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Coexistence of competing strategies in evolutionary games

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Document Version
Publisher's PDF, also known as Version of record

Publication date:
2015

Link to publication in University of Groningen/UMCG research database

Citation for published version (APA):
Zhang, J. (2015). *Coexistence of competing strategies in evolutionary games*. [Thesis fully internal (DIV), University of Groningen]. University of Groningen.

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Chapter 6

Effects of Opting Out and Insurance in Public Goods Games

Self-interest frequently causes individuals engaged in joint enterprises to choose actions that are counterproductive. Free-riders can invade a society of cooperators, causing a tragedy of the commons. In our previous works, we have investigated the influences of buying a policy that sequentially covers all punishment costs on the evolution of cooperation in public goods games with potential punishment on defectors, illustrating that insurance against punishment does not destabilize cooperation under realistic assumptions. There is compelling evidence that voluntary participation are effective mechanisms in ascertaining the evolution and stability of cooperation. As an extension form of evolutionary public goods game, competition among cooperative, defective with probabilistic punishment, speculation insured by some policy, and loner strategies is investigated here. By means of an evolutionary game theoretical approach, results suggest that our model displays complex dynamic behaviors. Depending on the initial condition, the state converges either to a domination of cooperators, or to a rock-scissors-paper type heteroclinic cycle of three strategies. Our model is, therefore, expected to shed light on the role of voluntary participation and speculation in solving the befuddling problem about the emergence of cooperative behaviors.

6.1 Introduction

Situations in which the private interest can be at odds with the public interest constitute an important class of societal problems. Evolutionary game theory is an interdisciplinary mathematical tool which seems to be able to embody several relevant features of the problem and, as such, is used in much cooperation-oriented research. In particular, the oft-cited public goods game (Hardin 1968, Axelrod and Dion 1988, Heckathorn 1996, Fehr and Gächter 2002, Brandt et al. 2006, Hauert et al. 2002b) is a paradigm example for investigating the emergence of cooperation in spite of the fact that self-interest seems to dictate defective behavior.

In typical public goods games, the so-called social dilemmas can be considered as binary situations in which two strategies are available: either choose alternative cooperation (C) in order to serve the public interest, or choose alternative defection (D), which serves the immediate private interest. The individual contributions are multiplied by a factor r and then divided equally among all players. With r smaller than the group size, this is an example of a riddle from the evolutionary viewpoint: individuals who do not contribute, but exploit the public goods, fare better than those who pay the cost by contributing. Thus, natural selection favors defection and leads to a social dilemma, because when all defect the mean payoff is lower than that when all cooperate.

A variety of solutions for this dilemma have been discussed in the past studies. The theory of kin selection focuses on cooperation among individuals that are genetically closely related, whereas theories of direct reciprocity focus on the selfish incentives for cooperation in bilateral long-term interactions (Imhof and Nowak 2010, Nowak 2006, Ohtsuki and Nowak 2007, Pacheco et al. 2008). The theories of indirect reciprocity and costly signalling indicate how cooperation in larger groups can emerge when the cooperators can build a reputation (Nowak and Sigmund 2005, Berger 2011, Brandt and Sigmund 2005). Current research has also highlighted two factors boosting cooperation in public goods interactions, namely, punishment of defectors (Gurerk et al. 2006, Hauert et al. 2007, Sigmund 2007, Brandt et al. 2003, Helbing et al. 2010, Gächter et al. 2008, Henrich 2006) and the option to abstain from the joint enterprise. Voluntary participation (Hauert et al. 2002b, Hauert et al. 2002a) allows individuals to adopt a risk-aversion strategy, termed loner. A loner refuses to participate in unpromising public enterprises and instead relies on a small but fixed payoff.

A strong body of theoretical and empirical evidence points to the importance of punishment as a major factor for sustaining cooperation in public goods games (Sigmund et al. 2010b, Fehr and Gachter 2005, Fowler et al. 2005). In addition, our previous work (Zhang et al. 2013) has studied a simple model to investigate the question whether stable cooperation can break down in the presence of speculation, a kind of risk-aversion strategy. Results indicate scenarios where speculation either leads to the reduction of the basin of attraction of the cooperative equilibrium or even the loss of stability of this equilibrium, if the costs of the insurance are lower than the expected fines faced by a defector. We reach the conclusion that an insurance of this type is not viable and under realistic assumptions speculation does not destabilize cooperation.

