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Adaptive Control for Flow and Volume Regulation in Multi-Producer District Heating Systems

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***Abstract*—Flow and storage volume regulation is essential for the adequate transport and management of energy resources in district heating systems. In this letter, we propose a novel and suitably tailored—decentralized—adaptive control scheme addressing this problem whilst offering closed-loop stability guarantees. We focus on a system configuration comprising multiple heat producers, consumers and storage tanks exchanging energy through a common distribution network, which are features of modern and prospective district heating installations. The proposed controller is based on passivity, backstepping and (indirect) adaptive control theory.**

***Index Terms*—Adaptive control, energy systems, uncertain systems.**

I. Introduction

The EFFECTIVE distribution of heat and management of energetic resources in DH systems strongly depends on the adequate regulation of the system temperatures, pressures and flows [1]. From the control viewpoint, these are significantly challenging tasks due to the nonlinear, networked, and uncertain nature of DH systems models [2], [3].

The control of DH systems with a *single* heat producer has received considerable attention. In [4] global asymptotic end-user (consumer) pressure regulation was addressed via decentralized proportional-integral controllers (see also [3]), whereas temperature and storage volume regulation was achieved in [2] via a novel internal model controller. Temperature control was also investigated using Lyapunov-Krasovskii theory in [5] for a system model which accounts for non-negligible delays in the heat transport from producer and consumers. For the case of DH systems with *multiple* heat producers, a number of works have focused on design and operational optimization, see, e.g., [6] and the references therein. The use of predictive control was investigated in [7] for optimal system operation. However, the implementation requires system-wide measurements (of temperatures) and no formal stability analysis is presented. The optimal regulation of flow networks was addressed in [8], including as a particular case a class of simplified DH systems with storage units. Even though closed-loop stability is guaranteed under some conditions, the system model neglects friction effects on pipelines, which are significant in these applications.

In this letter, we propose a novel adaptive controller for flow and storage volume regulation of a *multi*-producer DH system. The control scheme is decentralized, as each control input depends only on locally available information, and is based on backstepping, adaptive and passivity-based control design tools. We consider a nonlinear and uncertain system model which accounts the effects associated with friction in pipes and is suitable to describe general distribution network topologies. Conditions for closed-loop asymptotic stability are also presented.

Notation: $\mathbb{R}$ denotes the set of real numbers. For a vector $x \in \mathbb{R}^n$, x_i represents its ith component, i.e., $x = [x_1, \dots, x_n]^\top$ and $|x| = [|x_1|, \dots, |x_n|]^\top$. An $m \times n$ matrix with all-zero entries is written as $\mathbf{0}_{m\times n}$. An n-vector of ones is written as $\mathbf{1}_n$, whereas the identity matrix of size n is represented by I_n. For any vector $x \in \mathbb{R}^n$, we denote by $\langle x \rangle$ a diagonal matrix with elements x_i in its main diagonal. For any time-varying signal w, we represent by $\bar{w}$ its steady-state value, if exists.

II. Background and Problem Formulation

In this section we introduce the multi-producer DH system under consideration and formulate the flow and storage volume regulation control problem.

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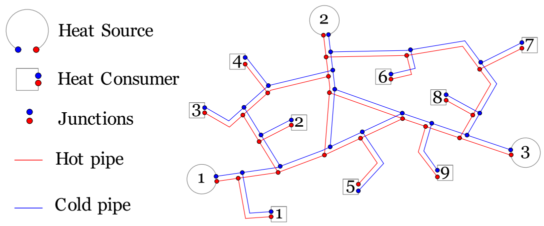

Fig. 1. Sketch of a simplified DH system (see [9]).

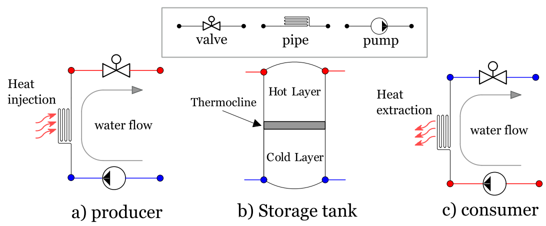

Fig. 2. Topologies of producers, consumers and storage tanks; see [2], [3]. Pipes of producers and producers represent heat exchangers.

A. System Model

We consider the hydraulic system of a water-based (leak free) DH system comprising n_{pr} heat producers, n_{c} consumers and n_{ST} storage tanks which are connected to a common distribution network. The latter is assumed to be symmetric in the sense that supply and return layers, which respectively transport hot and cold water, have the same topology. In Fig. 1 a simplified DH system with three producers and nine consumers is shown.[1]

In this work, producers, consumers and distribution network are assumed to be composed of elementary hydraulic devices, namely, valves, pipes and pumps. Producers are assisted by hydraulic pumps to deliver thermal power to the system by circulating and heating water through heat exchangers (viewed here as pipes): cold water is continuously drawn from the return layer of the distribution network which is then heated and injected back into the supply layer. The operation mode of consumers is analogous to that of producers. Storage tanks accumulate volumes of hot and cold water which are perfectly separated by a thermocline, i.e., hot water is on top and cold water at the bottom, and without heat exchange between them. In addition, each tank is considered to have four valves, two at the top and two at the bottom, which are used as inlets and outlets of hot and cold water, respectively. The foregoing description is schematically depicted in Fig. 2; see also [2].

The DH system is henceforth viewed as the connected graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$. The nodes $\mathcal{N}$ are all the system junctions as well as the hot and cold layers of the storage tanks. All two terminal devices, namely, pumps, pipes and valves, are represented by the set of edges $\mathcal{E}$. Each edge is assumed to have an arbitrary and fixed orientation and this is codified through the node-edge incidence matrix $\mathcal{B}_0$. For any edge $i \in \mathcal{E}$, $q_{\text{E},i}$ and $V_{\text{E},i}$ are the flow through it and the volume of water in it, respectively. Also, $V_{\text{N},k}$ and $p_{\text{N},k}$ are the volume and pressure of a given node $k \in \mathcal{N}$. The cardinalities of $\mathcal{E}$ and $\mathcal{N}$ are denoted by n_{E} and n_{N}, respectively.

