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Coexistence of competing strategies in evolutionary games

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Chapter 2

Crucial Role of Strategy Updating for Evolution in Interaction Networks

Here, we start the main blocks of this thesis. In the beginning, I want to ask some questions: what is the essential attribute of competing among different strategies? Do you think the statement "we are born selfish" (Dawkins 1976) is right or wrong? On the other hand, is the statement "evolution is constructive because of cooperation, and that we might add natural cooperation as a third fundamental principle of evolution beside mutation and natural selection"(Nowak 2006) right or wrong? In this chapter, what I want to show is that it is really difficult to out-compete another strategy. In other words, it is easy to get coexistence for different strategies. Cooperation is important for different individuals, but defective behaviours are also of importance to be investigated because every strategy is difficult to be eliminated by other strategies (Zhang, Chen, Zhang, Wang and Chu 2010b, Zhang, Zhang and Chu 2010).

In this chapter, we start from a simple model with structured populations. We use complex networks for describing the structures of investigated populations. Network models are useful tools for studying the dynamics of social interactions in a structured population. After a round of interactions with the players in their local neighbourhood, players update their strategy based on the comparison of their own payoff with the payoff of one of their neighbours. Here we show that the assumptions made on strategy updating are of crucial importance for the strategy dynamics. In the first step, we demonstrate that seemingly small deviations from the standard assumptions on updating have major implications for the evolutionary outcome of two cooperation games: cooperation can more easily persist in a Prisoner's Dilemma game, while it can go more easily extinct in a Snowdrift game. To explain these outcomes, we develop a general model for the updating of states

in a network that allows us to derive conditions for the steady-state coexistence of states (or strategies). The analysis reveals that coexistence crucially depends on the number of agents consulted for updating. We conclude that updating rules are as important for evolution on a network as network structure and the nature of the interaction.

2.1 Introduction

Network theory has provided important insights into the dynamics of interactions in a structured population. In this framework, population structure is represented by a network, the nodes of which represent the individual agents while the links correspond to the possible interactions (Albert and Barabási 2002, Newman 2003, Boccaletti et al. 2006, Lieberman et al. 2005). The agents can be molecules, individual organisms, or groups of individuals, and the interactions can also be highly diverse, ranging from chemical reactions among molecules to the exchange of goods or knowledge among groups (Nowak and May 1992, Santos et al. 2008, Bastolla et al. 2009, Ohtsuki et al. 2006). To fix ideas, we will here focus on the evolution of social interactions among individuals. In this context, network models typically assume that each agent is endowed with a certain strategy (corresponding to the agent's "state") that determines the agent's behaviour in interactions with their neighbours in the network and the resulting payoffs. After the interaction phase, agents can update their strategies by comparing their own accumulated payoffs with the payoff of one of their neighbours (Szabó and Fáth 2007, Perc and Szolnoki 2010, Gross and Blasius 2008a).

Network models have revealed that network structure plays an important role for the evolutionary dynamics of behaviour in a social interaction. Take, for example, the Prisoner's Dilemma game (PDG) (Weibull 1995, McNamara and Weissing 2010), where mutual cooperation is favoured to mutual defection by both players. Yet, cooperation is outcompeted by defection in a well-mixed population, since defection is a dominant strategy. When interactions take place on a network, however, cooperation can get established, but this strongly depends on the network structure; cooperation gets easily off the ground in heterogeneous networks (e.g., scale-free networks), while it will not easily evolve in homogeneous networks (e.g. random-

regular networks) (Santos and Pacheco 2005, Santos et al. 2006). In a Snowdrift game (SDG), another prototype example for the evolution of cooperation, the coexistence of cooperation and defection is expected in a well-mixed population, while network models predict the fixation of either cooperation or defection under a wide range of conditions (Axelrod 1984, Roca et al. 2009, Doebeli and Hauert 2005, Hauert and Doebeli 2004, Ramazi and Cao 2014).

Until now, the discussion on evolutionary games on networks has mainly focused on network structure and the nature of the game (Claussen and Traulsen 2008, Melbinger et al. 2010, Ohtsuki, Nowak and Pacheco 2007, Pacheco et al. 2006, Roca et al. 2006a, Riehl and Cao 2014, Zhang, Zhang, Chu and Perc 2011, Riehl and Cao 2015). Here we will scrutinize the role of strategy updating.