However, to our knowledge, past research paid little attention to the joint roles of punishment, voluntary participation and speculation in affecting the public goods provision. Actually, agents often have multiple choices in decision making due to the individual personality, especially when facing the potential punishment if defecting. Agents probably perform different behaviors due to the often observed different consciousness of risk prevention in real world. For example, resolute defectors will persist in their defection strategy, though taking the risk of being punished with a probability. Speculators incline to buy an insurance policy covering the costs of punishment when caught defecting. While timid loners will conservatively obtain an autarkic income independent of the other players' decision. These mentioned choices can better represents the possible attempts to raise money for public goods in complicated real-life situations. Thus, in most biological scenarios, the heterogeneity of competing individuals is an irrefutable fact and multiple choices is undeniably a part of biological reality. With this formulation, as an extension of our previous work proposing speculation (Zhang et al. 2013), we add the fourth strategy, called loner, which can refuse to participate and get some small but fixed income. As mentioned, it is based on the assumption that players can voluntarily decide whether to participate in the joint enterprise or not.

The four behavioral types in the population are: (a) the cooperators ready to join the group and to contribute their effort, (b) the defectors who join, but do not contribute, moreover, defectors are caught with a certain probability and a fine is imposed on them when caught. Here we are less interested in the specific establishment of an effective system of punishment, but the two additional options (speculation and loner) found in several systems. To address this question, we consider a public goods game with an external-agency punishment system indicated above.

(c) the speculators who purchase an insurance policy covering the costs of punishment when caught defecting. It means that by paying a fixed cost for their insurance policy, speculators can defect without paying any fine from punishment. (d) the loners unwilling to join the public goods game, but prefer to rely on a small but fixed payoff.

By means of a theoretical approach, we investigate the joint evolution of cooperation, defection, speculation and loner, focusing on the question whether such model will subsequently allow the stable establishment of sizable levels of cooperation.

6.2 The model setting

Our investigation is based on the public goods game (PGG), a paradigm to study the evolution of costly cooperation among selfish individuals, since it highlights the potential differences between individual interests and the social optimum. In the standard, obligatory public-goods model, the social dilemma can be considered as binary situations in which two strategies are available: either choose alternative C (cooperation) in order to serve the public interest, or choose alternative D (defection), which serves the immediate private interest.

To model this scenario with four strategies by evolutionary game theory, we assume a large population consisting of cooperators, defectors, speculators, and loners. To be precise, each participant receives an equal benefit rcx_c which is proportional to the fraction of cooperators (x_c) among the players. The costs associated with behaviors differ among strategies. Cooperators pay a fixed cost c as the contribution for the public goods game. Defectors contribute nothing, but will be possibly caught and then confronted with punishment. Their expected fine is α, which reflects the product of the probability of being detected and the fine in cost of detective. Speculators neither contribute for common goods nor pay a fine when caught, instead they pay an amount λ corresponding to the insurance policy. Loners obtain a fixed pay-off σ from a solitary pursuit without contribution.

We consider a very large, well-mixed population of players. From time to time, sample groups of N such players are chosen randomly and offered to join in a public goods game. Notably, the probability that two players in large populations ever encounter again can be neglected.

Within such a group, if N_c denotes the number of cooperators and N_l is the number of loners among the public goods players, the net payoffs of the four strategies are respectively given by

$$\begin{cases} P_c = \frac{rcN_c}{N-N_l} - c \\ P_d = \frac{rcN_c}{N-N_l} - \alpha \\ P_s = \frac{rcN_c}{N-N_l} - \lambda \\ P_l = \sigma \end{cases}, \tag{6.1}$$

where r denotes the amplification effect on the common pool. In this game, each unit of investment is multiplied by r and the product is distributed among all participants (except loners) irrespective of their strategies. The first term in the expression represents the benefit that the agent obtains from the public goods, while the second term denotes cost. For cooperators, the cost is the investment c to the public goods, and for speculators, the cost is the payment λ to the insurance. Selfish individuals will therefore always avoid the cost of altruism, i.e. a collective of selfish players will never cooperate.

In order to compute the payoff values for cooperators, defectors and speculators, we first derive the probability that n of the N sampled individuals are actually willing to join the public goods game. In the case $n = 1$ (no co-player shows up) we assume that the player has no other option than to play as a loner, and obtains payoff σ. This happens with probability x_l^{N-1}. Here, x_l is the fractions of loners. For a given player (C, D or S) willing to join the public goods game, the probability of finding, among the $N-1$ other players in the sample, $n-1$ co-players joining the group ($n > 1$), is given by

$$\binom{N-1}{n-1}(1-x_l)^{n-1}(x_l)^{N-n} \tag{6.2}$$

The probability that m of these players are cooperators is

$$\binom{n-1}{m}\left(\frac{x_c}{x_c+x_d+x_s}\right)^m\left(\frac{x_d+x_s}{x_c+x_d+x_s}\right)^{n-1-m} \tag{6.3}$$

where x_c, x_d, x_s respectively denote the fractions of cooperators, defectors and speculators in the population.