[1]It is assumed that water is incompressible and that its density ρ is constant. All system pipes are assumed to be cylindrical.

Basic models describing the dynamic behavior of the flows $q_{\text{E},i}$ and the volumes $V_{\text{N},k}$ are presented next.

1) Dynamics of Edges: For every edge $i \in \mathcal{E}$, with terminals $j, k \in \mathcal{N}$, $j \neq k$, we relate the rate of change of the flow through it with the pressure drop across it via the differential-algebraic equation [3]

$$p_{\text{N},j} - p_{\text{N},k} = J_{\text{E},i}\dot{q}_{\text{E},i} + f_{\text{E},i}(q_{\text{E},i}) - w_{\text{E},i}. \tag{1}$$

In (1), if i represents a pipe, then $J_{\text{E},i} > 0$ is a constant depending on its physical dimensions, such as length and cross-sectional area. If i is a pump, then w_i denotes the pressure difference that i produces across its terminals. The function $f_{\text{E},i}$ is assumed to be continuously differentiable, mononotically increasing and its explicit form may be unknown or depend on uncertain parameters. Indeed, for a given pipe $i \in \mathcal{E}$, the function $f_{\text{E},i}$ models the pressure drop caused by the friction between the pipe's interior wall and the stream of water; through $f_{\text{E},i}$ we also model the pressure drop caused by valves.

In Section II-A3 we recall results from [10] to associate (1) to an equivalent ODE-based model. Consider first the following additional details on $f_{\text{E},i}$.

Assumption 1: For each $i \in \mathcal{E}$ representing a pipe or a valve, the function $f_{\text{E,i}}$ in (1) is given by

$$f_{\text{E},i}(q_{\text{E},i}) = \theta_i |q_{\text{E},i}| q_{\text{E},i}, \tag{2}$$

where θ_i is an *unknown* positive scalar. If $i \in \mathcal{E}$ is a pump, then we take $\theta_i = 0$.

For pipes, the model (2) is related to the Darcy-Weisbach formula and is widely used in the literature (see, e.g., [9], [11]) to describe pressure drops associated to viscous friction. The coefficient θ_i notably depends on the pipe's roughness, which can be difficult to estimate accurately. We also use the model (2) to describe pressure drops in valves, as done, e.g., in [9] and [12]. In this case, θ_i depends on the opening degree of the valve and on parameters intrinsic to any type of valve, the values of which can become uncertain due to aging or corrosion. Out of simplicity, θ_i is assumed to be *constant*. See [13, Remarks 1 and 2] for more details.

2) Dynamics of Nodes: The volume $V_{\text{N},k}$ of every node $k \in \mathcal{N}$ in the DH system evolves according to the mass balance equation per node, which in view of the assumption of incompressibility and constant density of water, is equivalent to $\dot{V}_{\text{N},k} = \sum_{i \in \mathcal{I}_k} q_{\text{E},i}$, where $\mathcal{I}_k$ is the set of edges that are incident to node k. Considering the DH system's incidence matrix $\mathcal{B}_0$, the set of all these equations, for all $k \in \mathcal{N}$, can be written as

$$\dot{V}_{\text{N}} = \mathcal{B}_0 q_{\text{E}}, \tag{3}$$

where $q_{\text{E}} \in \mathbb{R}^{n_{\text{E}}}$ is a vector comprising the flow through every edge in the DH system. We assume that for any $k \in \mathcal{N}$ representing a simple junction, $V_{\text{N},k} = \delta_k$ for all time, with $\delta_k \geq 0$ constant. Then, (3) becomes a differential-algebraic equation. The latter assumption, which stems from the fact that simple junctions are of a much smaller dimension than storage tanks,

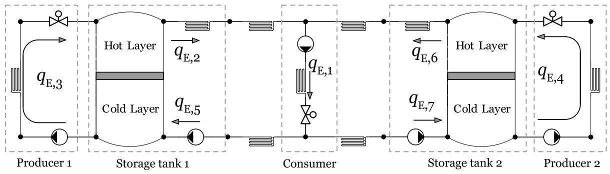

Fig. 3. Schematic diagram of a simplified DH system with two producers and one consumer. Each producer is interfaced to the distribution network through a storage tank. Only the flows through selected devices are shown.

is useful to write a reduced-order, ODE-based model equivalent to (3) that focuses on the volume dynamics of storage tanks; see Section II-A3. Out of simplicity we take $\delta_k = 0$.

3) A Reduced-Order ODE-Based Hydraulic Model: We recall first three instrumental assumptions from [10] to write equivalent ODE-based models of (1) and (3). These are: **(a)** every producer is interfaced to the distribution network through a storage tank as depicted in Fig. 3; **(b)** the total volume of water in each tank remains constant and at maximum capacity for all time; and **(c)** there are no standalone storage tanks in the system. The reader is referred to [10, Sec. 2.1] for details, however, we would like to add here that assumption **(b)** is commonly found in related literature and is usually understood as the desired operation mode for the considered type of storage units; see, e.g., [2], [14].

