

University of Groningen

Guiding Vector Fields for Robot Motion Control

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DOI:
[10.33612/diss.181475662](https://doi.org/10.33612/diss.181475662)

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Document Version
Publisher's PDF, also known as Version of record

Publication date:
2021

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Yao, W. (2021). *Guiding Vector Fields for Robot Motion Control*. [Thesis fully internal (DIV), University of Groningen]. University of Groningen. <https://doi.org/10.33612/diss.181475662>

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Guiding Vector Fields for Robot Motion Control

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The research described in this dissertation has been carried out at the Faculty of Science and Engineering, University of Groningen, the Netherlands.

disc

The research reported in this dissertation is part of the research program of the Dutch Institute of Systems and Control (DISC). The author has successfully completed the education program of the Graduate School DISC.

Cover design by Qian Wang

The thesis style is based on André Miede and Ivo Pletikosić's Classisc Thesis.



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Guiding Vector Fields for Robot Motion Control

PhD thesis

to obtain the degree of PhD at the
 University of Groningen
 on the authority of the
 Rector Magnificus Prof. C. Wijmenga
 and in accordance with the decision by the College of Deans.

This thesis will be defended in public on

Friday 8 October 2021 at 12.45 hours

by

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born on 14 November, 1992
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To my family

献给我的家人

ACKNOWLEDGMENTS

As I am writing the acknowledgments, I cannot help reflecting on the four years of my PhD journey. Perhaps one of the most unexpected events is the COVID-19 pandemic, and no one would predict that we have been socially distancing with each other for more than a year since the lock-down of the Netherlands in March, 2020. We also experienced the first nationwide curfew since World War II for more than three months. Nevertheless, I managed to keep advancing on the PhD journey, and I am finally reaching the destination! This would not have been possible without the help from many people, to whom I would like to pay my special regards.

First of all, I would like to express my eternal gratitude to my primary supervisor Prof. Ming Cao. Thank you for offering me the opportunity to pursue my PhD degree at the prestigious University of Groningen, and for helping me grow to be an independent researcher. In particular, I always enjoy being an audience of your group meetings, where everyone is encouraged to ask the speaker any kind of questions. On the other hand, it is never easy to be a speaker at the group meetings! When I gave a presentation at one of the meetings for the first time, I received perhaps more than sixty questions, and my presentation lasted for literally two hours! Thank you also for your valuable academic and life advice, as well as honest and helpful criticism. I have learned to clearly see both my strengths and weaknesses. In addition, I am particularly grateful for your detailed guidance when we were preparing the presentation to compete for the ICRA best conference paper award in 2021, and I could not appreciate it more when I saw you present during my (online) talk at 2:30 AM! (By the way, that is my first and last time sleeping in my office in Groningen.)

I would also like to express my heartfelt appreciation to my secondary supervisor Prof. Jacquélien M.A. Scherpen for her generous support and encouragement. Thank you for suggesting courses for my personal development, and for helping me out when I was in need. I appreciate the Dutch specialty “nieuwjaarsrolletjes” with cream you bring to the group at the beginning of every year (except 2021 due to COVID-19), and the annual (physical and virtual) group outings that you and many others organized.

My profound gratitude also goes to my collaborators, including Bohuan Lin, Prof. Brian D. O. Anderson, Dr. Yuri A. Kapitanjuk, Dr. Hector G. de Marina, Dr. Zhiyong Sun, LuLu Gong, and Binbin Hu. In particular, I owe my deepest gratitude to Bohuan Lin, who is not only my closest collaborator, but also my good friend, my mathematics mentor and my paronymph. Bohuan, thank you so much for spending a huge amount of time discussing the research conducted

