min} = \mathscr{Y}_{u_{p+}}(0,\gamma_{\min}) \qquad \text{and} \qquad \varsigma_{\max} = \mathscr{Y}_{u_{p-}}(0,\gamma_{\max}),$$

which correspond to both intersections of the axis $u = 0$ with the major loop parameterized by $(\upsilon,\mathscr{Y}_{u_{p+}}(\upsilon,\gamma_{\min}))$ and $(\upsilon,\mathscr{Y}_{u_{p-}}(\upsilon,\gamma_{\max}))$.

Consider that the input u_ς defined in (6.1) is being applied to a Duhem hysteresis operator whose gradient functions satisfy (6.19). Since in our formulation the periodic input-output phase plot oscillates around the axis $u = 0$ and crosses it two times, we associate the modulating amplitudes u_ς in consecutive pairs such that $\upsilon_{\min} \leq w_{2j+1} \leq$

$0 \leq w_{2j} \leq \upsilon_{\max}$ for every $j \in \mathbb{Z}_+$. In this form, we can define two functions

$$\mathscr{Y}_{+,j}(\upsilon)\colon [\upsilon_{\min}, \upsilon_{\max}] \to [\gamma_{\min}, \gamma_{\max}],$$
$$\mathscr{Y}_{-,j}(\upsilon)\colon [\upsilon_{\min}, \upsilon_{\max}] \to [\gamma_{\min}, \gamma_{\max}],$$

that parameterize the Duhem operator output as function of the instantaneous input value correspondingly for every monotonically increasing and monotonically decreasing subinterval of u_ς. In other words $\mathscr{Y}_{+,j}$ and $\mathscr{Y}_{-,j}$ are such that

$$\mathscr{Y}_{+,j}(u_\varsigma(t)) = Y_{u_\varsigma}(t, y_0) = \left[\mathscr{D}(u_\varsigma, y_0)\right](t), \quad \text{for } t \in \left[\left(2j - \frac{1}{2}\right)\tau,\ \left(2j + \frac{1}{2}\right)\tau\right],$$
$$\mathscr{Y}_{-,j}(u_\varsigma(t)) = Y_{u_\varsigma}(t, y_0) = \left[\mathscr{D}(u_\varsigma, y_0)\right](t), \quad \text{for } t \in \left[\left(2j + \frac{1}{2}\right)\tau,\ \left(2j + \frac{3}{2}\right)\tau\right].$$

with a initial condition $y_0 \in [\varsigma_{\min}, \varsigma_{\max}]$. Note that $\mathscr{Y}_{+,j}$ and $\mathscr{Y}_{-,j}$ satisfy

$$\mathscr{Y}_{+,j}(w_{2j}) = \mathscr{Y}_{-,j}(w_{2j}), \qquad \text{and} \qquad \mathscr{Y}_{-,j}(w_{2j+1}) = \mathscr{Y}_{+,j+1}(w_{2j+1}). \tag{6.20}$$

Furthermore, assume that both functions can be extended to the whole input interval such that

$$\mathscr{Y}_{+,j}(\upsilon) = \int_{\upsilon_{\min}}^{\upsilon} f_1\left(\nu, \mathscr{Y}_{+,j}(\nu)\right)\, \mathrm{d}\nu + \mathscr{Y}_{+,j}(\upsilon_{\min}), \tag{6.21}$$
$$\mathscr{Y}_{-,j}(\upsilon) = \int_{\upsilon_{\max}}^{\upsilon} f_2\left(\nu, \mathscr{Y}_{-,j}(\nu)\right)\, \mathrm{d}\nu + \mathscr{Y}_{-,j}(\upsilon_{\max}), \tag{6.22}$$

for every $\upsilon \in [\upsilon_{\min}, \upsilon_{\max}]$. Note that the output remnants, which corresponds to the instantaneous output values at the moments when u_ς is zero, are given by

$$\varsigma_{-,j} = \mathscr{Y}_{-,j}(0) = \left[\mathscr{D}(u_\varsigma, y_0)\right]((2j+1)\tau) \quad \text{and} \quad \varsigma_{+,j} = \mathscr{Y}_{+,j}(0) = \left[\mathscr{D}(u_\varsigma, y_0)\right](2j\tau).$$