Regarding the flow dynamics (1), we summarize the results in [10] as follows. Considering assumptions **(a)**, **(b)** and **(c)**, the overall DH system's flow vector q_E is completely determined by the flow through a selected number of devices. More precisely, there exists a constant matrix $\mathcal{F}$, $\mathcal{F}_{ij} \in \{-1, 0, 1\}$, such that

$$q_E = \mathcal{F}^\top \begin{bmatrix} q_{ch} \\ q_{pr} \end{bmatrix}, \tag{4}$$

where $q_{pr} \in \mathbb{R}^{n_{pr}}$ comprises the flows through each producer and $q_{ch} \in \mathbb{R}^{n_{ch}}$ stacks the flows through each consumer, through some pipes of the distribution network (only those generating fundamental loops [10]), and the flow at the hot layer's outlet pipe of each storage tank (except for one). Then, $n_{ch} = n_c + a + n_{pr} - 1$, where $a \geq 0$ denotes the number of (fundamental) loops in the distribution network. We remark that the components of q_{ch} and q_{pr} represent a set of independent variables. In the example of Fig. 3, $q_{ch} = (q_{E,1}, q_{E,2})$, $q_{pr} = (q_{E,3}, q_{E,4})$ and $a = 0$. The remaining flows are dependent on these as $q_{E,6} = q_{E,1} - q_{E,2}$ and, due to assumption **(b)**, $q_{E,5} = q_{E,2}$ and $q_{E,7} = q_{E,6}$.

Moreover, q_{ch} and q_{pr} satisfy the decoupled ODEs

$$J_{ch}\dot{q}_{ch} = f_{ch}(q_{ch}) + u_{ch}, \tag{5a}$$

$$J_{pr}\dot{q}_{pr} = f_{pr}(q_{pr}) + u_{pr}, \tag{5b}$$

where the matrix J_{ch} is symmetric, positive definite, and J_{pr} is diagonal and positive definite as well; both matrices are constant and depend on the parameters $J_{E,i}$ in (1). Also, $u_{ch,i}$ and $u_{pr,i}$ are independent control inputs and represent the pressure difference of hydraulic pumps in series with the devices whose flows are $q_{ch,i}$ and $q_{pr,i}$, respectively. Moreover, $-f_{ch}$ and $-f_{pr}$, which are associated with $f_{E,i}$ in (1) (see also (2)), are nonlinear, continuously differentiable and *monotone* mappings. Notably, each component of $f_{pr,i}$ can be written as

$$f_{pr,i}(q_{pr,i}) = -\theta_i |q_{pr,i}| q_{pr,i}, \quad i = 1, \ldots, n_{pr}. \tag{6}$$

This fact is fundamental to our developments in Section III. We remark on the other hand that the explicit form of $f_{ch,i}$ is not necessary to establish the main results and conclusions of this work.

We move on to reduce the nodes' volume dynamics (3). By considering (4) and assumptions **(a)**, **(b)** and **(c)**, then it can be shown (see [13, Sec. II-A3]) that (3) can be reduced to the following ODE:

$$\dot{V}_{sh} = q_{pr} - Bq_{ch}, \tag{7}$$

where $V_{sh} \in \mathbb{R}^{n_{ST}}$, with $n_{ST} = n_{pr}$, comprises the volumes of water in the hot layers of the storage tanks, and B is an appropriate sub-block of matrix $\mathcal{F}$ in (4) (thus $B_{ij} \in \{-1, 0, 1\}$). We remark that $(Bq_{ch})_i$ represents the flow at the hot layer's outlet of the storage tank to which we associate $V_{sh,i}$.

This section is concluded emphasizing that the validity of (5) relies on two additional assumptions concerning the topology of the distribution network and the placement of the system's pumps. All the details, which are not included here for space reasons, can be found in [10] (see also [13, Remarks 3–5]).

B. Problem Formulation

This letter is concerned with the objective of simultaneously regulating the DH system's flow and volume vectors q_{ch} and V_{sh} towards desired constant setpoints $q_{ch}^\star$ and $V_{sh}^\star$, respectively. More precisely, our goal is that the overall flow and volume dynamics conformed by (5) and (7) attain

$$\lim_{t\to\infty} q_{ch} = q_{ch}^\star, \text{ and } \lim_{t\to\infty} V_{sh} = V_{sh}^\star,$$

for an identifiable set of initial conditions.

Concerning the regulation of q_{ch}, we note that the heat transport from producers to consumers depends strongly on the flows through the distribution network; in particular, consumers usually regulate their heat demand and temperature indirectly by adjusting their flow. The regulation of V_{sh} on the other hand is relevant for augmenting or reducing the stored (useful) energy in a tank, as the latter is proportional to the volume in the hot layer of the tank [2].

To achieve the desired objective, we design for each of the system inputs $u_{ch,i}$ and $u_{pr,i}$, decentralized and dynamic control laws of the form

$$\begin{aligned} \dot{x}_{c,i} &= g_{c,i}(\xi_{c,i}, x_{c,i}), \\ u_{c,i} &= h_{c,i}(\xi_{c,i}, x_{c,i}), \end{aligned} \tag{8}$$

where $c \in \{ch, pr\}$, $x_{c,i}$ is the state of the controller and $\xi_{c,i}$ is a vector comprising signals and parameters available to $u_{c,i}$. By decentralized we mean that $u_{c,i}$ uses information that is locally available at its associated heat producer, consumer or distribution pipe. More precisely, we assume that

$$\begin{aligned} \xi_{ch,i} &= \begin{bmatrix} q_{ch,i}, & q_{ch,i}^\star \end{bmatrix}^\top, \\ \xi_{pr,i} &= \begin{bmatrix} q_{pr,i}, & V_{sh,i}, & (Bq_{ch})_i, & J_{pr,i}, & V_{sh,i}^\star \end{bmatrix}^\top. \end{aligned} \tag{9}$$

Remark 1: We recall that $(Bq_{\rm ch})_i$ represents the flow at the ith tank's hot layer outlet (see Fig. 3) and we assume that it can be measured locally by the ith producer. Then, we underscore that the knowledge of B is not needed to establish our main results in Section III and it is not needed to compute $\xi_{{\rm ch},i}$ nor $\xi_{{\rm pr},i}$. The same applies to each θ_i, as it has been assumed to be an unknown positive scalar (see (2) and (6)).

