Transport signatures and the origin of non-collinear spin textures
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Chapter 1

Introduction

1.1 Introduction

Electronic interactions are the key ingredient for many fundamental as well as technological applications. At low temperatures, these interactions favor organized states, such as magnetic ordering. Some magnetic orders survive thermal fluctuations even at room temperature. This makes magnets fit for data storage applications, such as the modern magnetic hard drives. A tiny magnetic domain is used as a single bit and millions of these bits are employed for data storage and access. The work on these memory devices was initiated by the discovery of Giant Magnetoresistance by Albert Fert [1] and Peter Grünberg [2] in 1988. The key concept is that a flowing current can change the direction of magnetization. It took only a few years from the discovery to a complete a rapid industrial revolution. This revolution prompted the quest for more storage efficient devices, required for the storage and access of Quintillions ($10^{18}$) bytes of data. A deep understanding of magnetism and related physical phenomena is crucial to handle the needs of future data devices.

In recent years, new physical phenomena have emerged as a result of the discovery of layered materials and the advancement of experimental techniques. Stability of two-dimensional crystals or long-range ordering in two dimensions was shown to be destroyed by the thermal fluctuations (a continuous symmetry cannot be broken spontaneously) [3] in the theoretical works of Peierls [4, 5], Landau [6, 7], Mermin[8] and Wagner[9]. Graphene being the first ever material to be isolated in two-dimensional morph [10] of a very well known and abundant material graphite, brings forth the discovery and prediction of a plethora of phenomena [11–13] and a zoo of more than 1000 other two-dimensional materials followed [14, 15].

The unique combination of stability, thickness, electronic properties and industrial production readiness,—the graphene discovery queue continues. The linear dispersion and a speed of $\frac{c}{300}$ leads to a slower QED [16–18], that can be performed on a tabletop. The Dirac fermions behave differently in magnetic field and are predicted to exhibit anomalous integer quantum Hall effect (IQHE) [19, 20], which was and verified experimentally [21, 22]. Because of the unusual electronic and structural properties, it is easy to tailor graphene in many different ways. Creating a
heterostructure of graphene and other materials provides a non-invasive avenue to engineer new physical properties [23].

A combination of a bulk or monolayer material with a thin metal or graphene makes it possible to manipulate electronic conduction properties at the film/monolayer material. This process of combination is known as heterostructure approach. Physical properties of film/monolayer and the participating bulk materials undergo transformations in heterostructures. Most importantly, these transformations are tunable. For a metal/graphene heterostructure, one of the most important properties is the electronic conduction. Therefore, the resistivity usually becomes a measure of how a heterostructure can change the property of a conducting layer material.

Creating a thin magnet at room temperature with graphene and a magnetic insulator is a significant achievement of this heterostructure approach. The anomalous Hall effect is a successful signature of the proximity-induced magnetism in graphene [24]. Interestingly these magnets may also show a periodic modulation of the magnetic moments. In presence of competing spin interactions, these moments create unusual spin structures that can be very different from collinear ferromagnetic and anti-ferromagnetic orders. Non-uniform magnetic structures also modify transport properties of normal metal and heterostructures. The origin of non-collinear magnetism can be understood from the materials symmetry properties. Minimization of magnetic free energy makes possible to find collinear and non-collinear ground states accessible by varying external parameters, such as pressure or magnetic field. Obtaining phase diagrams of materials containing such phases opens a new avenue towards experimental realizations of the simulated spin structures.

The aim of this thesis is twofold. First, we find the allowed competing interactions in a magnet. From a symmetry perspective, we find all possible interactions that are relevant and important. Using Landau-Lifshitz-Gilbert equation, we minimize the total magnetic energy of the system to obtain different magnetic ground states for different interaction strengths. Second, we find the change of resistivity of a thin metallic layer/graphene heterostructure with an insulating magnet. The resistivity of the conducting layer can be anisotropic due to interaction with the spatially varying magnetic structure in the magnetic insulator.