Instead of investigating the influences of cognitive processes or incomplete information on strategy dynamics in large populations, we introduce individual player's switching probabilities between competing strategies to investigate how strategies are being taken in structured populations. In the simplest scenario where two players interact with each other, say a cooperator and a defector, it is safe to assume that the cooperator has a larger probability to switch its strategy than that of the defector because of the advantage of defection over cooperation. In a real society, the probability that a defector turns into a cooperator is small but not zero. It is difficult to get the precise values of the two probabilities; however, it is relatively easier to know which is greater than the other. When a player interacts with more than one players, the problem of how she adjusts her strategy becomes complicated. In this study, we provide a framework to investigate the evolution of how players in large structured populations choose from two competing strategies after repeatedly playing games with their neighbors. The critical ingredient that enables us discover new mechanisms for coexistence of strategies based on players' contact patterns is exactly the introduction of each player's probability of switching strategies that we have just described.

2.2 Basic model

For simplicity, we consider games with two pure strategies, like the PDG or the SDG. At each point in time, an agent employs one of the two strategies. The payoff

obtained by an agent using strategy i in an interaction with an agent using strategy j is given by m_{ij} , where $M = (m_{ij})$ is the 2×2 payoff matrix characterizing the game. For example, the payoff matrices of a PDG and an SDG are given by

$$M_{PDG} = \begin{pmatrix} b-c & -c \\ b & 0 \end{pmatrix}, M_{SDG} = \begin{pmatrix} b-\frac{c}{2} & b-c \\ b & 0 \end{pmatrix}, \quad (2.1)$$

where b and c ($b > c$) indicate the benefits and costs of cooperation, respectively.

Typically, strategy updating is modelled as follows (Szabó and Fáth 2007, Perc and Szolnoki 2010, Gross and Blasius 2008a): an agent having used strategy A and accumulated payoff π_A in the previous interactions randomly selects another agent from her neighbourhood; if that agent happens to have used the alternative strategy B and accumulated payoff π_B , then the focal agent will switch from A to B with a probability $u_{A \rightarrow B}$ that reflects the payoff difference $\pi_B - \pi_A$. This probability may, for example, be given by the Fermi function

$$u_{A \rightarrow B} = \left(1 + e^{-\beta(\pi_B - \pi_A)}\right)^{-1}. \quad (2.2)$$

In the above equation, β controls the intensity of selection.

2.3 Evolutionary dynamic results

The red curves in Figure 1 show the evolution of cooperation in the PDG and the SDG for $b = 1$ and a spectrum of c -values for this updating rule. The results confirm that cooperation in a PDG can evolve in a scale-free network (for $c < 0.1$) but not in a random-regular network, and that cooperation in the SDG will spread to fixation more easily in a scale-free network (for $c < 0.2$) than in a random-regular network (for $c < 0.6$). The other curves in Figure 2.1 illustrate what happens if the strategy updating is not based on the consultation of one other agent, but on the consultation of two or more other agents. In these cases, a focal agent compares her payoff with that of m other agents and switches from A to B whenever any of these comparisons would result in such a switch in the standard updating scenario ($m = 1$) considered above.

Figure 2.1 clearly shows that such a change in strategy updating has a major effect on the evolutionary outcome. Now cooperation in the PDG can also get off

the ground in a random-regular network ($m = 2: c < 0.2; m = 4: c < 0.9; m = 10: \text{all } c$). Most strikingly, for larger values of m , fixation for either cooperation or defection gives rise to the stable coexistence of these strategies. Moreover, for large values of m , the evolutionary outcome is relatively independent of the type of interaction (i.e. PDG versus SDG) and the structure of the network (i.e. random-regular versus scale-free).

We also considered still another updating rule: agents interact sequentially with their neighbours (in random sequence) and update their strategy as above, but now updating takes place after each individual interaction. In other words, the switching probability is given by Eq. (2.2), but now the payoffs of the A - and the B -players are not accumulated over several interactions, but given by $\pi_A = m_{AB}$ and $\pi_B = m_{BA}$. In other words, the two switching probabilities ($u_{A \rightarrow B}$ and $u_{B \rightarrow A}$) only depend on the payoff matrix of a specific game. In addition, we can investigate the evolutionary dynamics of strategies in a more general scenario where the values of the two switching probabilities are given without a payoff matrix although we know that the switching probabilities are essentially determined by the payoff matrix. Many simulations for a large variety of payoff matrices M have revealed that - when $m = 1$, irrespective of the structure of the network - the evolutionary outcome is only dependent on the sign of $m_{12} - m_{21}$: if $m_{12} > m_{21}$, strategy 1 will spread to fixation; if $m_{12} < m_{21}$, strategy 2 will spread to fixation; and both strategies will coexist at equal frequencies if $m_{12} = m_{21}$. Hence, coexistence is very unlikely. But again, we arrive at the conclusion that the evolutionary outcome is more strongly affected by the updating rule than by the nature of the interaction (which is crucially dependent on the payoff parameters m_{12} and m_{21}) or the structure of the network. Next, we describe in detail this general model and the results from computer simulations and analytical approaches.