III. Volume Regulation Controller

In this section we detail the aspects of the proposed solution to the formulated problem. In view of the cascade structure of the open-loop flow and volume dynamics (5) and (7), we first present a decentralized, proportional-integral controller for the stabilization of the subsystem (5a). The latter controller is inspired by the results of [4] (see also [10, Sec. 3.1]) addressing end-user *pressure* regulation in *single*-producer DH systems *without* storage units. Afterwards, we focus on the remaining dynamics, i.e., in (5b) and (7), and propose a novel adaptive control scheme to achieve volume regulation of the storage tanks.

Proposition 1: Consider the following controller of the form introduced in (8):

$$\begin{aligned}\dot{x}_{\rm ch} &= -M_{\rm ch}\big(q_{\rm ch} - q^\star_{\rm ch}\big)\\ u_{\rm ch} &= -N_{\rm ch}\big(q_{\rm ch} - q^\star_{\rm ch}\big) + x_{\rm ch},\end{aligned} \tag{10}$$

where $M_{\rm ch}$ and $N_{\rm ch}$ are constant, positive definite diagonal matrices. Then, the subsystem (5a) in closed-loop with (10) admits a globally asymptotically stable equilibrium point. Moreover, $\lim_{t\to\infty} q_{\rm ch} = q^\star_{\rm ch}$.

Proof: Consider the closed-loop system (5a), (10). We note that $(\bar{q}_{\rm ch}, \bar{x}_{\rm ch}) = (q^\star_{\rm ch}, -f_{\rm ch}(q^\star_{\rm ch}))$ is its unique equilibrium point. Let us define the following function:

$$\begin{aligned}\mathcal{S}_{\rm ch}(q_{\rm ch}, x_{\rm ch}) = &\tfrac{1}{2}(q_{\rm ch} - \bar{q}_{\rm ch})^\top J_{\rm ch}(q_{\rm ch} - \bar{q}_{\rm ch})\\ &+ \tfrac{1}{2}(x_{\rm ch} - \bar{x}_{\rm ch})^\top M_{\rm ch}^{-1}(x_{\rm ch} - \bar{x}_{\rm ch}),\end{aligned} \tag{11}$$

which is positive definite and radially unbounded. The time derivative of $\mathcal{S}_{\rm ch}$ along the trajectories of the closed-loop system satisfies

$$\dot{\mathcal{S}}_{\rm ch} \le -(q_{\rm ch} - \bar{q}_{\rm ch})^\top N_{\rm ch}(q_{\rm ch} - \bar{q}_{\rm ch}),$$

where to obtain the latter inequality we have used the equilibrium identity $\bar{x}_{\rm ch} = -f_{\rm ch}(\bar{q}_{\rm ch})$ together with the fact that $(q_{\rm ch} - \bar{q}_{\rm ch})^\top (f_{\rm ch}(q_{\rm ch}) - f_{\rm ch}(\bar{q}_{\rm ch})) \le 0$, which holds by virtue of $-f_{\rm ch}$ being a monotone mapping (see [10, Lemma 4]). Since the only solution of the closed-loop system that can stay identically in the set $E = \{(q_{\rm ch}, x_{\rm ch}) : \dot{\mathcal{S}}_{\rm ch} = 0 \Leftrightarrow q_{\rm ch} = \bar{q}_{\rm ch}\}$ is the equilibrium $(\bar{q}_{\rm ch}, \bar{x}_{\rm ch})$, then by LaSalle's invariance principle (see [15, Th. 3.5]), it is thus concluded that $(\bar{q}_{\rm ch}, \bar{x}_{\rm ch})$ is globally asymptotically stable (see [13]). ■

Remark 2: The monotonicity of $-f_{\rm ch}$ recalled in the above proof implies that (5a) (in open-loop) is *shifted passive* with storage function $\mathcal{H}(q_{\rm ch}) = \frac{1}{2}(q_{\rm ch} - \bar{q}_{\rm ch})^\top J_{\rm ch}(q_{\rm ch} - \bar{q}_{\rm ch})$ and passive output $q_{\rm ch}$. Therefore, along any solution of (5a), the following inequality is satisfied $\dot{\mathcal{H}}(q_{\rm ch}) \le (u_{\rm ch} - \bar{u}_{\rm ch})^\top (q_{\rm ch} - \bar{q}_{\rm ch})$, for any equilibrium pair $(\bar{u}_{\rm ch}, \bar{q}_{\rm ch})$ (see [16]). Based on the results of [17] for the stabilization of nonlinear RLC circuits and of [4] for pressure regulation of single-producer DH systems, we propose the controller (10).

Now, we turn our attention to the problem of regulating the volume of hot water of each storage tank towards constant, specified setpoints. Then, we focus on the system (5b), (7), which we write next in a more suitable equivalent form. On the one hand, considering (6) (see also Assumption 1), the mapping $f_{\rm pr}$ in (5b) can be written as

$$f_{\rm pr}(q_{\rm pr}) = -W(q_{\rm pr})\theta, \tag{12}$$

where $\theta = \begin{bmatrix}\theta_1, & \theta_2, & \cdots & \theta_{n_{\rm pr}}\end{bmatrix}^\top$ and $W(q_{\rm pr}) = \langle |q_{{\rm pr},i}| q_{{\rm pr},i}\rangle_{i=1}^{n_{\rm pr}}$. On the other hand, (7) can be equivalently written as

$$\dot{V}_{\rm sh} = q_{\rm pr} - Bq^\star_{\rm ch} + \Psi(q_{\rm ch}), \tag{13}$$

where $\Psi(q_{\rm ch}) = B(q^\star_{\rm ch} - q_{\rm ch})$. Thus, the system of interest to address storage volume regulation is equivalent to

$$J_{\rm pr}\dot{q}_{\rm pr} = -W(q_{\rm pr})\theta + u_{\rm pr} \tag{14a}$$

$$\dot{V}_{\rm sh} = q_{\rm pr} - Bq^\star_{\rm ch} + \Psi(q_{\rm ch}), \tag{14b}$$

where $W(q_{\rm pr})\theta$, $Bq^\star_{\rm ch}$ and $\Psi(q_{\rm ch})$ act as disturbances. Indeed, we have considered that both θ and B are unknown to the DH system's input vectors $u_{\rm ch}$ and $u_{\rm pr}$ (see Remark 1). Also, the vector $q^\star_{\rm ch}$ is not necessarily available to each $u_{{\rm pr},i}$ as there is no communication among producers and consumers.[2]

