Multi-loop Hysteresis and Recursive Remnant Control

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DOI:
10.33612/diss.215199709

Publication date:
2022

Chapter 2

Preliminaries

I seem to have been only like a boy playing on the seashore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me.

—Isaac Newton

In this chapter, we introduce the general notation and theoretical preliminaries that will be used throughout this thesis. The notations table is included in Table 2.1. We also introduce the definition of hysteresis operator and auxiliary concepts such as time-transformation and rate-independent mapping, which assist us in the formalization. We will introduce two of the main hysteresis operators in literature: The Preisach hysteresis operator and the Duhem hysteresis operator.

2.1 Hysteresis operators

In this thesis, the word operator will always refer to a mapping between spaces of functions. There are different mathematical formulations of hysteresis operators in literature. A generic definition is presented in [38] which defines the hysteresis operator by causality and rate-independency properties. A stronger definition is presented in [47,51] where the consistency of the hysteresis loop is also part of the definition. While the latter definition captures the main nonlinear behaviors of hysteretic systems, it also admits linear systems described by partial differential equations as studied in [13]. Throughout this thesis, we define a hysteresis operator following the formulation of [38], which refers to it as a causal and rate-independent mapping between the space of continuous functions. Therefore, to state the definition of a hysteresis operator, we introduce below the three auxiliary concepts of time-transformation, rate-independent operator, and causal operator.

Definition 2.1. *A function* $\phi : \mathbb{R}_+ \to \mathbb{R}_+$ *is called a* time-transformation *if* $\phi(t)$ *is continuous and increasing with* $\phi(0) = 0$ *and* $\lim_{t\to\infty} \phi(t) = \infty$. △

Table 2.1: Notations Table

$\mathbb{R}$	Real numbers
$\mathbb{R}_+$	Positive real numbers including zero
$\mathbb{Z}$	Integer numbers
$\mathbb{Z}_+$	Positive integer numbers including zero
$\text{card}(U)$	Cardinality of the set U
$f\colon U \to Y$	Function from set U to set Y
$C_{\text{pw}}(U,Y)$	Space of piecewise continuous functions $f\colon U \to Y$
$C(U,Y)$	Space of continuous functions $f\colon U \to Y$
$AC(U,Y)$	Space of absolute continuous functions $f\colon U \to Y$
Φ	Mapping between two spaces (for instance $\Phi : AC(\mathbb{R}_+,\mathbb{R}) \to AC(\mathbb{R}_+,\mathbb{R})$)
$[\Phi(\cdots)](t)$	Evaluation of operator output function at time instance t
$\|\|x\|\|$	Euclidean norm o a vector
$[A]_{i,j}$	(i,j)-th element of a matrix A
$A^{\dagger}$	Moore-Penrose pseudo inverse of matrix A
$a_{[1:n]}$	Column vector with elements $a_1,\ldots,a_n$
$a_{[1:n,1:m]}$	Matrix with elements $a_{1,1},\ldots,a_{n,m}$
$\text{diag}\{d_1,\cdots,d_n\}$	Diagonal matrix whose diagonal elements are $d_1,\ldots,d_n$

A time-transformation is then a function that mantains the properties of monotonicity and unboudedness when it is applied in composition to another function.

Definition 2.2. *An operator Φ is said to be* rate independent *if*

$$\big[\Phi(u\circ\phi)\big] = \big[\Phi(u)\big]\circ\phi$$

holds for all $u \in AC(\mathbb{R}_+,\mathbb{R})$ and all admissible time transformation ϕ. △

Definition 2.3. *The operator Φ is said to be* causal *if for all $\tau > 0$ and all $u_1, u_2 \in AC(\mathbb{R}_+,\mathbb{R})$ it holds that*

$$u_1(t) = u_2(t) \quad \forall t \in [0,\tau] \qquad \Rightarrow \big[\Phi(u_1)\big](t) = \big[\Phi(u_2)\big](t) \quad \forall t \in [0,\tau].$$

△

We can now introduce formally what we mean by a hysteresis operator following the work of [38] on analysis of systems with hysteresis in the feedback loop.

