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## Guiding Vector Fields for Robot Motion Control

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# Guiding Vector Fields for Robot Motion Control

Weijia Yao  
姚伟嘉



university of  
 groningen

The research described in this dissertation has been carried out at the Faculty of Science and Engineering, University of Groningen, the Netherlands.

**disc**

The research reported in this dissertation is part of the research program of the Dutch Institute of Systems and Control (DISC). The author has successfully completed the education program of the Graduate School DISC.

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The thesis style is based on André Miede and Ivo Pletikosić's Classisc Thesis.



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# Guiding Vector Fields for Robot Motion Control

**PhD thesis**

to obtain the degree of PhD at the  
 University of Groningen  
 on the authority of the  
 Rector Magnificus Prof. C. Wijmenga  
 and in accordance with the decision by the College of Deans.

This thesis will be defended in public on

Friday 8 October 2021 at 12.45 hours

by

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*To my family*

献给我的家人



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Weijia Yao  
Groningen, the Netherlands  
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## ACRONYMS

VF-PF	Vector-Field Guided Path-Following
VF-CAPF	Vector-Field Guided Path-Following with Collision-Avoidance
UAV	Unmanned Aerial Vehicle
LOS	Line-of-sight
NLGL	Nonlinear Guidance Law
LQR	Linear Quadratic Regulator
ISS	Input-to-state Stable/Stability
GAS	Globally Asymptotically Stable / Global Asymptotic Stability
DOA	Domain of Attraction