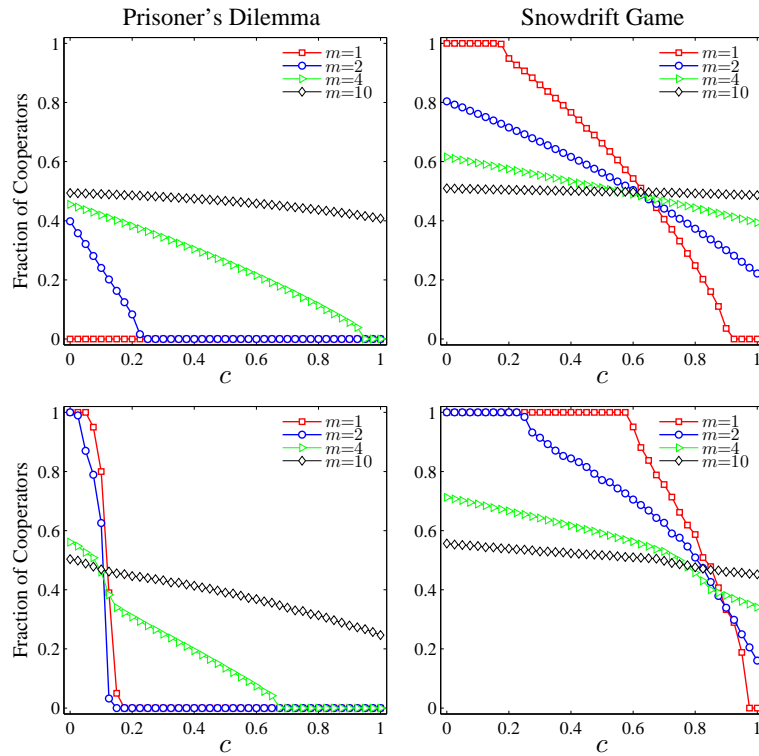


Figure 2.1: Equilibrium fraction of cooperators in the Prisoner's Dilemma game (PDG) and the Snowdrift game (SDG) as a function of the costs c of cooperation for four values of m , the number of agents consulted for strategy updating. Left panels: PDG; right panels: SDG; upper panels: random-regular network with degree 10; lower panel: Barabási-Albert scale-free network with average degree 10. The benefit of cooperation was kept constant at $b = 1$. In our simulations on scale-free networks, if a player's degree was smaller than m , she chose all of her neighbours for consultation.

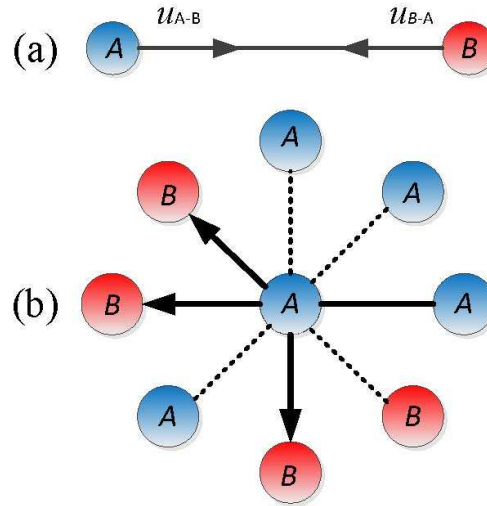


Figure 2.2: (a) A single updating when two different individuals encounter each other. Strategy A (blue) has a probability $u_{A \rightarrow B}$ to switch to B (red), while B switches to A with probability $u_{B \rightarrow A}$. (b) Diagrams illustrating an updating event in a network scenario where each player chooses m of her neighbours randomly for updating. Each arrow specifies a probabilistic switching because it is formed by different strategies. Dot lines indicate neighbours which are not selected at the current time step.

To explain the results mentioned above, we now take a more mathematical approach that is applicable beyond the context of evolutionary games. This approach is based on the two transition probabilities $u_{A \rightarrow B}$ and $u_{B \rightarrow A}$, which are viewed as given model parameters that do not necessarily reflect a fitness comparison. As above, $u_{A \rightarrow B}$ denotes the probability that an agent using strategy A will switch to the alternative strategy B when this agent happens to consult a B -player. Fig. 2.2(a) shows the setup for the case $m = 1$ where a single updating takes place between two players with different strategies. Fig. 2.2(b) represents a network scenario in which the focal player chooses m neighbours for updating. In this specific case, $m = 4$ and three of the four chosen neighbours maintain different strategies. If p_Ω denotes the probability that a neighbour of agent i uses strategy A , we can now calculate the probabilities $U_{A \rightarrow B}^i$ and $U_{B \rightarrow A}^i$ with which, after consulting m neighbours, agent i

would switch from A to B or from B to A , respectively:

$$\begin{cases} 1 - U_{A \rightarrow B}^i = (1 - u_{A \rightarrow B}(1 - p_{\Omega_i}))^m \\ 1 - U_{B \rightarrow A}^i = (1 - u_{B \rightarrow A}p_{\Omega_i})^m. \end{cases} \quad (2.3)$$