In this chapter we introduce the essential components and ideas that are extensively used in this thesis. In particular, the spin Hall and inverse spin Hall effects, spin Hall magneto-resistance, related electronic and transport properties of graphene, graphene heterostructures, Boltzmann equation, frustrated and chiral magnetism, Landau-Lifshitz-Gilbert equation.
1.2 Spin Hall and Inverse Spin Hall Effects

Spintronics, or spin based electronics, shows a huge potential to replace current silicon based transistor architecture that is becoming smaller and smaller. For example, the magnetic random access memory (MRAM) utilizes spin polarized currents to read and write information, which is not energy efficient. This process generates Joule heating due to the flowing charge current. Pure spin currents can solve this problem. Therefore, in modern electronic devices employing spin degrees of freedom, the mechanism of inter-conversion between charge and spin currents play an essential role. The spin Hall effect (SHE) refers to a transverse spin current induced by a charge current with spin polarization perpendicular to both charge and spin currents. The spin Hall effect is a key requirement for multitude of spintronics applications [25, 26], namely, the spin-orbit torques (SOT) and magnetic memories [27–31] in spin Hall magnetoresistance [32, 33]. The spin Hall effect is named in analogy to ordinary Hall effect and is conceptually related to the anomalous Hall effect [34]. The origin of the SHE is asymmetric deflection of electrons in a metal with strong spin-orbit coupling. The SHE was predicted by Dyakonov and Perel in 1971 [35] and Hirsch proposed an experiment in 1999 [36]. After rapid technological progress of device fabrication at nanoscale, the SHE was experimentally observed in 2004 [37].

In the SHE, the charge current $J_c$ flows through a non-magnetic metal with strong spin-orbit interaction. This interaction leads to a deflection of spins of opposite polarizations in opposite directions, causing a net spin current $J_s$ transverse to the charge current [25, 37, 38]. The inverse spin Hall effect follows from the Onsager’s reciprocity and is based on the same principles. An applied spin current produces a transverse charge current due to the spin-orbit coupling. The spin-charge conversion efficiency depends on the spin-orbit coupling strength and symmetry of the material under consideration, and is parametrized by the dimensionless spin Hall angle $\theta_{\text{SH}}$ [39]. The equation governing the spin Hall effect is

$$J_s = \theta_{\text{SH}} \sigma \times J_c,$$

where $\sigma$ represents the direction of spin polarization. The mechanisms for spin Hall effect can be extrinsic, i.e. the skew scattering of an impurity [40] or the side jump [41], or intrinsic originating from the spin Berry curvature in the reciprocal space [42, 43]. The spin Hall effect is universally present in all nonmagnetic materials with non-negligible spin-orbit coupling [44]. Pt and Ta are interesting candidate materials [45] that show large spin Hall conductivity with intrinsic mechanisms giving larger contribution than the extrinsic ones.
1.3 Spin Hall Magnetoresistance

Magnetoresistance (MR) is change of the electrical resistance under the application of a magnetic field. If this property depends on the angle between the directions of the electric current and applied magnetic field, then it is called Anisotropic Magnetoresistance (AMR). These effects are symmetric under the magnetization reversal unlike the Anomalous Hall effect (AHE), in which the magnetization reversal changes the sign of the transverse resistivity. Both AMR and AHE originate from the interplay between the magnetization and spin-orbit interaction [34, 46] in the same material. In multilayer metal/nonmagnetic insulator systems, the giant magnetoresistance (GMR) and tunnel magnetoresistance (TMR) are quantum-mechanical effects, in which resistivity is affected by alternating stacks of a metal and an insulator. The extraordinary magnetoresistance originates from the geometry of semiconductor-metal hybrid systems.