In the next proposition, by provisionally neglecting the effect of the disturbance $\Psi(q_{\rm ch})$, we present a stabilizing controller for (14). The proposed, suitably-tailored dynamic controller for $u_{\rm pr}$ attains asymptotic convergence of $V_{\rm sh}$ towards a desired constant value and estimates in real-time the unknown parameter vector θ. This result will be fundamental in Theorem 1 where, using cascade system arguments, we establish the asymptotic stability of the overall DH system's closed-loop dynamics, with $\Psi(q_{\rm ch})$ now acting on (14).

Proposition 2: Consider the system (14) and assume that $\Psi(q_{\rm ch}) = 0$ for all time. Define

$$z_{\rm pr} := q_{\rm pr} - x_{\rm a} + N_{\rm sh}\big(V_{\rm sh} - V^\star_{\rm sh}\big) \tag{15}$$

and consider the following dynamic controller

$$\dot{x}_{\rm a} = -M_{\rm a}\big(V_{\rm sh} - V^\star_{\rm sh}\big) \tag{16a}$$

$$\dot{x}_{\rm b} = -M_{\rm b}\tilde{W}(z_{\rm pr})z_{\rm pr} \tag{16b}$$

$$\begin{aligned}u_{\rm pr} = &\tilde{W}(z_{\rm pr})x_{\rm b} - \Big(J_{\rm pr}\Big(M_{\rm a} - N^2_{\rm sh}\Big) + I\Big)(V_{\rm sh} - V^\star_{\rm sh})\\ &- \big(J_{\rm pr}N_{\rm sh} + N_{\rm pr}\big)z_{\rm pr} + J_{\rm pr}N_{\rm sh}(Bq_{\rm ch} - x_{\rm a}),\end{aligned} \tag{16c}$$

where $N_{\rm pr}$, $N_{\rm sh}$, $M_{\rm a}$ and $M_{\rm b}$ are constant, positive definite diagonal matrices, and $\tilde{W}(z_{\rm pr}) := W(q_{\rm pr})|_{q_{\rm pr} = z_{\rm pr} + x_{\rm a} - N_{\rm sh}(V_{\rm sh} - V^\star_{\rm sh})}$. If the closed-loop system admits an equilibrium such that $\tilde{W}(\bar{z}_{\rm pr})$ is not identically zero, then said equilibrium is globally asymptotically stable. Moreover, $\lim_{t\to\infty} V_{\rm sh} = V^\star_{\rm sh}$, where $V^\star_{\rm sh}$ is a predefined, constant setpoint, and $\lim_{t\to\infty} x_{\rm b} = \theta$.[3]

Proof: Inspired by backstepping control design [15, Ch. 9], we propose first a change of variable from $q_{\rm pr}$ to $z_{\rm pr}$ as appears

[2] Henceforth we assume that (5a) is in closed-loop with (10), then $\Psi(q_{\rm ch})$ is a bounded and vanishing disturbance to (14).

[3] Notably, in order to show that $x_b \to \theta$, we do not need persistency of excitation of any signal.

in (15). Then, (14), with $\Psi(q_{\mathrm{ch}}) = 0$, is transformed into the equivalent system

$$\begin{aligned} J_{\mathrm{pr}}\dot{z}_{\mathrm{pr}} &= -\tilde{W}(z_{\mathrm{pr}})\theta + J_{\mathrm{pr}}\Big(M_{\mathrm{a}} - N_{\mathrm{sh}}^2\Big)\big(V_{\mathrm{sh}} - V_{\mathrm{sh}}^{\star}\big) \\ &\quad + J_{\mathrm{pr}}N_{\mathrm{sh}}\big(z_{\mathrm{pr}} + x_{\mathrm{a}} - Bq_{\mathrm{ch}}^{\star}\big) + u_{\mathrm{pr}} \\ \dot{V}_{\mathrm{sh}} &= z_{\mathrm{pr}} + x_{\mathrm{a}} - N_{\mathrm{sh}}\big(V_{\mathrm{sh}} - V_{\mathrm{sh}}^{\star}\big) - Bq_{\mathrm{ch}}^{\star} \\ \dot{x}_{\mathrm{a}} &= -M_{\mathrm{a}}\big(V_{\mathrm{sh}} - V_{\mathrm{sh}}^{\star}\big). \end{aligned} \tag{17}$$