For simplicity and without loss of generality, we set the cost c of cooperation equal to 1. In the above case, the payoff for a defector is $rm/n - \alpha$, while the payoffs for a cooperator and a speculator are respectively specified by $r(m+1)/n - 1$ and $rm/n - \lambda$. Hence, the expected payoff for a defector in such a group is:

$$
\begin{aligned}
&(\frac{rm}{n} - \alpha)\sum_{m=0}^{n-1}\binom{n-1}{m}(\frac{x_c}{1-x_l})^m(1-\frac{x_c}{1-x_l})^{n-m-1}\\
&= \frac{r}{n}\cdot(n-1)\frac{x_c}{1-x_l} - \alpha
\end{aligned}
$$

The payoff of a cooperator in a group of n players is:

$$
\begin{aligned}
&[\frac{r(m+1)}{n} - 1]\sum_{m=0}^{n-1}\binom{n-1}{m}(\frac{x_c}{1-x_l})^m(1-\frac{x_c}{1-x_l})^{n-m-1}\\
&= \frac{r}{n}\cdot(n-1)\frac{x_c}{1-x_l} + \frac{r}{n} - 1
\end{aligned}
$$

The payoff of a speculator in a group of n players is:

$$
\begin{aligned}
&(\frac{rm}{s} - \lambda)\sum_{m=0}^{N-1}\binom{n-1}{m}(\frac{x_c}{1-x_l})^m(1-\frac{x_c}{1-x_l})^{n-m-1}\\
&= \frac{r}{n}\cdot(n-1)\frac{x_c}{1-x_l} - \lambda
\end{aligned}
$$

The payoff of a loner is the constant value of σ.

Then, the expected payoff for a defector in the population is,

$$
\begin{aligned}
P_d &= \sigma x_l^{N-1} + \sum_{n=2}^{N}[\frac{r}{n}\cdot(n-1)\frac{x_c}{1-x_l} - \alpha]\binom{N-1}{n-1}\\
&\qquad (1-x_l)^{n-1}(x_l)^{N-n}\\
&= \sigma x_l^{N-1} + \frac{rx_c}{1-x_l}[1 - \frac{1-x_l^N}{N(1-x_l)}] - \alpha(1-x_l^{N-1}) \qquad (6.4)
\end{aligned}
$$

In the continuous time model, the evolution of the fractions of the four strategies proceeds according to

$$\dot{x}_i = x_i(P_i - \bar{P}), \qquad (6.5)$$

where i can be c, d, s, l, and $\bar{P} = x_cP_c + x_dP_d + x_sP_s + x_l\sigma$.

6.3 Evolutionary dynamics outcomes

We firstly focus on the replicator dynamics starting from a three-strategy state in the population, then we pay attention to analyzing the output when all the four strategies initially exist in the population.

For the replicator dynamics of three-strategy evolution, we comprehensively consider four scenarios depicted in Fig. 1 till Fig. 4 as follows. The advantage of one strategy over another depends on the payoff difference between them, hence

$$\begin{aligned} P_d - P_c &= \sum_{n=2}^{N}[1-\frac{r}{n}-\alpha]\binom{N-1}{n-1}(1-x_l)^{n-1}(x_l)^{N-n} \\ &= 1-\alpha+(r-1+\alpha)x_l^{N-1}-\frac{r}{N}\frac{1-x_l^N}{1-x_l}; \qquad (6.6) \\ P_d - P_s &= \sum_{n=2}^{N}[\lambda-\alpha]\binom{N-1}{n-1}(1-x_l)^{n-1}(x_l)^{N-n} \\ &= (\lambda-\alpha)(1-x_l^{N-1}); \qquad (6.7) \\ P_s - P_c &= 1-\lambda+(r-1+\lambda)x_l^{N-1}-\frac{r}{N}\frac{1-x_l^N}{1-x_l}. \qquad (6.8) \end{aligned}$$

□

In the above calculations, $N > 1$, $1 < r < N$ and $\alpha > 0$. The sign of $P_i - P_j$ in fact determines whether it pays to switch from cooperation to defection or vice versa, $P_i - P_j = 0$ being the equilibrium condition, where i, j can be strategy C, D, S, and L.