In this form, we formulate the control objective as driving one of the values $\varsigma_{-,j}$ or $\varsigma_{+,j}$ to the desired remnant value ς^*. Let us focus on finding the sequences of amplitudes $(w_{2j})_{j\in\mathbb{Z}_+}$ and $(w_{2j+1})_{j\in\mathbb{Z}_+}$ that yield $\varsigma_{-,j} \to \varsigma^*$ as $j \to \infty$ and consider that $\mathscr{Y}_{u_{p+}}(\upsilon, \gamma_{\min})$ is known for every $\upsilon \in [\upsilon_{\min}, \upsilon_{\max}]$. We can fix $\varsigma_{+,j} = \varsigma_{\min}$ for every $j \in \mathbb{Z}_+$ computing the amplitudes w_{2j+1} as the values that satisfy

$$\mathscr{Y}_{u_{p+}}(w_{2j+1}, \gamma_{\min}) = \mathscr{Y}_{-,j}(w_{2j+1}). \tag{6.23}$$

In other words, we fix the input-output phase plot to match with the major loop for every monotonically increasing sub-interval of the input of the form

$$\mathscr{Y}_{+,j+1}(\upsilon) = \mathscr{Y}_{u_{p+}}(\upsilon, \gamma_{\min}), \qquad \text{for every } \upsilon \in [\upsilon_{\min}, \upsilon_{\max}] \text{ and } j \in \mathbb{Z}_+. \tag{6.24}$$

6.6 The properties of the remnant rate of the Duhem hysteresis operator

We analyze the change of remnant when the input u_ς is applied to the Duhem hysteresis operator and the amplitudes w_{2j+1} for every $j \in \mathbb{Z}_+$ are computed to satisfy (6.23). For this, let us define the remnant difference

$$\begin{aligned}\Delta_{-,j}\varsigma :=& \varsigma_{-,j+1} - \varsigma_{-,j} \\ =& \mathscr{Y}_{-,j+1}(0) - \mathscr{Y}_{-,j}(0).\end{aligned} \tag{6.25}$$

In the next proposition, we introduce sector bounds for the change of remnant with respect to the change of amplitudes of triangular pulses.

Proposition 6.6. *Let w_{2j+1} satisfy* (6.23) *for every $j \in \mathbb{Z}_+$. Consider a compact and connected subdomain $Q \subset \mathbb{R}^2$ given by*

$$Q = \{\ (\upsilon, \gamma) \in \mathbb{R}^2 \mid (\upsilon, \gamma) \in [\upsilon_{\min}, \upsilon_{\max}] \times [\gamma_{\min} \gamma_{\max}]\ \}, \tag{6.26}$$

Then the change of remnant (6.25) *satisfies*

$$0 \leq \frac{\Delta_{-,j}\varsigma}{\Delta_{-,j}w} \leq \overline{f}_1, \qquad \textit{when } \Delta_{-,j}w \neq 0, \tag{6.27}$$

with $\Delta_{-,j}w = w_{2j+2} - w_{2j}$ and

$$\overline{f}_1 = \max_{(\upsilon,\gamma)\in Q} f_1(\upsilon, \gamma).$$

Proof. The remnant change after the application of the $(2j+2)$-th triangular pulse is given by

$$\begin{aligned}
\Delta_{-,j}\varsigma :=& \varsigma_{-,j+1} - \varsigma_{k,-} \\
=& \mathscr{Y}_{-,j+1}(0) - \mathscr{Y}_{-,j}(0) \\
=& -\int_0^{w_{2j+2}} f_2\left(\upsilon, \mathscr{Y}_{-,j+1}(\upsilon)\right)\, \mathrm{d}\upsilon + \mathscr{Y}_{-,j+1}(w_{2j+2}) \\
&\qquad + \int_0^{w_{2j}} f_2\left(\upsilon, \mathscr{Y}_{-,j}(\upsilon)\right)\, \mathrm{d}\upsilon - \mathscr{Y}_{-,j}(w_{2j}) \\
=& -\int_0^{w_{2j}} \left\{ f_2\left(\upsilon, \mathscr{Y}_{-,j+1}(\upsilon)\right) - f_2\left(\upsilon, \mathscr{Y}_{-,j}(\upsilon)\right)\right\}\, \mathrm{d}\upsilon \\
&\qquad - \int_{w_{2j}}^{w_{2j+2}} f_2\left(\upsilon, \mathscr{Y}_{-,j+1}(\upsilon)\right)\, \mathrm{d}\upsilon + \left(\mathscr{Y}_{-,j+1}(w_{2j+2}) - \mathscr{Y}_{-,j}(w_{2j})\right).
\end{aligned}$$