For example, $u_{B \rightarrow A}p_{\Omega_i}$ is the probability that agent i , when having played B , is consulting an A -playing neighbour that induces agent i to switch to A ; $1 - u_{B \rightarrow A}p_{\Omega_i}$ is the probability that any given neighbour does not induce agent i to switch when having played B , and $(1 - u_{B \rightarrow A}p_{\Omega_i})^m$ is the probability that none of m consulted neighbours will induce player i to switch to A . By definition, the latter probability corresponds to $1 - U_{B \rightarrow A}^i$.

We can now derive a recurrence equation for the probability $p_i(t)$ that a given agent i will employ strategy A at time t :

$$p_i(t+1) = p_i(t)[1 - U_{A \rightarrow B}^i(t)] + [1 - p_i(t)]U_{B \rightarrow A}^i(t). \quad (2.4)$$

2.3.1. THEOREM. *Under the dynamics of (2.4), it holds that*

$$\begin{cases} \text{A will persist if } u_{B \rightarrow A} > \frac{1 - (1 - u_{A \rightarrow B})^m}{m}; \\ \text{B will persist if } u_{A \rightarrow B} > \frac{1 - (1 - u_{B \rightarrow A})^m}{m}. \end{cases} \quad (2.5)$$

Proof: The first term on the right-hand side of Eq. (2.4) corresponds to the joint probability of having played A in the previous time step and not having switched to B , while the second term corresponds to the probability of having played B at time t but having switched to A . An equilibrium $p_i(t+1) = p_i(t) = \hat{p}_i$ of (2.4) is characterized by

$$\hat{p}_i \cdot \hat{U}_{A \rightarrow B}^i = (1 - \hat{p}_i) \cdot \hat{U}_{B \rightarrow A}^i. \quad (2.6)$$

For a homogeneous network, such as a random-regular network, it is plausible to assume that the probability to use strategy A will converge to the same value $\hat{p}_i = \hat{p}_{\Omega_i} = \hat{p}$ for all i . Inserting Eq. (2.3) in Eq. (2.6) yields an implicit equation for \hat{p} :

$$\begin{aligned} & \hat{p} \cdot [1 - (1 - u_{A \rightarrow B}(1 - \hat{p}))^m] \\ &= (1 - \hat{p}) \cdot [1 - (1 - u_{B \rightarrow A}\hat{p})^m]. \end{aligned} \quad (2.7)$$

When $u_{B \rightarrow A} \rightarrow u_{B \rightarrow A}^*$, the probability that a player i adopts strategy A is \hat{p}_i , where $0 < \hat{p}_i \ll 1$. Assume that after a long time evolution, $\hat{p}_i \approx \hat{p} \approx \hat{p}_{\Omega_i}$, then after substituting this into Eq. (2.6), and neglecting the high-order terms in \hat{p}_i , we get

$$m\hat{p}u_{B \rightarrow A}^* = \hat{p} - \hat{p}(1 - u_{A \rightarrow B})^m. \quad (2.8)$$

For fixed values of $u_{A \rightarrow B}$ and m , we have

$$u_{B \rightarrow A}^* = \frac{1 - (1 - u_{A \rightarrow B})^m}{m}. \quad (2.9)$$

Similarly, strategy B will persist for a given value of $u_{B \rightarrow A}$ whenever $u_{A \rightarrow B}$ is larger than a threshold value $u_{A \rightarrow B}^*$, which can be obtained from Eq. (2.6) by taking the limit $\hat{p} \rightarrow 1$. The result is:

$$\left\{ \begin{array}{l} \text{A will persist if } u_{B \rightarrow A} > \frac{1 - (1 - u_{A \rightarrow B})^m}{m}; \\ \text{B will persist if } u_{A \rightarrow B} > \frac{1 - (1 - u_{B \rightarrow A})^m}{m}. \end{array} \right.$$

□

For $m = 1$, this immediately implies that equilibrium coexistence of both strategies (i.e. $0 < \hat{p} < 1$) is possible if, and only if, $u_{A \rightarrow B} = u_{B \rightarrow A}$. This explains our earlier results that strategy updating after each individual interaction will only lead to the coexistence of the two strategies if $m_{12} = m_{21}$. It also implies that in a homogeneous network strategy coexistence requires that, at equilibrium, both strategies have the same payoffs: $\pi_A(\hat{p}) = \pi_B(\hat{p})$.