Definition 2.4. *An operator Φ is called a* hysteresis operator *if Φ is causal and rate-independent.* △

2.2 The clockwise and counterclockwise relay operator

One of the simplest operators that can be found in literature is the relay operator which we introduce in this section. We define the counterclockwise relay operator $\mathscr{R}^{\circlearrowleft}_{\alpha,\beta} : AC(\mathbb{R}_+,\mathbb{R}) \times \{-1,1\} \to C_{\mathrm{pw}}(\mathbb{R}_+,\mathbb{R})$ with switching parameters $\alpha > \beta$ and initial condition r_0 by

$$\left[\mathscr{R}^{\circlearrowleft}_{\alpha,\beta}(u,r_0)\right](t) := \begin{cases} +1 & \text{if } u(t) > \alpha, \\ -1 & \text{if } u(t) < \beta, \\ \left[\mathscr{R}^{\circlearrowleft}_{\alpha,\beta}(u,r_0)\right](t_-) & \begin{array}{l}\text{if } \beta \le u(t) \le \alpha, \\ \quad \text{and } t > 0,\end{array} \\ r_0 & \begin{array}{l}\text{if } \beta \le u(t) \le \alpha, \\ \quad \text{and } t = 0.\end{array} \end{cases} \tag{2.1}$$

Formally, to create a hysteresis operator Φ in the sense of Definition 2.4, one must fix the initial condition $r_0 \in \{-1,+1\}$ of the counterclockwise relay operator and then note that

$$\Phi(u) = \mathscr{R}^{\circlearrowleft}_{\alpha,\beta}(u,\, r_0).$$

Analogous to its counterclockwise version, a clockwise relay operator $\mathscr{R}^{\circlearrowright}_{\alpha,\beta} : AC(\mathbb{R}_+,\mathbb{R}) \times \{-1,1\} \to C_{\mathrm{pw}}(\mathbb{R}_+,\mathbb{R})$ with switching parameters $\alpha > \beta$ and initial condition r_0 can be defined by

$$\left[\mathscr{R}^{\circlearrowright}_{\alpha,\beta}(u,r_0)\right](t) := \begin{cases} -1 & \text{if } u(t) > \alpha, \\ +1 & \text{if } u(t) < \beta, \\ \left[\mathscr{R}^{\circlearrowright}_{\alpha,\beta}(u,r_0)\right](t_-) & \begin{array}{l}\text{if } \beta \le u(t) \le \alpha, \\ \quad \text{and } t > 0,\end{array} \\ -r_0 & \begin{array}{l}\text{if } \beta \le u(t) \le \alpha, \\ \quad \text{and } t = 0.\end{array} \end{cases} \tag{2.2}$$

Similarly, a hysteresis operator Φ can be created from a clockwise relay operator $\mathscr{R}^{\circlearrowright}_{\alpha,\beta}$ fixing its initial condition $r_0 \in \{-1,+1\}$ and observing that

$$\Phi(u) = \mathscr{R}^{\circlearrowright}_{\alpha,\beta}(u,\, r_0).$$

For explanatory purposes, an illustration of the input-output phase plot of both relay operators is included in Figures 2.1 and 2.2. We also remark an important property of both relay operators below.

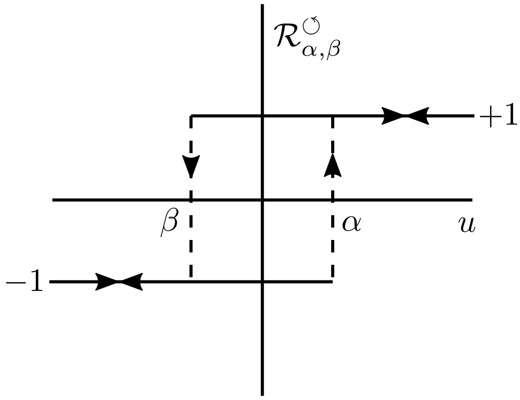

Figure 2.1: Input-output phase plot of the counterclockwise relay operator $\mathscr{R}^{\circlearrowleft}_{\alpha,\beta}$ as defined in (2.1)

