Blending of mathematics and physics

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Appendix A

Interview study 1

This is the complete interview that we developed and conducted in the context of study 1, which we reported on in Chapter 2.

What is a PDE?

Main question: Can you explain what a partial differential equation is?

Follow up questions:

- What is a differential equation?
- What is the meaning of the word ’partial’ in the name?
- How does a PDE differ from an ODE?
- How does a PDE differ from an equation like $x^2 - 5x + 6 = 0$?
- What can you say about the unknown in a PDE?
- What is the difference between the solution of an equation like $x^2 - 5x + 6 = 0$ and of a differential equation?

Conceptual understanding of the heat equation

Show the student the following equation:

$$\boxed{\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}}$$

Main question 1: This PDE is used in physics to model some physical phenomena, which phenomena can you think of? Explain as clearly as possible.

Follow up:

- What do you mean by heat? What happens exactly?
- What do you mean by diffusion? What happens exactly?
- There are other phenomena very similar to ... (heat/diffusion depending on what they say) that are also represented by this equation, can you think of another phenomenon?
- Do you know what diffusion is? Can you explain this to me?
- Do you see similarities between heat transfer and diffusion? Which ones? / Does diffusion suit this PDE? How and why?

Main question 2: What is the physical meaning of the letters in this equation? What are the dimensions of these physical quantities and constants?

Follow up:

- if student says that alfa is a parameter he doesn't know: indeed, but when you check the dimensions of left and right you can find the dimensions of alfa.

Main question 3: Link the physical situations to the mathematical description.

Follow up:

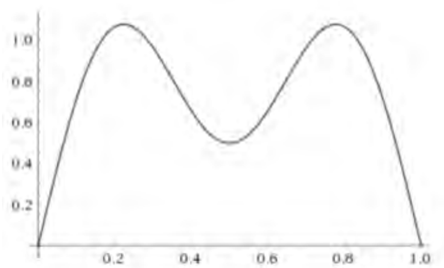
- What does the left hand side mean? What does the right hand side mean?
- Phys answer: Why is this described mathematically like this? Why a first and a second derivative for example?
- Why is this a first derivative and this a second? What is the meaning of a second derivative to position?

Conceptual interpretation of the heat equation

Main question: In the figure, a temperature profile is given. This shows the temperature of a bar in function of position on time $t = 0$. This situation is modelled by the heat equation as given here with these two boundary conditions:

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2} \text{ with } 0 < x < 1 \text{ and } 0 < t < \infty$$
$$u(0, t) = 0 \text{ with } 0 < t < \infty$$
$$u(1, t) = 0 \text{ with } 0 < t < \infty$$

How will the temperature evolve in time and position?



Follow up:

- Can you make sketches of the temperature profile at different times?
- How does the temperature profile look like when the time goes to infinity? and at times in between zero and infinity?
- What do the boundary conditions mean?
- What will happen to the maxima/minimum?
- How can you see that from the heat equation? (when they give a physical and graphical interpretation but don't use the differential equation)
- Why is that happening? How do you know this?
- Is this the only thing that happens?
- Can you give an interpretation of the second derivative and how this is linked to the graphical representation and the process?

Concrete example: Diffusion

Boundary and initial conditions

Main question: In a tube with a length of one meter there are u_0 particles. At time $t = 0$ they are distributed as the function $f(x) = u_0(1 - cos(2\pi x))$. The left and right end of the tube are closed so no particles can flow in or out of the tube. Write down the mathematical description of this physical situation (PDE, boundary and initial conditions). Also make a sketch of the initial distribution of the particles.

Follow up:

- In the boundary condition you choose to take the derivative with respect to time/position, can you explain this to me?
- What is on the axes of the graph?
- When doubting if the derivative should be to t or x or if it should be a derivative or not: What does each option mean?
- You chose the derivative to be to position/time, why?
- Maybe: What is the meaning of the term u_0 in the initial condition? Why is it there?

Interpreting the solution

Main question: We solved this diffusion problem for you. The analytical solution is:

$$u(x, t) = u_0 - u_0 e^{-(2\pi)^2 \alpha t} cos(2\pi x)$$

Describe the particle flow.

Follow up:

- Can you make sketches of the particle distribution at different times t?
- What are the characteristics of the physical situation after a long time?
- What is the influence of α?
- What is the physical meaning of α? What factors can influence α?

Extra: Heat

If students have problems with the context of diffusion the same problem can be solved in the context of heat.

Main question: A bar with a length of one meter has an initial temperature distribution with the shape of the function $f(x) = 1 - cos(2\pi x)$. The left and right end of the tube are isolated so no heat can flow in or out of the system. Write down the mathematical description of this physical situation (PDE, boundary and initial conditions).. Also make a sketch of the initial distribution of the temperature.