Figure 2.3 illustrates that for $m > 1$ the coexistence of A and B is easy to achieve. For two values of $u_{A \rightarrow B}$, this figure shows the equilibrium frequency \hat{p} of strategy A for a spectrum of values $u_{B \rightarrow A}$ and the outcome of simulations that are in excellent agreement with the equilibrium value predicted by Eq. (2.6). For a given value of $u_{A \rightarrow B}$, strategy A will persist in the population whenever $u_{B \rightarrow A}$ is larger than a certain threshold value $u_{B \rightarrow A}^*$. This minimum value for $\hat{p} > 0$ can be calculated by taking the limit $\hat{p} \rightarrow 0$ in Eq. (2.6).

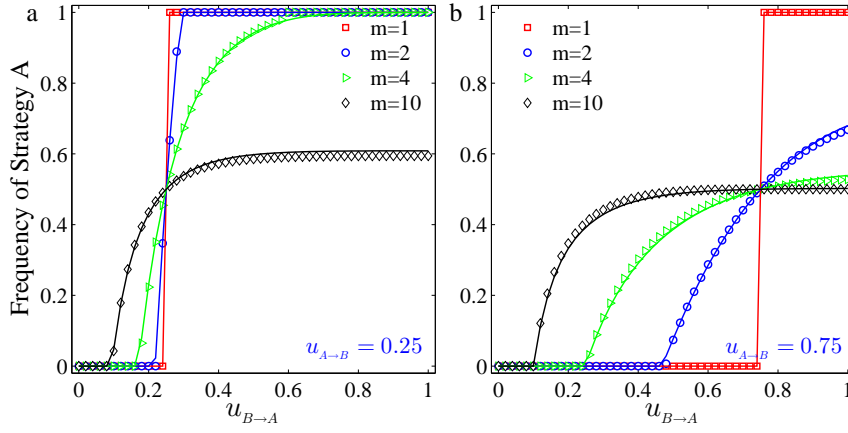


Figure 2.3: Equilibrium frequency of strategy A in a random-regular network of degree 10 for four values of m , the number of agents consulted for strategy updating. For (a) $u_{A \rightarrow B} = 0.25$ and (b) $u_{A \rightarrow B} = 0.75$, the analytical predictions based on Eq. (2.6) (solid lines) and the outcome of simulations (symbols) are shown for a spectrum of values of $u_{B \rightarrow A}$. Both panels clearly indicate that a larger value of m favours the equilibrium coexistence of both strategies. In our simulations time is discretized in time steps and in each step players choose to be an A or a B -player with the probability determined when finishing the previous step. We start from a configuration in which each player adopts strategy A with a probability chosen uniformly from the range $[0, 1]$. In each round, player i updates her strategy and is correspondingly associated with a probability that she is an A -player in the next round. Each simulation result corresponds to a result of averaging over 10^3 generations after a transient period of 10^4 rounds in 100 independent realizations with the population size 10^4 .

Coexistence of A and B will occur if both conditions of Eq. (2.5) are satisfied. Figure 2.4 illustrates that simulations in random-regular networks are in excellent agreement with this prediction and that the coexistence region becomes very large already for moderate values of m (e.g. $m = 4$).

Our analytical results do not directly apply to heterogeneous networks, since the equilibrium value \hat{p}_i of the probability to use strategy A will depend on the degree of player i . As a rule, \hat{p}_i will more likely be between 0 and 1 when the degree of player i is higher. Qualitatively, however, our basic insight that a larger value of m favours polymorphism for a broad range of values of $u_{A \rightarrow B}$ and $u_{B \rightarrow A}$ also applies to heterogeneous networks. This is illustrated by Figure 2.5 that indicates for

a Barabási-Albert scale-free network that the conditions for the coexistence of competing strategies are even less stringent than in a random-regular network. Here, the critical values of $u_{B \rightarrow A}$ for a given $u_{A \rightarrow B}$ in scale-free networks are smaller than that in random-regular networks.

To show the validity of the approach here discussed, we have also performed more computer simulations on different networks for the sake of comparison. To model the evolutionary dynamics on the described topologies we incorporate a model in which, at each time step, each node has a probability of strategy A . In our simulations time is discretized in time steps and each simulation starts with a random probability P_i ranging in $[0, 1]$. The simulation runs until a stationary state for the average frequency of strategy A (f_A) of the whole population. Notably, the network topology employed on top of which the evolutionary game is evolved is regular ring, thus enabling an efficient comparison of different average degrees \bar{k} .