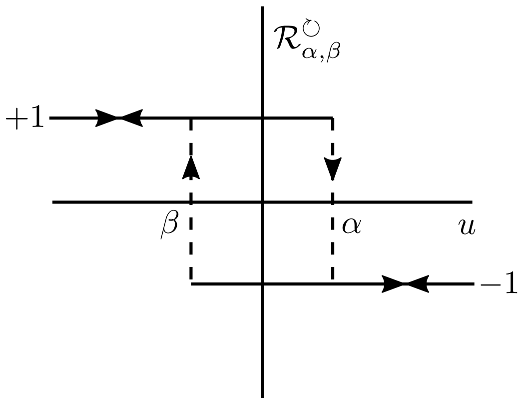

Figure 2.2: Input-output phase plot of the clockwise relay operator $\mathscr{R}^{\circlearrowright}_{\alpha,\beta}$ as defined in (2.2)

Remark 2.5. *For the same initial condition $r_0 \in \{-1,+1\}$, we have that*

$$\mathscr{R}^{\circlearrowleft}_{\alpha,\beta} = -\mathscr{R}^{\circlearrowright}_{\alpha,\beta}. \tag{2.3}$$

In other words, the output of a counterclockwise relay operator is equal to the negative output of a clockwise operator given that the same input function $u \in AC(\mathbb{R}_+,\mathbb{R})$ is applied to both of them.

2.3 The Preisach hysteresis operator

The Preisach hysteresis operator is one of the well-known hysteresis operators in the literature. The Preisach hysteresis operator definition takes inspiration from the idea that separate magnetic domains or particles within a ferromagnetic material can switch between two states. It also considers that different domains or particles have different distributions of reversal fields. Following this idea, the Preisach hysteresis operator is, roughly speaking, the weighted integral of all infinitesimal relay operators, which are also called *hysterons*, with switching parameters satisfying $\alpha > \beta$. The formal definition of the Preisach hysteresis operator requires the introduction of two more concepts. Firstly, we introduce the admissible plane of relay operators given by

$$P := \{(\alpha, \beta) \in \mathbb{R}^2 \mid \alpha > \beta\},$$

which is commonly referred to as the Preisach plane. Secondly, we denote the set of *interfaces* which roughly speaking is the set of monotonically decreasing staircase curves $\mathscr{I}$ where every element $L \in \mathscr{I}$ can be paremeterized by a function $\sigma(\gamma) \in C(\mathbb{R}_+, P)$ in the form

$$L = \{\sigma(\gamma) \in P \mid \gamma \in \mathbb{R}_+\},$$

which satisfies $\lim_{\gamma\to\infty}\|\sigma(\gamma)\| = \infty$ and $\sigma(0) = (\alpha, \alpha)$ for some $\alpha \in \mathbb{R}$. By monotonically decreasing L we mean that for every pair $(\alpha_1, \beta_1), (\alpha_2, \beta_2) \in L$ we have that $\beta_1 \leq \beta_2$ implies $\alpha_1 \geq \alpha_2$. Based on these concepts, the Preisach hysteresis operator $\mathscr{P} : AC(\mathbb{R}_+, \mathbb{R}) \times \mathscr{I} \to AC(\mathbb{R}_+, \mathbb{R})$ is formally expressed by

$$\big[\mathscr{P}(u, L_0)\big](t) := \iint\limits_{(\alpha,\beta)\in P} \mu(\alpha, \beta)\ \big[\mathscr{R}^{\circlearrowleft}_{\alpha,\beta}(u, r_{\alpha,\beta}(L_0))\big](t)\ \mathrm{d}\alpha \mathrm{d}\beta$$

where $\mu \in C_{\mathrm{pw}}(P, \mathbb{R})$ is a weighting function, $L_0 \in \mathscr{I}$ is the initial interface and $r_{\alpha,\beta} : \mathscr{I} \to \{-1, +1\}$ is an auxiliary function that determines the initial condition of every relay $\mathscr{R}^{\circlearrowleft}_{\alpha,\beta}$ according to its position $(\alpha, \beta) \in P$ with respects to the initial interface L_0. This last function is defined by

$$r_{\alpha,\beta}(L_0) := \begin{cases} +1 & \text{if } L_0 \cap \{(\alpha_1, \beta_1) \in P \mid \alpha \leq \alpha_1,\, \beta \leq \beta_1\} \neq \emptyset, \\ -1 & \text{otherwise.} \end{cases}$$

From the previous definition, it can be observed that the value of the function $r_{\alpha,\beta}$ will be $+1$ if the point $(\alpha, \beta) \in P$ is above the initial interface L_0, and will be -1 if the point $(\alpha, \beta) \in P$ is below the initial interface L_0.