We now proceed to the study of evolutionary dynamics when $\lambda \neq \alpha$ where four strategies coexist in the population, being referred to an interior point. We make the following three assumptions and will get the results that: at least one strategy will become extinct with the evolution of the system initialized from an interior point.

6.3.1. THEOREM. *If $\lambda \neq \alpha$, at least one strategy will become extinct with the evolution of the system initialized from an interior point. Here, an interior point means that the fraction of every strategy is larger than zero.*

Proof: We now analyze the system in different situations.
(1) when $\lambda \neq \alpha$, supposing $\lambda > \alpha$ (i.e. $P_d > P_s$), when $x_l \neq 0$. We suppose that there

is a closed set, meaning that the subsequent evolving state of each initial state in this set also belongs to this set. So $x_c > 0$, $x_d > 0$, $x_s > 0$ and $x_l > 0$ in this closed set.

(1.1) We first suppose only one point $(x_c^*, x_d^*, x_s^*, x_l^*)$ in this closed set, and satisfying $x_c^* > 0, x_d^* > 0, x_s^* > 0, x_c^* > 0$, and $\dot{x}_c^* = \dot{x}_d^* = \dot{x}_s^* = \dot{x}_l^* = 0$, thus

$$\begin{cases} \dot{x}_d^* = x_d^*(p_d^* - \bar{p}^*) \\ \dot{x}_s^* = x_s^*(p_s^* - \bar{p}^*) \end{cases} \tag{6.9}$$

Herein, the result $\dot{x}_d^* = \dot{x}_s^* = 0$ needs $\dot{p}_d^* = \bar{p}^* = \dot{p}_s^*$, which contradicts with $\dot{p}_d^* - \dot{p}_s^* > 0$. Therefore we can safely get the conclusion that there is no interior stable point.

(1.2) We next assume that the interior domain is a limit cycle. In this case, the four strategy players will gain the same average payoffs driven by the replicator equation, where $\bar{p}_c = \bar{p}_d = \bar{p}_s = \bar{p}_l$. However, $\bar{p}_d = \bar{p}_s$ contradicts with $p_d > p_s$, indicating that the closed set is not a limit cycle.

(1.3) We then verify whether the interior domain is a chaos, where also $x_c > 0, x_d > 0, x_s > 0, x_l > 0$. By introducing the fraction of defections in a population consisting of defectors and speculators, $f = \frac{x_d}{x_d + x_s}$, thus

$$\dot{f} = (\frac{x_d}{x_d + x_s})' = \frac{\dot{x}_d x_s - x_d \dot{x}_s}{(x_d + x_s)^2} = \frac{x_d x_s (p_d - p_s)}{(x_d + x_s)^2} > 0. \tag{6.10}$$

Then, $\lim_{t \to \infty}(\frac{x_d}{x_d + x_s}) = 1$ and $x_s \to 0$.

The above mentioned results suggest that, when $\lambda > \alpha$ there is no such a closed set, in which the evolving state of each initial state which consist of these four strategies in this set also belongs to this set.

(2) When $\lambda < \alpha$ and according to the results in (1), there is no internal domain.

(3) When $\lambda = \alpha$ and thus $p_d = p_s$, the four-strategy system was reduced to the simplex $T = (C, D, L)$ or $T = (C, S, L)$. We will discuss this situation in the following.

Summing up the above dynamics, we can safely get the following conclusions: $\lambda = \alpha$ reduce the system to a three-strategy game, and $\lambda \neq \alpha$ will lead to the distinction of at least one strategy.

6.3.1 Scenario 1: the corners of the simplex $T = (C, D, L)$

6.3.2. THEOREM. *If $r > 2 - 2\alpha$ holds, there there exists a threshold value of x_l in the interval $(0, 1)$, above which $P_d - P_c < 0$.*

Proof: Here, we employ the function $G(x_l) = (1 - x_l)(P_d - P_c)$ which has the same roots as $P_d - P_c$. For $x_l \in (0, 1)$,

$$\begin{aligned} G(x_l) &= (1 - x_l)(P_d - P_c) \\ &= (1 - \frac{r}{N} - \alpha) - (1 - \alpha)x_l + (r - 1 + \alpha)x_l^{N-1} + (\frac{r}{N} + 1 - \alpha - r)x_l^N \end{aligned} \tag{6.11}$$

$$G'(x_l) = (\alpha - 1) + (N - 1)(r - 1 + \alpha)x_l^{N-2} + N(\frac{r}{N} + 1 - \alpha - r)x_l^{N-1} \tag{6.12}$$