Our results on ring networks with different $u_{B \rightarrow A}$, where individuals share homogeneous interaction degree. From Fig. 2.6-2.8, we can see that the simulation results are in accordance with the theoretical analysis, where larger average degrees are beneficial for the maintenance and diversity of the two strategies. For example, when $k = 100$, f_A can be close to 0.5 at high $u_{A \rightarrow B}$ no matter the value of $u_{B \rightarrow A}$. This result also transfer a clue that interactions play a relatively high impact on the coexistence of different strategies.

Until now, we have shown that the essential factor is the interaction number for each player. However, we still believe that the two probabilities which indicate the shifting rates to each other are also important for describing a specific game. For example, if we consider a Prisoner's Dilemma game (PDG), the probability of defection to cooperation should be much smaller than the opposite. Whereas, in a Snowdrift game (SDG), the two probabilities should have smaller difference than that in a PDG. In other words, the two probabilities should not be independent of each other. Motivated by the above considerations, we now study a very simple situation where $u_{B \rightarrow A} = 1 - u_{A \rightarrow B}$.

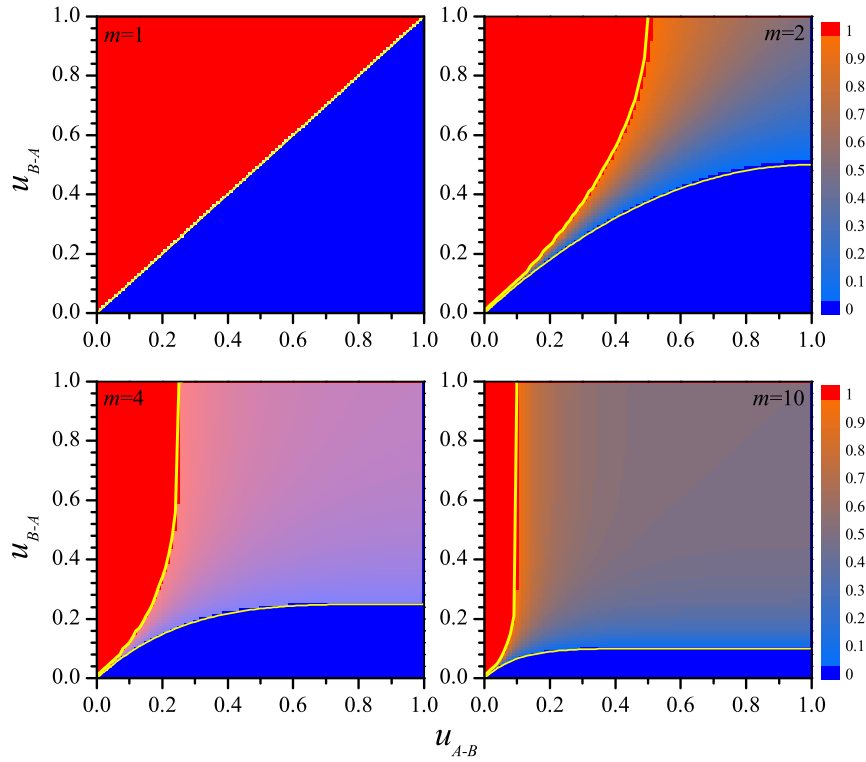


Figure 2.4: Equilibrium coexistence of strategies A and B as a function of the updating probabilities $u_{A \rightarrow B}$ and $u_{B \rightarrow A}$ for four values of m , the number of agents consulted for strategy updating: (a) $m = 1$; (b) $m = 2$; (c) $m = 4$; (d) $m = 10$. Red: fixation of strategy A ; blue: fixation of strategy B ; yellow lines: boundaries of coexistence region based on Eq. (2.7); all other colours: frequency of A ($0 < \hat{p} < 1$) resulting from Eq. (2.6).

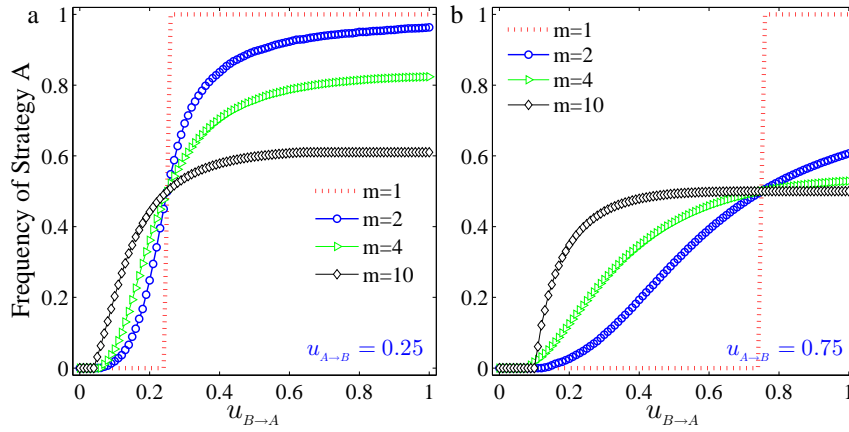


Figure 2.5: Equilibrium frequency of strategy A in a Barabási-Albert scale-free network with average degree 10 for four values of m , the number of agents consulted for strategy updating. For (a) $u_{A \rightarrow B} = 0.25$ and (b) $u_{A \rightarrow B} = 0.75$, the outcome of simulations is shown for a spectrum of values of $u_{B \rightarrow A}$. As in case of a random-regular network (Fig. 2.3), a larger value of m favours the equilibrium coexistence of both strategies.