The Spin Hall magnetoresistance (SMR) occurs in a nonmagnetic heavy metal | magnetic insulator systems. This unconventional magnetoresistance is a non-equilibrium phenomenon caused by simultaneous action of the spin Hall effect (SHE) and inverse spin Hall effect (ISHE) [47]. A metal with strong spin-orbit coupling, such as Pt, is necessary for the spin Hall magneto-resistance. The SMR effect was discovered in 2012 in the heterostructure of Yttrium Iron Garnet and Pt [48–50] and almost simultaneously explained theoretically [47]. The SMR was also found in YIG|Pt [51, 52] and YIG|Ta [53] heterostructures and in more complex systems, such as CoFe$_2$O$_4$|Pt [54], CoCr$_2$O$_4$|Pt [55] and Cu$_2$OSeO$_3$|Pt (Chapter 2).

The SHE and ISHE act simultaneously in a synchronized fashion to produce the SMR response. When an in-plane electrical current flows through a nonmagnetic metal with strong spin-orbit coupling, electrons with opposite spin polarizations accumulate on the opposite interfaces as a result of the SHE. The spin accumulation ($\mu_s$) due to the SHE can be partially absorbed or fully reflected at the FM|NM interface depending on the interfacial magnetization ($M$) of the FM. If $\mu_s \perp M$, the spins in NM arriving at the interface, are partially absorbed, resulting in higher NM resistance. If $\mu_s \parallel M$, spins are fully reflected. The reflected spin current generates an extra charge current through the ISHE, resulting in a reduced resistance. A rotation of the magnetization in FM modifies magneto-resistance and is measured as SMR in the NM. The transverse and
1.4. Graphene

Graphene responses are given by,

$$\rho_{\text{trans}} = \Delta \rho_1 m_x m_y + \Delta \rho_2 m_z, \quad \rho_{\text{long}} = \rho + \Delta \rho_0 + \Delta \rho_1 (1 - m_y^2),$$

(1.2)

where $\rho$ is the electrical resistivity of NM, $m_x$, $m_y$, $m_z$ are the components of the magnetization in FM along the $\hat{x}$, $\hat{y}$, $\hat{z}$ directions, respectively. The out-of-plane magnetization and anomalous Hall effect is taken into account in $\rho_{\text{trans}}$ [56]. The $\Delta \rho_0$, $\Delta \rho_1$, $\Delta \rho_2$ are [47],

$$\frac{\Delta \rho_0}{\rho} = -\theta_{\text{SH}}^2 \frac{2\lambda}{t} \tanh \frac{t}{2\lambda},$$

$$\frac{\Delta \rho_1}{\rho} = \theta_{\text{SH}}^2 \frac{\lambda}{t} \text{Re} \left( \frac{2\lambda G_{\uparrow \downarrow} \tanh^2 \frac{t}{2\lambda}}{\sigma + 2\lambda G_{\uparrow \downarrow} \coth \frac{t}{\lambda}} \right),$$

$$\frac{\Delta \rho_2}{\rho} = -\theta_{\text{SH}}^2 \frac{\lambda}{t} \text{Im} \left( \frac{2\lambda G_{\uparrow \downarrow} \tanh^2 \frac{t}{2\lambda}}{\sigma + 2\lambda G_{\uparrow \downarrow} \coth \frac{t}{\lambda}} \right),$$

where $\lambda, t, \theta_{\text{SH}}, \sigma$ and $G_{\uparrow \downarrow}$ represent the spin relaxation length, film thickness, spin Hall angle, bulk conductivity of NM and spin mixing conductance of the NM|FM interface, respectively.

1.4 Graphene

Graphene is a two-dimensional form of graphite[10]. Each time we write with a pencil, we produce a few layered graphene stacks. Although this material was abundant in our daily uses in disguise, it was discovered only very recently [10]. The discovery is a surprising use of scotch tape, or micromechanical cleavage. Graphene or monolayer graphites are loosely stacked upon each other and are bound by van der Waals force. When a scotch tape sticks to and then detaches from graphite, one or a few graphene layers get separated and are subsequently transferred to a substrate [3].