Note that $G(1) = G'(1) = 0$,

$$G''(1) = (N - 1)(N - 2)(r - 1 + \alpha)x_l^{N-3} + N(N - 1)(\frac{r}{N} + 1 - \alpha - r)x_l^{N-2} \tag{6.13}$$

$$G''(1) = (N - 1)(2 - 2\alpha - r) \tag{6.14}$$

We have

$$\begin{aligned} G(x_l) &\simeq G(1) + G'(1)(z - 1) + \frac{1}{2}G''(1)(z - 1)^2 \\ &= \frac{1}{2}(N - 1)(2 - 2\alpha - r)(1 - x_l)^2. \end{aligned} \tag{6.15}$$

For $r > 2 - 2\alpha$, $\lim_{x_l \to 1^-} G(x_l) < 0$,

$$G''(x_l) = x_l^{N-3}(N - 1)[(N - 2)(r - 1 - \alpha) + x_l(r + N - N\alpha - Nr). \tag{6.16}$$

Since $G''(x_l)$ changes sign at most once in the interval $(0, 1)$, we claim that there exists a threshold value of x_l in the interval $(0, 1)$, above which $P_d - P_c < 0$.

From the above analysis, we get

$$\begin{cases} G(x_l) = (1 - x_l)(P_d - P_c) \\ G(0) = 1 - \frac{r}{N} - \alpha \\ G(1) = 0 \end{cases} \tag{6.17}$$

As illustrated in Fig. 1, the game dynamics takes on three qualitatively different cases, which will be discussed one by one.

Case 1.1($1 - r/N - \alpha > 0$, i.e. $G(0) > 0$):

$$\lim_{x_l \to 1^-} G(x_l) = \frac{1}{2}(N-1)(2-2\alpha-r)(1-x_l)^2. \quad (6.18)$$

When $r < 2 - 2\alpha$, $G(x_l) > 0$, $x_l \in (0,1)$, the three corners represent a rock-scissors-paper type heteroclinic cycle, and there is no stable equilibrium of the game dynamics in this case.

Case 1.2 ($1 - r/N - \alpha > 0$,$r > 2 - 2\alpha$, $G(1^-) > 0$): the three corners represent a heteroclinic cycle. It is a center surrounded by closed orbits. Similar to case 1.1, there is no stable equilibrium of the game dynamics in this case.

Case 1.3 ($1 - r/N - \alpha < 0$, i.e. $r > 2 - 2\alpha$): In this case, for all x_s, pure speculation (S) and pure defection (D) are both unstable equilibria of the game dynamics. The cooperation equilibrium (C) is stable and in fact a global attractor.

Summarizing the three cases in this scenario corresponding to the simplex $T = (C, D, L)$, we can conclude that the three corners represent a rock-scissors-paper type heteroclinic cycle if $1 - r/N - \alpha > 0$ (cases 1.1 and 1.2) while pure cooperation is a global attractor if $1 - r/N - \alpha < 0$ (case 1.3). Hence, the outcome of the game dynamics depends on the model parameters. □

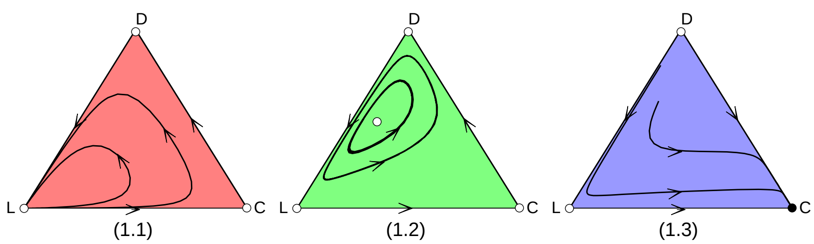

Figure 6.1: The evolution dynamics results of $T = (C, D, L)$, where in the absence of speculation. (1.1): $r < 2 - 2\alpha$. (1.2): $r > 2 - 2\alpha$; and (1.3): $1 - r/N - \alpha < 0$. Parameters: $N = 5, \delta = 0.3$, and $r = 1.6, \alpha = 0.1$ for (1.1); $r = 3, \alpha = 0.1$ for (1.2); $r = 3, \alpha = 0.5$ for (1.3). Open dots are unstable equilibrium points and closed dots are stable equilibrium points. It suggests that three corners represent a rock-scissors-paper type heteroclinic cycle if $1 - r/N - \alpha > 0$ (cases 1.1 and 1.2) while pure cooperation is a global attractor if $1 - r/N - \alpha < 0$ (case 1.3).