Remark 2.6. *It is important to note from* (2.1) *that the actual initial state of a relay operator* $\mathscr{R}^{\circlearrowleft}_{\alpha,\beta}$ *is determined by* r_0 *only when* $\beta \leq u(0) \leq \alpha$*. This can produce an inconsistency between the values of the function* $r_{\alpha,\beta}(L_0)$ *and the actual initial state of relay operators* $\left[\mathscr{R}^{\circlearrowleft}_{\alpha,\beta}(u, r_{\alpha,\beta}(L_0))\right](0)$ *with* $\alpha < u(0)$ *or* $\beta > u(0)$*. Therefore, for well-posedness, we assume that the initial interface* L_0 *in the Preisach hysteresis operator* (2.3) *always satisfies* $(u(0), u(0)) \in L_0$*.*

As with the relay operator, the Preisach hysteresis operator is a hysteresis operator in the sense of Definition 2.4 for specified initial conditions $L_0 \in \mathscr{I}$, in the form

$$\Phi(u) = \mathscr{P}(u, L_0).$$

In simple words, it can be said that the output of the Preisach hysteresis operator is determined instantaneously with the variations of the input as all the relays in P react instantaneously and simultaneously to the applied input u. For this reason, as explained in [42], the initial interface L_0 evolves continuously and at every time instance $t \geq 0$ there exists an interface $L_t \in \mathscr{I}$ that divides the Preisach plane into two subdomains P_t^+ and P_t^- where all relays $\mathscr{R}_{\alpha,\beta}$ with $(\alpha, \beta) \in P_t^+$ are in state $+1$ while all relays $\mathscr{R}_{\alpha,\beta}$ with $(\alpha, \beta) \in P_t^-$ are in state -1 [42].

Remark 2.7. *We remark that although* L_t *is dependent on the input signal* u*, we remove its dependence to* u *in its notation for conciseness.*

The memory behavior and geometric interpretation of the Preisach hysteresis operator defined by (2.3) has been studied well in literature. An illustration of the Preisach plane P and its partition into two subdomains P_t^+ and P_t^- at a time instant t by an interface $L_t \in \mathscr{I}$ is presented in Figure 2.3. A similar formulation where the Preisach plane P is rotated by $-3\pi/4$ can be found in [11].

2.4 The Duhem hysteresis operator

The Duhem model [23] is a hysteresis model that consists of integro-differential equations. Its general form encompasses many other hysteresis models based on integro-differential equations such as the Dahl model, the Bouc-Wen model, and the Maxwell-slip model [40]. Roughly speaking, the Duhem model maps input signals to output signals via switched nonlinear differential equations where the switching signal depends on the sign of the derivative of its input signal. Control systems properties, where the Duhem hysteresis operator is feedback interconnected with other nonlinear systems, have been studied in the literature. For instance, the study of dissipativity in a class of Duhem hysteresis operators is presented in [27–29], where the associated storage

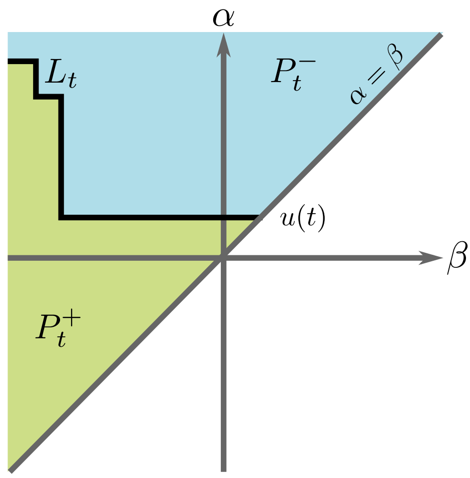

Figure 2.3: Preisach plane P with an interface L_t at time instance t partitioning it into the regions P_t^+ (indicated in green) and P_t^- (indicated in blue), respectively, where relays are in state $+1$ and -1, respectively.