Carbon atoms in graphene arrange in a hexagonal structure and give rise to Dirac electrons in two dimensions at low energies [57]. This specialty gives altogether different electronic properties [11] to graphene which attracted a large body of research.

1.4.1 Dirac Electrons in Graphene

To understand the emergence of Dirac electrons in graphene, consider the tight-binding model of graphene with nearest-neighbour hopping. Hexagonally arranged carbon atoms in graphene can be described as a triangular lattice of two atoms per
unit cell. The nearest-neighbour vectors $\delta_1, \delta_2, \delta_3$ are (see Fig. 1.1),

$$\delta_1 = \frac{a}{2}(1, \sqrt{3}), \quad \delta_2 = \frac{a}{2}(1, -\sqrt{3}), \quad \delta_3 = -a(1, 0) \quad (1.4)$$

where $a \sim 1.42$ Å is the distance between the carbon atoms. The tight-binding Hamiltonian has the form,

$$H = -t \sum_{<i,j>,\sigma} \left( a_{i\sigma}^\dagger b_{j\sigma} + b_{j\sigma}^\dagger a_{i\sigma} \right), \quad (1.5)$$

where $a_{i\sigma}(a_{i\sigma}^\dagger)$ are the creation (annihilation) operators of the electron with spin $\sigma$ on site $r_i$ of sublattice A (similarly, for sublattice B). $t \sim 2.8$ eV is the hopping energy between nearest neighbours. Fourier transform of the operators is given by,

$$\begin{pmatrix} a_{i\sigma} \\ b_{i\sigma} \end{pmatrix} = \frac{1}{\sqrt{N}} \sum_k e^{ik \cdot r_i} \begin{pmatrix} a_{k\sigma} \\ b_{k\sigma} \end{pmatrix}. \quad (1.6)$$

The band energies derived from this Hamiltonian are [57],

$$E_{\pm}(k) = \pm t\sqrt{3 + f(k)}, \quad f(k) = 2\cos(\sqrt{3}k_ya) + 4\cos(\frac{\sqrt{3}}{2}k_ya)\cos(\frac{3}{2}k_xa) \quad (1.7)$$

and are shown in Figure 1.1.

The + sign denotes the upper ($\pi^*$) band and the - sign denotes the lower ($\pi$) band. This spectrum is symmetric around zero energy. If we expand Eq.(1.7) around the
Figure 1.2: Dirac cones for electrons and holes are denoted by red and blue color, respectively (left). The Fermi surface at a finite electronic density is a circle drawn in black (right). Flat cyan surface denotes the Fermi energy.

zero energy, we obtain,

$$E_{\pm}(k) = \pm \hbar v_F |k| + O[(k/K)^2]$$

(1.8)

which is the Dirac dispersion as shown in Figure 1.2. Here, the momentum $k$ is measured from the zero point and the Fermi velocity $v_F = 3at/2\hbar = 1 \times 10^6 \text{ m/s}$. This zero point of energy is called the Dirac point and is located at

$$K = \left( \frac{2\pi}{3a}, \frac{2\pi}{3\sqrt{3}a} \right) \quad \text{and} \quad K' = \left( \frac{2\pi}{3a}, -\frac{2\pi}{3\sqrt{3}a} \right).$$

(1.9)

Equation (1.8) is the dispersion of massless Dirac particle in two-spatial dimensions, which provides a fertile playground for tabletop QED experiments and numerous interesting phenomena [11].

To study transport properties of graphene [12, 58], it is useful to derive the electric current operator in the tight-binding model. To obtain the current, we define another quantity, namely, the polarization operator,

$$P = e \sum_{i\sigma} r_i n_{i\sigma} = e \left( \sum_{i \in A, \sigma} r_i a_i^{\dagger} a_i + \sum_{i \in B, \sigma} r_i b_i^{\dagger} b_i \right), \quad e < 0,$$

(1.10)

where $n_{i\sigma}$ is the electron number operator at site $i$. The tight-binding current is
obtained as the rate of change of the polarization operator,

\[ j = \dot{P} = \frac{-ie\ell}{\hbar} \sum_{<i,j>\sigma} (r_i - r_j)(b_j^{\dagger}\sigma a_i\sigma - a_i^{\dagger}\sigma b_j\sigma). \]  

(1.11)

It describes a network-like flow of electrons within the graphene lattice along the carbon-carbon bonds.