6.3.3. PROPOSITION. *When $T = (C, D, L)$, under the replicator dynamics of (6.5), it holds*

that
if $1 - r/N - \alpha > 0$ *and* $r < 2 - 2\alpha$*, there is no inner fixed point in* T*;*
if $1 - r/N - \alpha > 0$ *and* $r > 2 - 2\alpha$*, there is one inner fixed point in* T*;*
if $1 - r/N - \alpha < 0$*, full* C *is only stable fixed point in* T*.*

Proof: When $r > 2 - 2\alpha$, there exists a fixed point $x_l \in (0, 1)$ that $P_d = P_c$. From Eq. (6.4), we can get the only x_c and $x_d = 1 - x_l - x_c$, hence there is one inner fixed point in T. If $1 - r/N - \alpha > 0$ and $r < 2 - 2\alpha$, $P_d > P_c$ for all $x_l \in (0, 1)$, so there is no fixed point in T. If $1 - r/N - \alpha < 0$, we have $r > 2 - 2\alpha$, $(N > 2)$. Then it must be true that $P_c > P_d$, so full C is only stable fixed point in T. □

6.3.2 Scenario 2: the corners of the simplex $T = (C, D, S)$

$$\begin{cases} P_d - P_c = 1 - \alpha - \frac{r}{N} \\ P_d - P_s = \lambda - \alpha \\ P_c - P_s = \lambda + \frac{r}{N} - 1 \end{cases} \tag{6.19}$$

Case 2.1 ($\lambda - \alpha > 0$, $1 - \alpha - r/N > 0$ and $1 - \lambda - r/N > 0$): Here, pure cooperation and pure speculation are both unstable equilibria of the game dynamics. Full defection equilibrium (D) is stable and in fact a global attractor.

Case 2.2 ($\lambda - \alpha > 0$, $1 - \alpha - r/N > 0$ and $1 - \lambda - r/N < 0$): In this case, pure cooperation and pure speculation are both unstable equilibria of the game dynamics. Pure defection equilibrium (D) is stable and a global attractor. The difference between case 2.1 and case 2.2 is that when there are only cooperators and speculators in the population, pure cooperation is the attractor in case 2.2 while pure speculation is the attractor in case 2.1.

Case 2.3 ($\lambda - \alpha > 0$, $1 - \alpha - r/N < 0$, and $1 - \lambda - r/N < 0$): Herein, pure defection and pure speculation are both unstable equilibria of the game dynamics. Pure cooperation is a stable and global attractor.

Case 2.4 ($\lambda - \alpha < 0$, $1 - \alpha - r/N > 0$, and $1 - \lambda - r/N > 0$): In this case, pure speculation is the only stable and global attractor.

Case 2.5 ($\lambda - \alpha < 0$, $1 - \alpha - r/N < 0$, and $1 - \lambda - r/N < 0$): Pure cooperation is thus the only stable and global attractor.

Case 2.6 ($\lambda - \alpha < 0, 1 - \alpha - r/N < 0$, and $1 - \lambda - r/N > 0$): Pure speculation is the only stable and global attractor. The difference between case 2.6 and 2.4 is that when the population consists of only cooperators and defectors, pure cooperation is the attractor in case 2.6 while pure defection is the attractor in case 2.4.

Summarizing the six cases in scenario 2 corresponding to the simplex $T = (C, D, S)$, we can see that there is always a global attractor in the system. And similar with scenario 1, the outcome of the game dynamics depends on model parameters.