Substituting (16c) into (17), which in addition brings the variable x_{b} satisfying (16b), illustrates the IDA-PBC [18] feature of the controller by virtue of attaining a closed-loop system that we write in Hamiltonian form as follows:

$$\dot{X} = \underbrace{\begin{bmatrix} -N_{\mathrm{pr}} & -I & 0 & \tilde{W}(z_{\mathrm{pr}}) \\ I & -N_{\mathrm{sh}} & I & 0 \\ 0 & -I & 0 & 0 \\ -\tilde{W}(z_{\mathrm{pr}}) & 0 & 0 & 0 \end{bmatrix}}_{=:F(X)} \nabla\tilde{H}(X), \tag{18}$$

with state vector $X = (J_{\mathrm{pr}}z_{\mathrm{pr}}, V_{\mathrm{sh}}, M_{\mathrm{a}}^{-1}x_{\mathrm{a}}, M_{\mathrm{b}}^{-1}x_{\mathrm{b}})$ and Hamiltonian

$$\tilde{H} = \frac{1}{2}\big(X - \bar{X}\big)^{\top}\text{block.diag}\Big(J_{\mathrm{pr}}^{-1}, I, M_{\mathrm{a}}, M_{\mathrm{b}}\Big)\big(X - \bar{X}\big),$$

where

$$\bar{X} = (0, V_{\mathrm{sh}}^{\star}, M_{\mathrm{a}}^{-1}Bq_{\mathrm{ch}}^{\star}, M_{\mathrm{b}}^{-1}\theta) \tag{19}$$

is a constant vector. We underscore that the *real-time* estimation of θ, which is obtained from x_{b}, represents the adaptive aspect of the proposed controller (*see*, [19, Example 4]), provided that (18) is asymptotically stable. Next we show, using LaSalle's invariance principle, that $\bar{X}$ is globally asymptotically stable. Consider the Hamiltonian $\tilde{H}$ and observe that it is positive definite with respect to $\bar{X}$ and radially unbounded with respect to X. Moreover, along the solutions of (18) we have that:

$$\dot{\tilde{H}} = -z_{\mathrm{pr}}^{\top}N_{\mathrm{pr}}z_{\mathrm{pr}} - (V_{\mathrm{sh}} - V_{\mathrm{sh}}^{\star})^{\top}N_{\mathrm{sh}}(V_{\mathrm{sh}} - V_{\mathrm{sh}}^{\star}) \leq 0.$$

Since no solution of (18) can stay in $S = \{X : \dot{\tilde{H}}(X) = 0 \Leftrightarrow z_{\mathrm{pr}} = 0,\ V_{\mathrm{sh}} = V_{\mathrm{sh}}^{\star}\}$ other than $\bar{X}$, then we conclude, invoking LaSalle's invariance principle (see [15, Th. 3.5]), that $\bar{X}$ is *globally* asymptotically stable (see [13] for additional details). ∎

Before presenting the next result, which concerns the asymptotic stability of the overall DH system hydraulic dynamics, consider the following:

Remark 3: The assumption about the equilibrium of the closed-loop system (14), (15), (16) satisfying $\tilde{W}(\bar{z}_{\mathrm{pr}}) \neq 0$, which in view of $\tilde{W}(z_{\mathrm{pr}}) = W(q_{\mathrm{pr}})$ is equivalent to $\bar{q}_{\mathrm{pr},i} \neq 0$ (for all i), notably guarantees that $x_{\mathrm{b}} \to \theta$ asymptotically, i.e., θ can be accurately estimated via the proposed adaptive scheme (*see*, [19, Example 4]), overcoming the challenging unknown and time-varying disturbance $-W(q_{\mathrm{pr}})\theta$ acting on (14), which we recall stems from Assumption 1. Considering (7), it is clear that a necessary and sufficient condition for $\bar{q}_{\mathrm{pr}} \neq 0$ is that $B\bar{q}_{\mathrm{ch}} \neq 0$, which is a condition that can potentially be enforced through an adequate choice of the setpoint $q_{\mathrm{ch}}^{\star}$ (see Proposition 1). We note also that a steady-state condition in which $\bar{q}_{\mathrm{pr},i} = 0$, for some index i, implies that the associated heat producer is not in operation.

Theorem 1: The overall closed-loop flow and volume dynamics of the DH system, described by (5), (7), (10), (15) and (16) has a (locally) asymptotically stable equilibrium, and

$$\lim_{t\to\infty} q_{\mathrm{ch}} = q_{\mathrm{ch}}^{\star}, \text{ and } \lim_{t\to\infty} V_{\mathrm{sh}} = V_{\mathrm{sh}}^{\star}, \tag{20}$$

where $q_{\mathrm{ch}}^{\star}$ and $V_{\mathrm{sh}}^{\star}$ are pre-specified setpoints, provided that $\bar{q}_{\mathrm{pr},i} \neq 0$, for all $i = 1, 2, \ldots, n_{\mathrm{pr}}$.

Proof: Let Σ_{ch} denote the dynamics (5a) in closed-loop with (10). Also, let $\Sigma_{\mathrm{pr,sh}}$ represent (5b), (7) (see also (14)) in closed-loop with (16). In view of (15) and (18), $\Sigma_{\mathrm{pr,sh}}$ is equivalent to (18), but perturbed by the additive disturbance $[0_{n_{\mathrm{pr}}}^{\top}, \Psi^{\top}(q_{\mathrm{ch}}), 0_{2n_{\mathrm{pr}}}^{\top}]$. Then, the overall DH system's closed-loop flow and volume dynamics is given by $\Sigma_{\mathrm{ch}} \circ \Sigma_{\mathrm{pr,sh}}$, which has a cascade structure as Σ_{ch} is independent of the states of $\Sigma_{\mathrm{pr,sh}}$. Moreover, $\Sigma_{\mathrm{ch}} \circ \Sigma_{\mathrm{pr,sh}}$ has a unique equilibrium point. Indeed, on the one hand $(\bar{q}_{\mathrm{ch}}, \bar{x}_{\mathrm{ch}}) = (q_{\mathrm{ch}}^{\star}, -f_{\mathrm{ch}}(q_{\mathrm{ch}}^{\star}))$ is the unique equilibrium of Σ_{ch} (see Proposition 1). On the other, it is clear that if $q_{\mathrm{ch}} = \bar{q}_{\mathrm{ch}}$, which implies $\Psi(q_{\mathrm{ch}}) = 0$, then $\bar{X}$, as given in (19), is a unique equilibrium of $\Sigma_{\mathrm{pr,sh}}$, provided that $F(\bar{X})$ is non singular, which holds by assumption (see equation (18) and Remark 3). It follows that $(\bar{q}_{\mathrm{ch}}, \bar{x}_{\mathrm{ch}}, \bar{X})$ is a unique equilibrium point of $\Sigma_{\mathrm{ch}} \circ \Sigma_{\mathrm{pr,sh}}$.