### 1.4.2 Boltzmann transport of electrons in Graphene

The Boltzmann equation provides an intuitive point of view on transport [59] in graphene. Although this approach is not valid at the Dirac point, at a finite electronic density it gives a very clear picture of scattering mechanisms present in the system[12]. It is known from experiments that the conductivity of graphene varies linearly with the gate potential [10] and the linear dependence results mainly from screened Coulomb impurities [12, 60]. For simplicity, we consider only random neutral scatterers e.g. due to defects in presence of a substrate. The Boltzmann equation has the form,

\[ -v_k \cdot \nabla f(k) - e(E + v_k \times H) \cdot \nabla_k f(k) = -\frac{\partial f_k}{\partial t}_{\text{scatt}} \]  

(1.12)

An analytical solution of Boltzmann equation is, in general, difficult to obtain, which is why one often resorts to approximations. To solve this equation, we assume that the distribution function \( f(k) = f^0(\varepsilon_k) + g(k) \), where \( f^0(\varepsilon_k) \) is the equilibrium state distribution function and \( g(k) \) is a small change in the steady state. For simplicity, consider spatially uniform distribution functions in zero magnetic field and in finite electric field \( E \), in which case Boltzmann equation reads,

\[ -eE \cdot \nabla_k f(\varepsilon_k) = -\frac{\partial f_k}{\partial t}_{\text{scatt}}, \]  

(1.13)

The Fermi surface shifts as a result of the applied electric field as shown in Figure 1.3. In simplest case, one can approximate the scattering term by, \( -\frac{\partial f_k}{\partial t}_{\text{scatt}} \approx \frac{g_k}{\tau_k} \), where \( \tau_k \) is the scattering time. This approximation is known as the scattering time approximation. The non-equilibrium distribution is then, \( g_k = eE \cdot v \tau_k \delta(\varepsilon_k - E_F) \). The inverse scattering time for random non-magnetic impurities \( \Gamma(k) \), is given by,

\[ \Gamma(k) = N_i A \int \frac{dk'}{(2\pi)^2} S(k, k') = n_i v_0^2 \frac{\hbar^2 v_F}{k}; \quad \tau_0 = \frac{\hbar^2 v_F}{n_iv_0^2} \frac{1}{k}, \]  

(1.14)

where \( S(k, k') = (2\pi/\hbar^2v_F)\langle |V_{kk'}|^2 \rangle \delta(k' - k) \) is the transition probability obtained from the Fermi golden rule. \( \langle |V_{kk'}|^2 \rangle \) is the averaged matrix element squared with
1.4. Graphene

Figure 1.3: Effect of an uniform electric field on the Fermi surface of graphene. The Fermi surface shifts as the electric field is applied.

a potential strength $v_0$. Using non-equilibrium distribution obtained from the Boltzmann equation, we obtain the electric current and conductivity:

$$ J = \frac{4}{A} \int dk \ e \ v_k g_k, \ \text{and} \ J = \sigma E $$

For random scatterers, the conductivity is energy independent,

$$ \sigma_{xx} = 4 \frac{e^2}{h} \left( \frac{\hbar^2 v_F^2}{n_i v_0^2} \right), $$

which is why one has to consider charged Coulomb impurity scattering in order to obtain the linear energy dependence of conductivity observed in many experiments. Both the white noise and screened Coulomb disorder potentials are used for accurate description of the experimentally observed resistivity [60].