Consider the case $\Delta_{-,j}w \geq 0$ and note by (6.24) we have

$$\mathscr{Y}_{+,j+1}(w_{2j+2}) - \mathscr{Y}_{+,j}(w_{2j}) = \int_{w_{2j}}^{w_{2j+2}} f_1\left(\upsilon, \mathscr{Y}_{u_{p+}}(\upsilon, \gamma_{\min})\right)\, \mathrm{d}\upsilon \geq 0.$$

It follows from (6.20) and Lemma 5.1 that we have $\mathscr{Y}_{-,j+1}(\upsilon) - \mathscr{Y}_{-,j}(\upsilon) \geq 0$ for every $\upsilon \in [0, w_{2j+2}]$. Therefore, we can bound $\Delta_{-,j}\varsigma$ as follows

$$\begin{aligned}
\Delta_{-,j}\varsigma \leq & -\int_0^{w_{2j}} \left\{ f_2\left(\upsilon, \mathscr{Y}_{-,j+1}(\upsilon)\right) - f_2\left(\upsilon, \mathscr{Y}_{-,j}(\upsilon)\right)\right\}\, \mathrm{d}\upsilon \\
&\qquad - \int_{w_{2j}}^{w_{2j+2}} f_2\left(\upsilon, \mathscr{Y}_{-,j+1}(\upsilon)\right)\, \mathrm{d}\upsilon + \overline{f}_1 \Delta_{-,j}w
\end{aligned}$$

and note that by condition (5.14) the integrals in the previous expression are positive for every $\upsilon \in [\upsilon_{\min}, \upsilon_{\max}]$. Consequently, removing the negative terms we obtain

$$\Delta_{-,j}\varsigma \leq \overline{f}_1 \Delta_{-,j}w.$$

Consider now the case $\Delta_{-,j}w \leq 0$ and note again that by (6.24) we have in this case

$$\mathscr{Y}_{+,j+1}(w_{2j+2}) - \mathscr{Y}_{+,j}(w_{2j}) = \int_{w_{2j}}^{w_{2j+2}} f_1\left(\upsilon, \mathscr{Y}_{u_{p+}}(\upsilon, \gamma_{\min})\right)\, \mathrm{d}\upsilon \leq 0.$$

Again, it follows from (6.20) and Lemma 5.1 that we have $\mathscr{Y}_{-,j+1}(\upsilon) - \mathscr{Y}_{-,j}(\upsilon) \leq 0$ for every $\upsilon \in [0, w_{2j+2}]$. Therefore, we can bound $\Delta_{-,j}\varsigma$ as follows

$$\Delta_{-,j}\varsigma \geq -\int_0^{w_{2j}} \left\{ f_2\left(\upsilon, \mathscr{Y}_{-,j+1}(\upsilon)\right) - f_2\left(\upsilon, \mathscr{Y}_{-,j}(\upsilon)\right) \right\} \,\mathrm{d}\upsilon \\ - \int_{w_{2j}}^{w_{2j+2}} f_2\left(\upsilon, \mathscr{Y}_{-,j+1}(\upsilon)\right) \,\mathrm{d}\upsilon + \overline{f}_1 \Delta_{-,j} w$$

and note that by condition (5.14) the integrals in the previous expression are negative for every $\upsilon \in [\upsilon_{\min}, \upsilon_{\max}]$. Consequently, removing the positive terms we obtain

$$\Delta_{-,j}\varsigma \geq \overline{f}_1 \Delta_{-,j} w.$$

Finally, combining the result for both cases yields (6.27). □

6.7 The recursive algorithm for the remnant control of the Duhem hysteresis operator

In this section, we present the proof that a recursive algorithm with the update rule similar to (6.17) can guarantee the convergence of the Duhem hysteresis operator remnant to a desired reference value under mild assumptions. The algorithm is presented in the form of a proposition below.

Proposition 6.7. *Let $\varsigma_* \in [\varsigma_{\min}, \varsigma_{\max}]$ and assume w_{2j+1} satisfy* (6.23) *for every $j \in \mathbb{Z}_+$. Consider the following update rule for the amplitude w_{2j} with $j \in \mathbb{Z}_+$ of the triangular pulses*

$$w_{2j+2} = w_{2j} - \kappa \varepsilon_{-,j}, \tag{6.28}$$

where $\varepsilon_{-,j} = \varsigma_{-,j} - \varsigma^*$ and $\kappa > 0$ is the adaptation gain. If κ satisfies

$$0 < \kappa < \frac{2}{\overline{f}_1}, \tag{6.29}$$

then $\varepsilon_{-,j} \to 0$ as $k \to \infty$.