in this thesis with me, and for sharing the happiness and bitterness of pursuing a PhD. Additionally, I am constantly inspired by your thought-provoking ideas regarding academia, mathematics and also life. I would also like to show my gratitude to Brian. Brian, you are always a role model for me. I admire your persistent passion for research, your work ethic and your encyclopedic knowledge. It is a great pleasure working with you and I have benefited enormously from the technical and non-technical discussions, and the meticulous comments on my manuscripts. I would like to recognize the invaluable assistance that Yuri provided during my first year. Thank you, Yuri, for leading me to this research topic and working with me on my first academic paper. Hector, thank you for the technical discussions and for conducting fixed-wing aircraft experiments. I also enjoy many late-night Google chats with you on different aspects of academia. Zhiyong, thank you for spending time on our technical discussions and for giving me tips on academic writing. Thank you, Lulu, for inviting me to the wonderful world of evolutionary game theory, and Binbin, for extending the guiding vector field to the ordering-free case.

My special thanks go to the reading committee: Prof. Arjan J. van der Schaft, Prof. Kristin Y. Pettersen and Prof. Magnus B. Egerstedt. I greatly appreciate your spending precious time reading my thesis and providing valuable comments, especially during the summer break. I also received generous support from Sha Luo, Bohuan Lin, Marco A. Vasquez Beltran, Rafael F. Cunha and Zhiyuan Liu for proofreading chapters of my thesis. The advice and comments from you helped me improve the thesis significantly. I am particularly grateful to Dr. Nelson Chan for translating the English summary to Dutch and Matthijs C. de Jong for proofreading it.

I thank all my colleagues from the ENTEG institute and the Bernoulli Institute. Specifically, I am deeply grateful to Marco A. Vasquez Beltran for agreeing to be my paranymph, and for the joyful and fruitful discussions on DISC homework. On top of that, Marco, thank you for letting me distract you whenever you were concentrating on your research. I am thankful to my office-mates, Carlo Cenedese, Lulu Gong, Fangzhou Liu, Lanlin Yu, Anja Schmerbauch, Amirreza Silani, Tinghua Li, Rafael F. Cunha, Miao Guo, Ningbo Li and Agung Prawira Negara, for accompanying me and discussing interesting topics. Moreover, I cannot forget the days I spent with Carlo and Yuri on preparing a robotics demonstration for the royal family on King's day in 2018. Neither can I forget the time we worked together as a TA team for the robotics course. In addition, I would like to express my gratitude to our thoughtful, courteous secretary Frederika G. Fokkens, and our kind and helpful technicians Simon Busman and Martin Stokroos for everything that they helped me.

During the preparation of my ICRA talk for the best paper award, I have received valuable feedback from many people in addition to my collaborators of the ICRA paper. These people include Prof. Paolo Robuffo Giordano, Prof. Guido de Croon, Prof. Vinicius M. Gonçalves, Prof. Kristin Y. Pettersen, Dr. Chen Wang,

Dr. Mengbin Ye, Dr. Fabrizio Schiano, Dr. Qingkai Yang, Dr. Lorenzo Zino, Sha Luo, Rafael F. Cunha, Kathinka Frieswijk, Zhiyuan Liu and Emin Martirosyan. I greatly appreciate your help in improving the quality of my talk.

Finally, I want to thank my friends and family. My PhD life would not have been so special without the company of my friends in Groningen, especially those once stayed in the SSH student apartment on Plutolaan. Special thanks go to Qian Wang for designing the amazing cover of this thesis. I would also like to address some words to my beloved girlfriend Sha Luo. You are being supportive, thoughtful and understanding, especially when I was diving into the boundless ocean of scientific exploration. I always enjoy every second with you, and I appreciate your company these years!

Last but not least, I owe my beloved family members a deep debt of gratitude for their selfless and endless love and support!

Weijia Yao
Groningen, the Netherlands
30 August, 2021

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ACRONYMS

VF-PF	Vector-Field Guided Path-Following
VF-CAPF	Vector-Field Guided Path-Following with Collision-Avoidance
UAV	Unmanned Aerial Vehicle
LOS	Line-of-sight
NLGL	Nonlinear Guidance Law
LQR	Linear Quadratic Regulator
ISS	Input-to-state Stable/Stability
GAS	Globally Asymptotically Stable / Global Asymptotic Stability
DOA	Domain of Attraction