We see next that $(\bar{q}_{\mathrm{ch}}, \bar{x}_{\mathrm{ch}}, \bar{X})$ is (locally) asymptotically stable. We recall first that $(\bar{q}_{\mathrm{ch}}, \bar{x}_{\mathrm{ch}})$ is globally asymptotically stable for Σ_{ch} (see Proposition 1). Secondly, it was established in Proposition 2 that if $q_{\mathrm{ch}} = \bar{q}_{\mathrm{ch}}$ ($\Rightarrow \Psi(q_{\mathrm{ch}}) = 0$), then $\bar{X}$ is globally asymptotically stable for $\Sigma_{\mathrm{pr,sh}}$. Thus, we can invoke [20, Proposition 4.1] to conclude that the overall (coupled) system $\Sigma_{\mathrm{ch}} \circ \Sigma_{\mathrm{pr,sh}}$, with $\Psi(q_{\mathrm{ch}})$ acting as an exogenous *vanishing* disturbance on $\Sigma_{\mathrm{pr,sh}}$, admits $(\bar{q}_{\mathrm{ch}}, \bar{x}_{\mathrm{ch}}, \bar{X})$ as a unique, locally asymptotically stable equilibrium point (see [13] for additional details). ∎

Remark 4: In the preceding proof, the subsystems Σ_{ch} and $\Sigma_{\mathrm{pr,sh}}$ were shown to be *globally* asymptotically stable if they are decoupled, i.e., if $\Psi(q_{\mathrm{ch}}) = 0$ for all time. Notwithstanding, the result [20, Proposition 4.1] allows us only to claim *local* stability of the overall coupled system $\Sigma_{\mathrm{ch}} \circ \Sigma_{\mathrm{pr,sh}}$, including the effect of $\Psi(q_{\mathrm{ch}})$. In view of this drawback, part of our current research efforts are aimed at providing estimates of the system's domain of attraction.

Remark 5: Since the matrices M_{ch}, N_{ch}, N_{pr}, N_{sh}, M_{a} and M_{b} are all diagonal, then dynamic controllers (10) and (16) are fully decentralized.

IV. Numerical Simulations

In this section the performance of the DH system model in closed-loop with the proposed controller is illustrated via numerical simulations. We have used the configuration and data of the case study reported in [10, Sec. 4], which corresponds to a DH system with three heat producers ($n_{\mathrm{pr}} = 3$), nine consumers ($n_c = 9$) and with the same topology as the sketch shown in Fig. 1. Thus, $n_{\mathrm{ch}} = 17$. All producers are interfaced to the distribution network through storage tanks; each tank is assumed to have a total capacity of 1000 m^3.

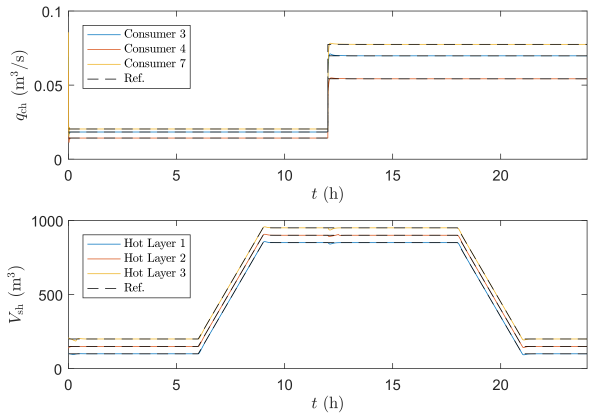

Fig. 4. Evolution of the flow vector q_{ch} (top) and of the volume of hot water in the storage tanks (bottom). The tuning gains of the dynamic controllers (10) and (16) are taken as $M_{\mathrm{ch}} = N_{\mathrm{ch}} = 10^5 I_{n_{\mathrm{ch}}}$, and $N_{\mathrm{pr}} = 7.11 \times 10^4 I_{n_{\mathrm{pr}}}$, $N_{\mathrm{sh}} = 7.5 \times 10^{-3} I_{n_{\mathrm{pr}}}$, $M_{\mathrm{a}} = 14.06 \times 10^{-5} I_{n_{\mathrm{pr}}}$ and $M_{\mathrm{b}} = 7.11 \times 10^7 I_{n_{\mathrm{pr}}}$, respectively. This selection is based on a trial-and-error procedure aimed at attaining a fair balance between settling time and overshoot for the signals of interest (see [13] for additional details).

Consider Fig. 4. The system is initialized in the vicinity of system's equilibrium representing a context of low consumer demand (25% w.r.t. full demand) and with relatively small set-points for each component of V_{sh}; the consumers' heat demand is proportional to their flow setpoints. Convergence is observed after a short transient. At $t = 6$h all storage tanks switch to a charging mode and attain their respective desired volume at approximately $t = 9$h. At $t = 12$h the reference value $q^\star_{\mathrm{ch}}$ is changed to represent a context of high consumer demand (from 25% to 95% w.r.t. full demand). The plot of q_{ch} shows that convergence is achieved relatively quickly. At $t = 18$h the tanks switch now to a discharging mode that ends at approximately $t = 21$h. The new, lower values for the entries of $V^\star_{\mathrm{sh}}$ are maintained until the end of the simulation.