Proof. The remnant error after the application of the $(2j+2)$-th triangular pulse is given by

$$\begin{aligned} \varepsilon_{-,j+1} &= \varsigma_{-,j+1} - \varsigma^* \\ &= \varsigma_{-,j+1} - \varsigma^* + \varsigma_{-,j} - \varsigma_{-,j} \\ &= \varepsilon_{-,j+1} + \Delta_{-,j}\varsigma. \end{aligned}$$

Introducing $\Delta_{-,j}w = -\kappa\varepsilon_{-,j}$ we obtain

$$\varepsilon_{-,j+1} = \left(\varepsilon_{-,j} + \frac{\Delta_{-,j}\varsigma}{\Delta_{-,j}w}\Delta_{-,j}w\right) = \left(1 - \kappa\frac{\Delta_{-,j}\varsigma}{\Delta_{-,j}w}\right)\varepsilon_{-,j}$$

which by (6.27) in Proposition 6.6 is a contraction mapping if κ is chosen to satisfy (6.29). □

6.8 Numerical experiment: Miller hysteresis model remnant control

In this section, we present a simulation of the remnant control algorithm for a particular class of Duhem hysteresis operator. The Miller model introduced in [44] is a class of Duhem model that is used to described the relation between electric field and polarization in ferroelectric capacitors, and which is given by

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \begin{cases} \Xi^+(E,P)\dfrac{\partial P^+_{\mathrm{sat}}}{\partial E}\dfrac{\mathrm{d}E}{\mathrm{d}t}, & \text{if } \frac{\mathrm{d}E}{\mathrm{d}t} \geq 0, \\ \Xi^-(E,P)\dfrac{\partial P^-_{\mathrm{sat}}}{\partial E}\dfrac{\mathrm{d}E}{\mathrm{d}t}, & \text{if } \frac{\mathrm{d}E}{\mathrm{d}t} < 0, \end{cases} \tag{6.30}$$

where P is polarization, E is electric field, P^+_{sat} and P^-_{sat} are functions that describe the saturation polarization, and

$$\Xi^+(E,P) = 1 - \tanh\left(\tfrac{P-P^+_{\mathrm{sat}}(E)}{P_s-P_d}\right), \qquad \Xi^-(E,P) = 1 - \tanh\left(\tfrac{P-P^-_{\mathrm{sat}}(E)}{-P_s-P_d}\right),$$
$$P^+_{\mathrm{sat}}(E) = P_s\tanh\left(\tfrac{E-E_c}{2\delta}\right), \qquad P^-_{\mathrm{sat}}(E) = -P_s\tanh\left(\tfrac{-E-E_c}{2\delta}\right),$$
$$\delta = E_c\left(\ln\frac{1+\frac{P_r}{P_s}}{1-\frac{P_r}{P_s}}\right)^{-1}.$$

We can re-express (6.30) as a Duhem model of the form (2.4) whose input and output are the electric field and the polarization, respectively, and whose gradient functions f_1 and f_2 are given by

$$f_1(E,P) := \Xi^+(E,P)\frac{\partial P^+_{\mathrm{sat}}}{\partial E}, \tag{6.31}$$

$$f_2(E,P) := \Xi^-(E,P)\frac{\partial P^-_{\mathrm{sat}}}{\partial E}. \tag{6.32}$$