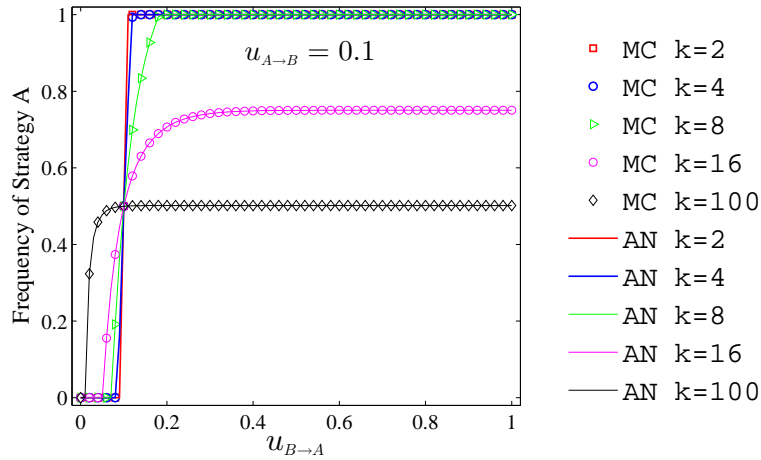


Figure 2.6: Phase diagram for the interaction model in Ring networks for different values of $u_{B \rightarrow A}$ and average degree k when values of $u_{A \rightarrow B} = 0.1$. The networks are made up of $N = 10^4$ nodes. MC results are averages over 100 realizations. Solid lines corresponds to the theoretical prediction and symbols to MC results.

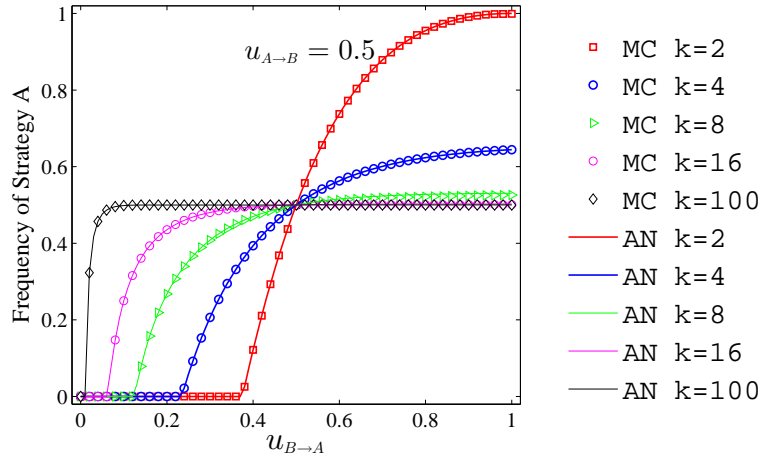


Figure 2.7: Phase diagram for the interaction model in Ring networks for different values of $u_{B \rightarrow A}$ and average degree k when values of $u_{A \rightarrow B} = 0.5$. The networks are made up of $N = 10^4$ nodes. MC results are averages over 100 realizations. Solid lines corresponds to the theoretical prediction and symbols to MC results.

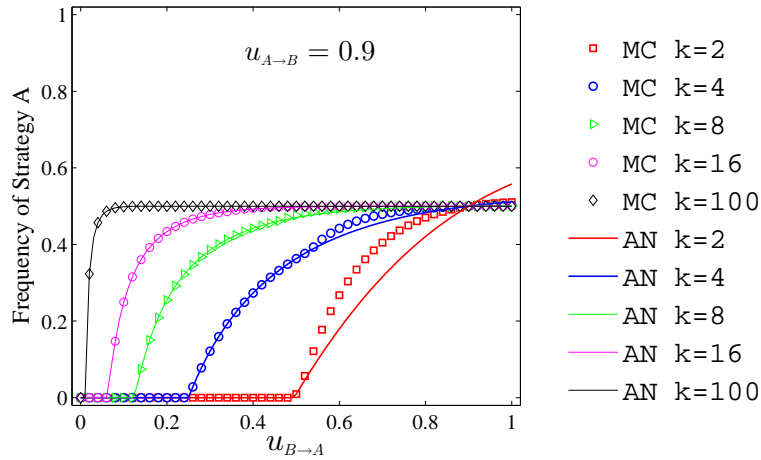


Figure 2.8: Phase diagram for the interaction model in Ring networks for different values of $u_{B \rightarrow A}$ and average degree k when values of $u_{A \rightarrow B} = 0.9$. The networks are made up of $N = 10^4$ nodes. MC results are averages over 100 realizations. Solid lines corresponds to the theoretical prediction and symbols to MC results.