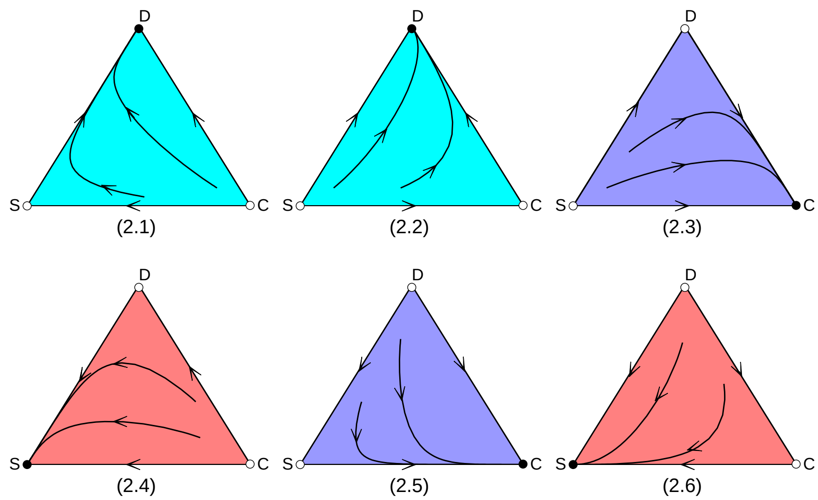

Figure 6.2: The evolution dynamics results of $T = (C, D, S)$, where in the absence of defection. We consider six cases, which are discussed in cases 2.1 till 2.3 in the upper panel of Fig. 6.2. Fig. 6.2 focuses on the situation $\lambda - \alpha > 0$ implying that the fine for defectors is higher than the costs of cooperation. Lower panels of Fig. 6.2 considers the opposite case $\lambda - \alpha < 0$, where defection is the dominating strategy. Results show that there is always a global attractor in the system, and the outcome of the game dynamics depends on model parameters. Parameters: $N = 5, r = 3, \delta = 0.3$, and $\alpha = 0.1, \lambda = 0.2$ for (2.1); $\alpha = 0.1, \lambda = 0.8$ for (2.2); $\alpha = 0.5, \lambda = 0.8$ for (2.3); $\alpha = 0.1, \lambda = 0.2$ for (2.4); $\alpha = 0.8, \lambda = 0.5$ for (2.5); $\alpha = 0.8, \lambda = 0.1$ for (2.6).

6.3.4. PROPOSITION. *When $T = (C, D, S)$, under the replicator dynamics of (6.5), it holds that*

if $\lambda - \alpha > 0$ and $1 - \alpha - r/N > 0$: full D is only stable fixed point in T;

if $1-\alpha-r/N<0$ *and* $1-\lambda-r/N<0$*: full* C *is only stable fixed point in* T*;*
if $\lambda-\alpha<0$ *and* $1-\lambda-r/N>$*: full* S *is only stable fixed point in* T*;*

Proof: When $x_l=0$, if $1-\alpha-r/N>0$, $P_d>P_c$; if $\lambda-\alpha>0$, $P_d>P_s$, therefore if $x_d>0$, $P_d>\bar{P}$. That means full D ($x_d=1$) is only stable fixed point in T.
When $x_l=0$, if $1-\alpha-r/N<$, $P_c>P_d$; if $1-\lambda-r/N<0$, $P_c>P_s$, therefore if $x_c>0$, $P_c>\bar{P}$. That means full C ($x_c=1$) is only stable fixed point in T.
When $x_l=0$, if $\lambda-\alpha<0$, $P_s>P_d$; if $1-\lambda-r/N>0$, $P_s>P_c$, therefore if $x_s>0$, $P_s>\bar{P}$. That means full S ($x_s=1$) is only stable fixed point in T. □

6.3.3 Scenario 3: the corners of the simplex $T=(C,L,S)$

It is easily observed that $x_l=0$ leads to $P_c-P_s=\lambda-1<0$. Thus, the three corners represent a rock-scissors-paper type heteroclinic cycle. There is no stable equilibrium in this case.

6.3.5. PROPOSITION. *When* $T=(C,S,L)$*, under the replicator dynamics of (6.5), it holds that*
if $1-r/N-\lambda>0$ *and* $r<2-2\lambda$*, there is no inner fixed point in* T*; if* $1-r/N-\lambda>0$ *and* $r>2-2\lambda$*, there is one inner fixed point in* T*; if* $1-r/N-\lambda<0$*, full* C *is only stable fixed point in* T*.*

Proof: By using λ takes the place of α, we can get the similar results with proposition 6.3.3. □

6.3.4 Scenario 4: the corners of the simplex $T=(D,L,S)$

Case 4.1 ($\lambda-\alpha<0$): In this case, pure loners is the only stable and in fact the only global attractor.
Case 4.2 ($\lambda-\alpha<0$): Still, pure loners remains the only stable and in fact the only global attractor. The difference between case 4.1 and 4.2 is that when there are only speculators and defectors in the population, pure speculation is the attractor in case 4.1 while pure defection is the attractor in case 4.2.

Summarizing the two cases in scenario 4 corresponding to the simplex $T = (C, D, S)$, we can conclude that pure loners is the only global attractor in the system.