1.4.3 Heterostructure modification of electronic properties

Graphene as a two-dimensional host of Dirac electrons is an ideal candidate to explore exotic phenomena, such as the room temperature quantum Hall effect and the anomalous integer quantum Hall effect [61]. But apart from pure graphene, behaviour of modified graphene can also be very interesting [62]. Chemical modification, adding adatoms and vacancy defects [11] introduces the spin-orbit coupling [63], local magnetic moments [64] or edge states. This comes at the cost of reduced electronic quality. But intercalation [65], heterostructures [66] or putting graphene on top of a substrate [67] is an ideal way to separately preserve the crystallographic structure of constituent materials, while modifying specific electronic properties according to desire [23]. Particularly, putting graphene on top of an insulating substrate or creating an heterostructure with a insulating material is most interesting, as it does not shunt away current from graphene, but changes the properties of con-
duction electrons. Here, we explore a few cases of heterostructure modification of graphene.

**FeCl$_3$ intercalation and magnetic ordering**

FeCl$_3$ intercalated graphene or Exeter graphene (first made in Exeter, UK) provides the first evidence for a magnetic ordering [65] in the monolayer limit of graphene. The longitudinal resistivity of graphene $\rho_{xx}$ shows a monotonous decrease with temperature from 175 K to 25 K. However, below 25 K, the resistivity increases again due to two-dimensional ordering in FeCl$_3$ layers of the Exeter graphene[65].

**Anomalous Hall effect in Graphene**

The anomalous Hall effect, i.e. the transverse resistance linearly proportional to the magnetization of a ferromagnet, is a phenomenon indicating the presence of magnetism [34]. It is observed in graphene placed on top of the yttrium iron garnet substrate [24]. Proximity coupling to YIG induces an exchange interaction and also enhances the spin-orbit coupling in graphene, which is intrinsically weak ($\sim$ 1.4 K) [68]. Gate tunability of the anomalous Hall effect, which cannot be achieved in bulk ferromagnetic metals, is observed in graphene—YIG heterostructure. The Exchange field for this heterostructure is estimated to be 0.2 T [69].

**Anti-hysteresis in graphene on strontium titanate**

Graphene being a surface sensitive two-dimensional material can sense the out-of-plane polarization at the interface of TiO$_2$ terminated strontium titanate (STO) used as a substrate for graphene [67]. Anti-hysteresis with respect to gate voltage is observed due to the electric polarization at the interface and quantum confined states.

1.5 **Frustrated and chiral magnetism**

If there are several competing interactions between spins, then the minimal-energy spin configuration can be non-collinear or degenerate. There are several interactions that can give rise to spin frustration. Frustration and chirality often give rise to non-collinear spin textures. Below we consider non-collinear magnetic states that will be encountered in this thesis. Then we give a brief introduction to Landau-Lifshitz-Gilbert equation used to minimize the magnetic energy and find the ground state for a set of parameters.
1.5. Frustrated and chiral magnetism

1.5.1 Frustration by Exchange

To understand frustration by exchange, we consider a spin chain with nearest-neighbour and next-nearest-neighbour Heisenberg interactions. The Hamiltonian for this system is,

$$H = \sum_n \left( J_1 S_n \cdot S_{n+1} + J_2 S_n \cdot S_{n+2} \right).$$  \hspace{1cm} (1.17)

Assuming a spiral of the form $S_n = (\cos qn, \sin qn)$ as depicted in Figure 1.4 and minimizing the energy, we find the optimal wave vector, $q = \cos^{-1} (-J_1/4J_2)$. Classical ground states of this system are: a collinear anti-ferromagnetic state for $J_1 > 4J_2$, which coincides with the spiral with $q = \pi$ at $J_1 = 4J_2$, the spiral state for $-4J_2 < J_1 < 4J_2$ and the ferromagnetic state for $J_1 < -4J_2$, which coincides with the $q = 0$ spiral at $J_1 = -4J_2$.

