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## Coexistence of competing strategies in evolutionary games

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# **Coexistence of Competing Strategies in Evolutionary Games**

Jianlei Zhang



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 groningen**

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