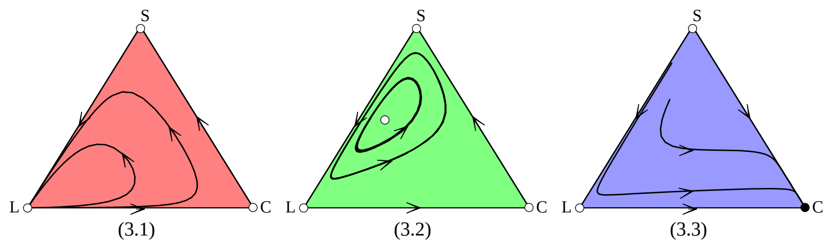

Figure 6.3: The evolution dynamics results of $T = (C, S, L)$, where in the absence of speculation. (3.1): $r < 2 - 2\lambda$. (3.2): $r > 2 - 2\lambda$; and (3.3): $1 - r/N - \lambda < 0$. Parameters: $N = 5, \delta = 0.3$, and $r = 1.6, \lambda = 0.1$ for (3.1); $r = 3, \lambda = 0.1$ for (3.2); $r = 3, \lambda = 0.5$ for (3.3). It suggests that three corners represent a rock-scissors-paper type heteroclinic cycle if $1 - r/N - \lambda > 0$ (cases 3.1 and 3.2) while pure cooperation is a global attractor if $1 - r/N - \lambda < 0$ (case 3.3).

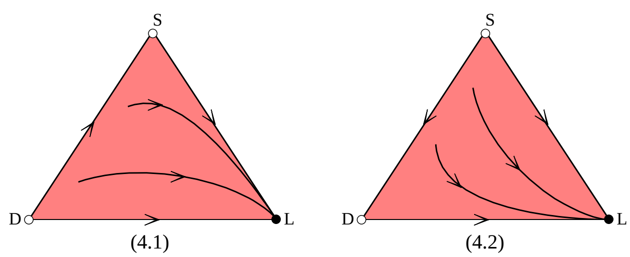

Figure 6.4: The evolution dynamics results of $T = (D, L, S)$ where in the absence of cooperation.(4.1) resulting game dynamics in the absence of speculation, where pure loners is the only global attractor in the system. Parameters: $N = 5, r = 3, \delta = 0.3$, and $\alpha = 0.4, \lambda = 0.1$ for (3); $\alpha = 0.4, \lambda = 0.1$ for (4.1); $\alpha = 0.1, \lambda = 0.4$ for (4.2).

6.3.6. PROPOSITION. *When $T = (S, D, L)$, under the replicator dynamics of (6.5), it holds that*

full L is only stable fixed point in T;

Proof: When $x_c = 0$, $P_l - P_d = (\alpha + \sigma)(1 - N_l^{N-1}) > 0$ and $P_l - P_s = (\lambda + \sigma)(1 -$

$N_l^{N-1}) > 0$, therefore full L ($x_l = 1$) is only stable fixed point in T. □

6.4 Conclusions

Public goods pose a riddle from the evolutionary viewpoint. The extensive research here is mainly focused on the exploration of strategic options. In the standard, obligatory public-goods model, the two simplest strategies choose always defection and cooperation, and the corresponding players are called defectors (shortly D) and cooperators (C), respectively. There is a growing evidence that the threat of punishment can induce self-interested players to prefer actions that sustain the public goods, and turn away from free riding. When facing potential punishment as a defector, speculation and optional participation are also feasible choices driven by individual diversity in a wide range of real-world situations.

Our previous work find scenarios where speculation either leads to the reduction of the basin of attraction of the cooperative equilibrium or even the loss of stability of this equilibrium, if the costs of the insurance are lower than the expected fines faced by a defector. As an extension of our study proposing speculation strategy(Zhang et al. 2013), here we base our analysis of the evolutionary game on replicator dynamics for four strategies: C (cooperators), D (defectors), S (speculators) and L (nonparticipants). For simplicity we assumed that punishment of a given effectiveness is externally imposed upon the defectors in a public goods game. We do not consider the question how the punishment system was established or who carries the costs of punishment.

Here, we show that the evolutionary fate of the system depends on special assumptions about model parameters. When starting from the three-strategy state, the observed domination of some strategy or a rock-paper-scissors type of cycle suggests that the additional strategic options can radically alter the evolution of cooperation. Specifically, larger multiplication factor r and punishment α on defectors can facilitate cooperation to be a dominant strategy in the absence of speculation (scenario 1). Here, we show that the option to abstain from the joint enterprise offers an escape from the social trap. This leads to the decline of exploiters and allows the reemergence of cooperators. Further, public goods cooperation can also be favored to be an equilibrium by moderate values of punishment α and cost of insurance λ in the absence of loner (scenarios 2). It is also intriguing that cooperation fails to dominate the population in the competition with speculation and loner strategy, even though in the absence of defection (scenarios 3). When the initial state consists of the four strategies, at least one strategy will evolve to vanish within the evolution.

Summarizing, we show that this conclusion depends on the particular assumptions of the proposed model here. An interesting future direction would be to address whether the presence of more strategy options altogether affect the dynamics of behaviors in the field of human cooperation.