1.5.2 Frustration by Geometry

Geometric frustration arises when no ordering of spins on a lattice can simultaneously satisfy all exchange interactions. Consider the simplest example of three spins arranged in an antiferromagnetic triangle. Once two spins are pointing opposite to each other, the third spin experiences confusion (see Fig. 1.5). The whole system becomes frustrated as the other two spins have the same problem [70]. For these three spins, the energy of the classical Heisenberg model,

$$S_1 \cdot S_2 + S_2 \cdot S_3 + S_3 \cdot S_1 = \frac{1}{2}(S_1 + S_2 + S_3)^2 - \frac{3}{2},$$  \hspace{1cm} (1.18)

finds minimum when $S_1 + S_2 + S_3 = 0$. For a triangular lattice (see Fig. 1.5), this holds true for all triangles and leads to a unique three-sublattice ground state up to a global rotation of spins and the sign of a vector chirality. On the other hand, if we consider Kagome lattice consisting of corner sharing triangles, the condition $S_1 + S_2 + S_3 = 0$ for each triangle can be satisfied in an infinite number of ways leading to a huge degeneracy.
1.5.3 Frustration by Dzyaloshinskii-Moriya Interaction

The red iron rust or $\alpha$-Fe$_2$O$_3$, which is an anti-ferromagnetic insulator with a small magnetic moment, led to controversy [71]. Responsible for the weak ferromagnetism is the anti-symmetric interaction between spins. This interaction was first obtained by Igor Dzyaloshinskii in 1958. For an interaction between two antiferromagnetic sublattices $S_1$ and $S_2$, it has the form,

$$D \cdot (S_1 \times S_2),$$

obtained from the symmetry arguments [72]. Later, Toru Moriya derived this interaction microscopically considering virtual electron hopping between two magnetic sites [73], which is why this interaction is called Dzyaloshinskii-Moriya interaction [74]. To illustrate spin frustration by Dzyaloshinskii-Moriya (DM) interactions, we consider a spin chain with the nearest-neighbour Heisenberg exchange constant $J_1$. For a bond constituting magnetic and non-magnetic ions, the DM vector is denoted as $D_{ij}$, and the bond Hamiltonian has the form,

$$H = \sum_n (J_1 S_n \cdot S_{n+1} + D \cdot [S_n \times S_{n+1}]).$$

Assuming the same spiral as in section 1.5.1 and the DM vector along the $z$ direction $D = D\hat{z}$, we minimize the energy of this system and obtain a spiral with the wave vector $q = \tan^{-1}(D/J)$. 

Figure 1.5: Spin frustration on a triangle with antiferromagnetically interacting spins (left) and spins on a triangular lattice showing a non-collinear $120^\circ$ ordering as a result of geometrical frustration (right).
1.5. Frustrated and chiral magnetism

1.5.4 Landau-Lifshitz-Gilbert Equation

Figure 1.6: Landau-Lifshitz-Gilbert equation describing the time dependence of the magnetization $\mathbf{M}$ in an effective field $\mathbf{H}_{\text{eff}}$ and under the influence of the damping term proportional to $\mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t}$.

In 1935, Landau and Lifshitz proposed an equation describing precession of magnetization in ferromagnets:

$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma \mathbf{M} \times \mathbf{H}_{\text{eff}}, \quad (1.21)$$

where $\mathbf{H}_{\text{eff}}$ is an effective field, which includes the sum of the external, exchange and anisotropy fields and $\gamma$ is the electron gyromagnetic ratio. In 1955, Gilbert proposed an additional torque term,

$$\frac{\alpha}{M_s} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t}, \quad (1.22)$$

where $\alpha > 0$ is the dimensionless Gilbert damping constant which depends on the material, $M_s$ is the saturation magnetization, the magnitude of which is preserved at each point within a ferromagnet both by the precession and damping. The additional Gilbert term preserves the amplitude of magnetization. The sum of the two terms gives the Landau-Lifshitz-Gilbert (LLG) equation:

$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma \mathbf{M} \times \mathbf{H}_{\text{eff}} + \frac{\alpha}{M_s} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t}. \quad (1.23)$$