functions and supply rate functions depend on the specific hysteresis loops obtained from the Duhem models. The Duhem hysteresis operator, which is the mapping that assigns the solutions of the switched differential equation for a given input, is given by $\mathscr{D}: AC(\mathbb{R}_+,\mathbb{R}) \times \mathbb{R} \to AC(\mathbb{R}_+,\mathbb{R})$ such that $y = \mathscr{D}(u,y_0)$ satisfies

$$\begin{aligned} \dot{y}(t) &= \begin{cases} f_1(u(t),y(t))\dot{u}, & \text{if } \dot{u}(t) \geq 0, \\ f_2(u(t),y(t))\dot{u}, & \text{if } \dot{u}(t) < 0, \end{cases} \\ y(0) &= y_0, \end{aligned} \tag{2.4}$$

at almost every $t \geq 0$ and with $f_1, f_2 \in C^0(\mathbb{R}^2,\mathbb{R})$. Given an arbitrary input $u \in AC(\mathbb{R}_+,\mathbb{R})$ and initial condition $y_0 \in \mathbb{R}$, the existence and uniqueness of $y \in AC(\mathbb{R}_+,\mathbb{R})$ satisfying (2.4) at almost every $t \in [0,T]$ with $T > 0$ is studied in [40, 78] and guaranteed when f_1 and f_2 satisfy

$$(f_1(\upsilon,\gamma_1) - f_1(\upsilon,\gamma_2))(\gamma_1 - \gamma_2) \leq \lambda_1(u)(\gamma_1 - \gamma_2)^2, \tag{2.5}$$

$$(f_2(\upsilon,\gamma_1) - f_2(\upsilon,\gamma_2))(\gamma_1 - \gamma_2) \geq -\lambda_2(u)(\gamma_1 - \gamma_2)^2, \tag{2.6}$$

for every $\upsilon, \gamma_1, \gamma_2 \in \mathbb{R}$ and some for non-negative functions $\lambda_1, \lambda_2 \in C(\mathbb{R},\mathbb{R}_+)$.

As defined in (2.4), the Duhem hysteresis operator satisfies the conditions to be a hysteresis operator in the sense of Definition 2.4, in the form

$$\Phi(u) = \mathscr{D}(u,y_0).$$

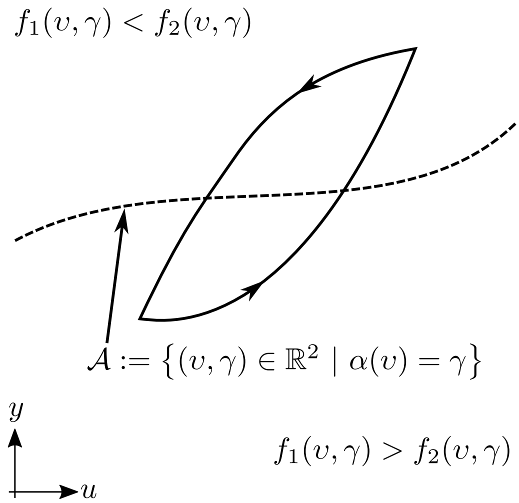

Figure 2.4: Illustrative example of hysteresis loop from a Duhem hysteres model where the *anhysteresis curve* $\mathscr{A}$ is indicated by the dashed line with the two regions below and above it where the model functions satisfy $f_1(\upsilon,\gamma) > f_2(\upsilon,\gamma)$ and $f_1(\upsilon,\gamma) < f_2(\upsilon,\gamma)$, respectively.

Furthermore, throughout this thesis we assume that the implicit function $\upsilon \mapsto \{\gamma \in \mathbb{R} \mid f_1(\upsilon,\gamma) - f_2(\upsilon,\gamma) = 0\}$ admits an explicit solution

$$\gamma = \alpha(\upsilon) \tag{2.7}$$

with $\alpha \in C^0(\mathbb{R},\mathbb{R})$, which we call the *anhysteresis function* and the corresponding curve (generated by α) given by

$$\mathscr{A} = \left\{(\upsilon,\gamma) \in \mathbb{R}^2 \mid \gamma = \alpha(\upsilon)\right\}, \tag{2.8}$$

is called the *anhysteresis curve*. By definition, the curve $\mathscr{A}$ divides the input-output plane into two regions where $f_1(\upsilon,\gamma_1) - f_2(\upsilon,\gamma_1) \geq 0$ whenever $\gamma_1 \geq \gamma = \alpha(\upsilon)$, and $f_1(\upsilon,\gamma_1) - f_2(\upsilon,\gamma_1) \leq 0$ whenever $\gamma_1 \leq \gamma = \alpha(\upsilon)$. An example of a Duhem model *anhysteresis curve* is illustrated in Figure 2.4.