Here, a big $u_{B \rightarrow A}$ (larger than 0.5) means that strategy A has an advantage over strategy B , and vice versa. Thus, we get

$$2p - p(1 - u_{B \rightarrow A}p)^m + p(u_{B \rightarrow A} + (1 - u_{B \rightarrow A})p)^m + (1 - u_{B \rightarrow A}p)^m - 1 = 0 \quad (2.10)$$

Figure 2.9 presents the results on regular random networks and BA scale-free networks respectively. In BA scale-free networks, we still suppose that each player uses all of her neighbours for updating. Still, simulation results are in great agreement with that obtained through theoretical analysis. The results also confirm that it is easier for achieving coexistence of different strategies when players choose more neighbours for updating. Different than the results shown in Fig. 2.2, the fractions of A -players are much larger than 0.5 even when the value of m is large (e.g. $m = 10$). The reason is that the two probabilities are correlated. This plays a positive feedback effect on the dynamics of strategies.

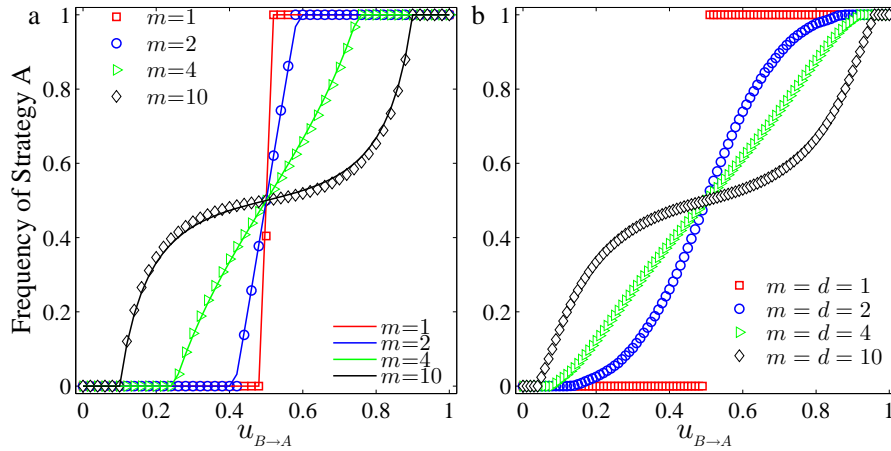


Figure 2.9: Frequency of A -players for the interaction model in random regular (RR) networks (left panel) and BA scale-free (SF) networks (right panel), with $u_{B \rightarrow A} + u_{A \rightarrow B} = 1$. In RR networks, each player chooses m neighbours for updating while in SF networks, one chooses all of her neighbours for updating. The settings are the same as in Fig 2.2. Solid lines correspond to the theoretical prediction and symbols are for simulation results.

2.4 Conclusion remarks

In conclusion, we have shown that evolution on an interaction network can be as strongly affected by the strategy updating procedure as by the network structure and the payoff matrix. In this paper for two-strategy evolutionary games in structured populations, we follow a different approach, bypassing the requirement for explicit knowledge of the exact payoffs, by encoding the payoffs into the willingness of any player to switch from her current strategy to the competing one.

Theoretical computations and numerical simulations show that the evolutionary dynamics are intrinsically regulated by contact relationships specified by the network topologies of the populations. We demonstrate that updating rules are of crucial importance for the steady state distribution of states. On the basis of general arguments, we show that the coexistence of different states strongly depends on the number m of agents that determine the updating of a given agent: if $m = 1$, as typically assumed, coexistence is difficult to achieve, while coexistence occurs under mild conditions when $m > 1$. By means of two cooperation games, we show that this general insight has important implications for the strategy dynamics of games on a network. In comparison to earlier models, cooperation can more easily persist in a Prisoner's Dilemma game, while it can go more easily extinct in a Snowdrift game. This implies that strategy updating deserves more attention in empirical and theoretical studies.

The main conclusion of this chapter is that updating rules are crucial for the evolution of competing strategies and every strategy is difficult to be eliminated by other strategies. In other words, it is difficult to say which strategy is better than the others because even a strategy earns lower payoff, it can spread in the population in some scenarios.