Substituting $\frac{\partial \mathbf{M}}{\partial t}$ in Eq.(1.23) back into the third term of itself, we obtain,

$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma' \mathbf{M} \times \mathbf{H}_{\text{eff}} - \lambda' \mathbf{M} \times \mathbf{M} \times \mathbf{H}_{\text{eff}} \quad (1.24)$$

where $\gamma' = \frac{\gamma}{1+\alpha^2\gamma^2}$ and $\lambda' = \frac{\gamma\alpha/M_s}{1+\alpha^2\gamma^2}$. 
We solve this equations for spin configurations defined on a lattice and find local energy minima (competing ground states) of a magnetic material for various sets of parameters.

### 1.6 Thesis outline

This thesis deals with three different problems, which can be classified in two main directions: the origin of non-collinear magnetism and the detection of non-collinear magnetism via transport.

To understand the origin of non-collinear magnetism, we resort to a phenomenological analysis. From symmetry arguments, we find the Heisenberg exchange and the Dzyaloshinskii-Moriya interactions. We minimize the total energy of the magnetic system including various competing interactions and find the ground state of the system for specified values of the interactions parameters. By varying two interactions we plot a phase diagram including several competing ground states.

The detection of magnetic phases by means of transport deals with the changes in resistivity of a thin metal layer or graphene in presence of a magnetic interaction with an insulating magnetic substrate. The magnetic insulator does not shunt away the electrical current. The conduction electrons of the flowing current interacts with the magnetic moments of the magnet. This interaction alters the resistivity behaviour of the metallic layer or graphene.

In chapter 1 **Theory of electrical detection of spiral spin structures in a Pt | Cu$_2$OSeO$_3$ heterostructures**, we study the spin Hall magnetoresistance (SMR) in Pt|non-collinear insulating magnet heterostructure. We show that SMR, which until recently was investigated only for collinear magnetic structures, can also be used as an all-electric probe for complex spin states exhibited by a ferromagnet in an applied magnetic field. Inspired by experiments, we show that the magnetic field dependence of SMR and discontinuous (or non-monotonic) nature of the SMR signal are due to magnetic phase transitions in the Cu$_2$OSeO$_3$. We theoretically explain that the sign change of the SMR signal in the conical spiral state, as well as the discontinuity resulting from the transition between the conical to ferromagnetic states. We show that the observed magnetic field dependence of SMR is well described in the framework developed earlier for collinear magnets.

In chapter 2 **Electrically tunable anisotropic resistivity of graphene on spiral magnet** we discuss how two-dimensional materials and proximity-induced magnetism provide a new avenue to explore the physics of next generation spintronic devices. Proximity-induced ferromagnetism in graphene provides a rich playground to explore electrically tunable magnetic effects. We consider graphene put on a magnetic insulator hosting a spin spiral. This heterostructure splits the Dirac dispersion
of graphene in two cones separated by the ordering wave vector of the magnetic spiral. The splitting of the Fermi surface changes electrical resistivity below the magnetic phase transition. Electrically tunable anisotropic resistivity makes possible to detect the spiral ordering in magnetic insulator by means of transport. The magnetic control of electrical resistivity in graphene, provides a new direction in heterostructure spintronics.

In chapter 3 Unusual 90° state and Non-collinear Spin Ordering in Pb₂MnO₄, we study the magnetic states of the acentric, non-polar, multi-ferroic magnet Pb₂MnO₄. We establish relations between different exchange couplings and Dzyaloshinskii-Moriya interactions using symmetry arguments. An unusual 90° ordering of Mn spins, observed in powder neutron diffraction, is shown to originate from competing exchange and Dzyaloshinskii-Moriya interactions. Several noncollinear spin textures, such as the helical and conical spiral, meron anti-meron lattice and the unusual 90°-phases are found in a phase diagram with a variable exchange coupling and magnetic anisotropy.

Bibliography


1. Introduction


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