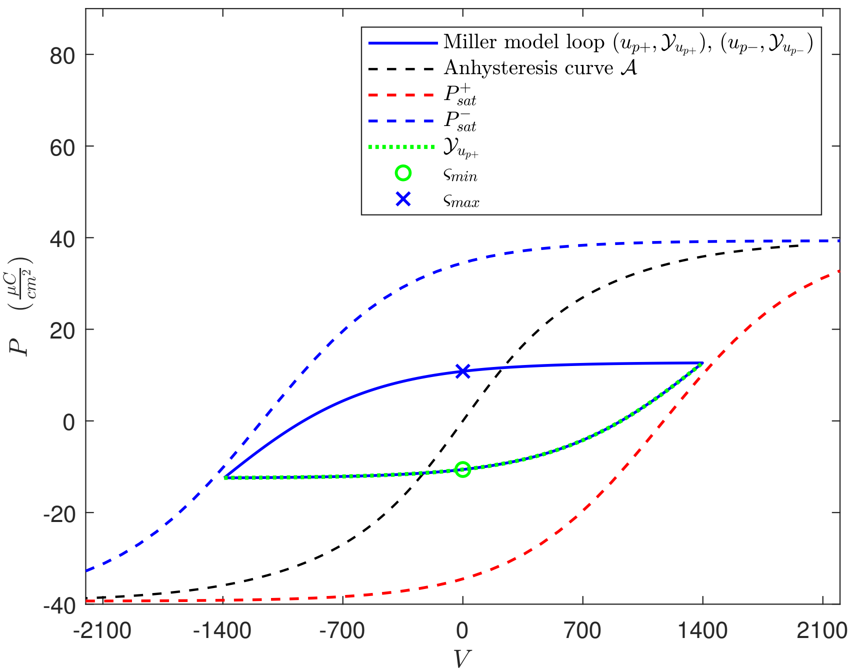


Figure 6.4: Hysteresis loop of the identified Miller model that relates the Voltage $(E \cdot cm)$ and Polarization $\left(\frac{\mu C}{cm^2}\right)$ of a piezoelectric material.

In this numerical example, we used real data of the relation between electric-field and polarization of the same piezoelectric material sample as used in Section 6.4, which was made of doped Lead Zirconate Tinate (PZT). The measurements were taken by a laser interferometer applying triangular periodic inputs of 1400V of amplitude at constant low frequency of 1Hz, which is significantly lower than the resonant frequency of the system for obtaining the rate-independent hysteresis measurement as in [59], and we identified the Miller model parameters $P_s = 39.33$, $P_r = 34.50$, $E_c = 1171.40$. For the simulation, we took $\kappa = 30$ and $\varsigma^* = 6$ and used an input u_ς whose triangular pulses length was $\tau = 1$. We truncated it to zero after 18 steps once the output was sufficiently close to ς^* and the error was negligible. It can be observed in the simulation results of Fig. 6.5 that the output value $y(t) \approx \varsigma^*$ is maintained for $t \geq 18$.

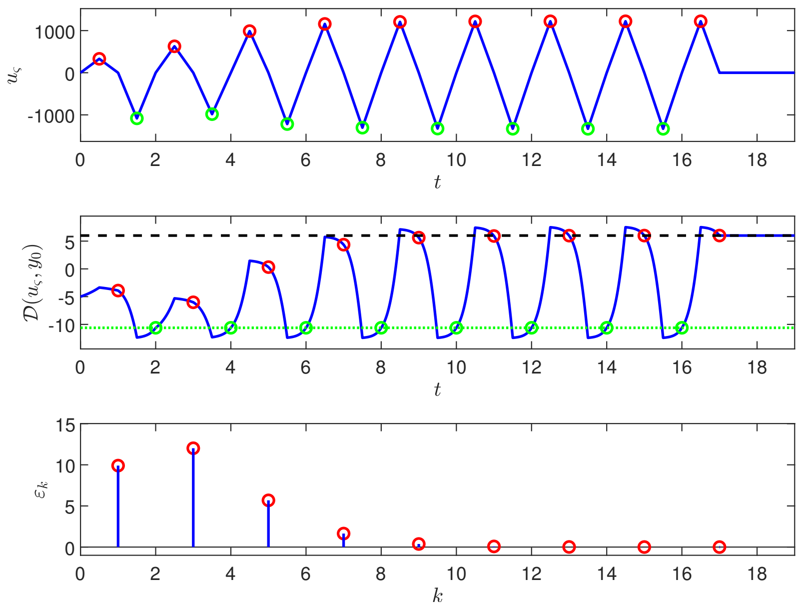


Figure 6.5: Simulation results for the first 18 steps of the algorithm controlling the remnant of a Duhem hysteresis operator with an input u_ς whose triangular pulses have time length $\tau = 1$. The upper plot shows the input $u_\varsigma(t)$ where the amplitudes w_{2j} are marked in red and w_{2j+1} are marked in green, for every $k = 2j \in [0, 18]$. The middle plot corresponds to output $y(t)$ where the remnant after the application of the $(2j)$-th triangular pulse is marked in red and the reference ς^* is indicated by a black dashed line. Moreover, the remnant after every $(2j+1)$-th triangular pulse is marked in green and $\varsigma_{\min}$ indicated by a green dotted line. The bottom plot shows the remnant error ε_{2j}.

6.9 Conclusions

In this chapter, we presented a formulation for the problem of controlling the *remnant* of a system with hysteresis modeled by a Preisach hysteresis operator and by a Duhem hysteresis operator. Using a train of triangular pulses as the kernel of the remnant control input u, we analyzed the properties of output remnant sequences due to the application of this family of input signals to both hysteresis operators. Moreover, we presented recursive algorithms for both operators to update the amplitude of the triangular pulse sequences and guarantee the convergence of the output remnant sequence to a desired remnant value under some mild conditions.