V. Conclusion

In this work we have addressed the flow and storage volume regulation of a multi-producer DH system via a novel adaptive decentralized control scheme which offers closed-loop (local) stability guarantees and overcomes the nonlinear, networked and uncertain characteristics of the considered system model, notably stemming from our consideration of frictional effects in pipes. Part of our ongoing research is related to the following: (i) the establishment of estimates of the closed-loop system's domain of attraction; (ii) the identification of gain tuning rules to improve performance, e.g., the speed of convergence of x_{b}; and (iii) the compatibility of our control design procedure with other, more general, parameter and disturbance estimation schemes (see [21]), to formally study and address the effects of measurement noise, which were not considered in the present work.

References

[1] A. Vandermeulen, B. van der Heijde, and L. Helsen, "Controlling district heating and cooling networks to unlock flexibility: A review," *Energy*, vol. 151, pp. 103–115, May 2018. [Online]. Available: https://doi.org/10.1016/j.energy.2018.03.034

[2] T. Scholten, C. De Persis, and P. Tesi, "Modeling and control of heat networks with storage: The single-producer multiple-consumer case," *IEEE Trans. Control Syst. Technol.*, vol. 25, no. 2, pp. 414–427, Mar. 2017.

[3] C. De Persis and C. S. Kallesøe, "Pressure regulation in nonlinear hydraulic networks by positive and quantized controls," *IEEE Trans. Control Syst. Technol.*, vol. 19, no. 6, pp. 1371–1383, Nov. 2011.

[4] C. De Persis, T. N. Jensen, R. Ortega, and R. Wisniewski, "Output regulation of large-scale hydraulic networks," *IEEE Trans. Control Syst. Technol.*, vol. 22, no. 1, pp. 238–245, Jan. 2014.

[5] J. Bendtsen, J. Val, C. Kallesøe, and M. Krstic, "Control of district heating system with flow-dependent delays," *IFAC-PapersOnLine*, vol. 50, no. 1, pp. 13612–13617, 2017.

[6] H. Wang, H. Wang, Z. Haijian, and T. Zhu, "Optimization modeling for smart operation of multi-source district heating with distributed variable-speed pumps," *Energy*, vol. 138, pp. 1247–1262, Nov. 2017.

[7] G. Sandou *et al.*, "Predictive control of a complex district heating network," *Proc. 44th IEEE Conf. Decis. Control Eur. Control Conf. (CDC-ECC)*. Seville, Spain, 2005, pp. 7372–7377.

[8] S. Trip, T. Scholten, and C. De Persis, "Optimal regulation of flow networks with transient constraints," *Automatica*, vol. 104, pp. 141–153, Jun. 2019.

[9] Y. Wang, S. You, H. Zhang, W. Zheng, X. Zheng, and Q. Miao, "Hydraulic performance optimization of meshed district heating network with multiple heat sources," *Energy*, vol. 126, pp. 603–621, May 2017. [Online]. Available: http://dx.doi.org/10.1016/j.energy.2017.03.044

[10] J. E. Machado, M. Cucuzzella, and J. M. A. Scherpen, "Modeling and passivity properties of district heating systems," 2021. [Online]. Available: arXiv:2011.05419.

[11] S.-A. Hauschild *et al.*, "Port-hamiltonian modeling of district heating networks," in *Progress in Differential-Algebraic Equations II*. Cham, Switzerland: Springer, 2020, pp. 333–355. [Online]. Available: https://link.springer.com/chapter/10.1007/978-3-030-53905-4_11

[12] P. Vladimarsson, "District heat distribution networks," in *United Nations University Geothermal Training Programme*, vol. 30, United Nations Univ., Tokyo, Japan, 1898, pp. 239–240.

[13] J. E. Machado, M. Cucuzzella, N. Pronk, and J. M. A. Scherpen, "Adaptive control for flow and volume regulation in multi-producer district heating systems," 2021. [Online]. Available: arXiv:2103.06568.

[14] V. Verda and F. Colella, "Primary energy savings through thermal storage in district heating networks," *Energy*, vol. 36, no. 7, pp. 4278–4286, 2011. [Online]. Available: http://dx.doi.org/10.1016/j.energy.2011.04.015

[15] H. Khalil, *Nonlinear Control, Global Edition*. Harlow, U.K.: Pearson, 2015.

[16] N. Monshizadeh, P. Monshizadeh, R. Ortega, and A. van der Schaft, "Conditions on shifted passivity of port-Hamiltonian systems," *Syst. Control Lett.*, vol. 123, pp. 55–61, Jan. 2019. [Online]. Available: https://doi.org/10.1016/j.sysconle.2018.10.010

[17] B. Jayawardhana, R. Ortega, E. García-Canseco, and F. Castaños, "Passivity of nonlinear incremental systems: Application to PI stabilization of nonlinear RLC circuits," *Syst. Control Lett.*, vol. 56, pp. 618–622, Sep./Oct. 2007.

[18] R. Ortega and E. García-Canseco, "Interconnection and damping assignment passivity-based control: A survey," *Eur. J. Control*, vol. 10, no. 5, pp. 432–450, 2004.

[19] S. P. Nageshrao, G. A. Lopes, D. Jeltsema, and R. Babuška, "Port-Hamiltonian systems in adaptive and learning control: A survey," *IEEE Trans. Autom. Control*, vol. 61, no. 5, pp. 1223–1238, May 2016.

[20] R. Sepulchre, M. Jankovíc, and P. V. Kokotovic, *Constructive Nonlinear Control*. London, U.K.: Springer, 1997.

[21] I. Kolmanovsky, I. Sivergina, and J. Sun, "Simultaneous input and parameter estimation with input observers and set-membership parameter bounding: Theory and an automotive application," *Int. J. Adapt. Control Signal Process.*, vol. 20, no. 5, pp. 225–246, 2006.