Follow up:

- In the boundary condition you choose to take the derivative with respect to time/position, can you explain this to me?
- What is on the axes of the graph?
- When doubting if the derivative should be to t or x or if it should be a derivative or not: What does each option mean?
- You chose the derivative to be to position/time, why?

Main question: We solved this heat problem for you. The analytical solution is:

$$u(x,t) = 1 - e^{-(2\pi)^2 \alpha t} cos(2\pi x)$$

Describe the evolution in temperature.

Follow up:

- Can you make sketches of the temperature profile at a couple of times t?
- What are the characteristics of the physical situation after a long time?
- What is the physical meaning of α? What is the influence of α? What factors can influence α?

Appendix B

Interview study 2

This is the complete interview that we developed and conducted in the context of study 2, which we reported on in Chapters 3 and 4.

Part 1

A metal rod is 1 meter long. Assume that heat can only flow along the rod. The initial temperature distribution in the rod is given in the graph in Figure B.1. Both ends of the rod are isolated, which means that no heat can flow in or out of the rod.

A. Describe the situation the graph is referring to.

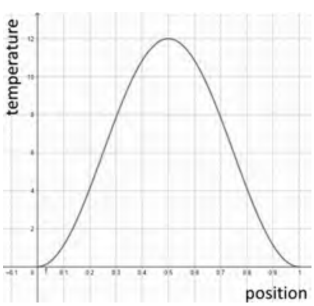


Figure B.1: Initial temperature distribution in the rod.

B. What is going to happen after some time? Sketch the temperature distribution in the rod at different times and explain important features of your graphs.

C. Explain what happens at the boundaries. How do you express that mathematically? Explain your reasoning.

Part 2

Below you see a partial differential equation (PDE) that describes a physical process.

$$\frac{\partial u}{\partial t}(x,t) = \alpha \frac{\partial^2 u}{\partial x^2}(x,t).$$

1. What kind of physical processes are modelled by this PDE?

2. This PDE in combination with an initial condition (IC) and boundary conditions (BC) describes a specific physical situation and its evolution.

In what follows, we will show you four sets of PDE, IC and BCs. For each set answer the following three questions:

1. Describe a physical situation that matches these BC and describe it in a couple of sentences.
2. Discuss what the BC mean for the evolution of the values of temperature/concentration and heat/particle flow.
3. Discuss what the BC mean for the values of temperature/concentration and heat/particle flow after a very long time.

The four sets:

A.

$$\frac{\partial u}{\partial t}(x,t) = \alpha \frac{\partial^2 u}{\partial x^2}(x,t).$$

$$u(x,0) = f(x)$$

with $f(x)$ an unspecified, continuous function on [0,1]

$$u(0,t) = 0$$

$$u(1,t) = 0$$

B.

$$\frac{\partial u}{\partial t}(x,t) = \alpha \frac{\partial^2 u}{\partial x^2}(x,t).$$
$$u(x,0) = f(x)$$

with $f(x)$ an unspecified, continuous function on [0,1]

$$u(0,t) = 0$$
$$u(10,t) = 50$$

C.

$$\frac{\partial u}{\partial t}(x,t) = \alpha \frac{\partial^2 u}{\partial x^2}(x,t).$$
$$u(x,0) = f(x)$$

with $f(x)$ an unspecified, continuous function on [0,1]

$$\frac{\partial u}{\partial x}(0,t) = 0$$
$$\frac{\partial u}{\partial x}(1,t) = 0$$

D.

$$\frac{\partial u}{\partial t}(x,t) = \alpha \frac{\partial^2 u}{\partial x^2}(x,t).$$
$$u(x,0) = f(x)$$

with $f(x)$ an unspecified, continuous function on [0,1]

$$\frac{\partial u}{\partial x}(0,t) = -2$$
$$\frac{\partial u}{\partial x}(1,t) = 4$$

Appendix C

Tutorial study 3

This is the complete tutorial that we developed in the context of study 3, which we reported on in Chapter 5. When implemented in the interviews, there was place provided between the different tasks for students to write down their answers.

Introduction

In the lectures, you have discussed the following partial differential equation. It models the time evolution of the temperature in a 1D system along the x-axis:

$$\frac{\partial T}{\partial t}(x,t) = \alpha \frac{\partial^2 T}{\partial x^2}(x,t).$$

Herein, T is the temperature, x the position along the system ($0 < x < L$), and t the time ($0 < t < \infty$). α is the thermal diffusivity ($\alpha > 0$).

Exercise 1

The graph in Figure C.1 shows the initial temperature distribution of the physical system that we study here

a) Given the heat equation and the initial temperature distribution, is it possible to predict how the temperature distribution will evolve when t goes from zero to infinity? If yes, do so. If not, explain why not.

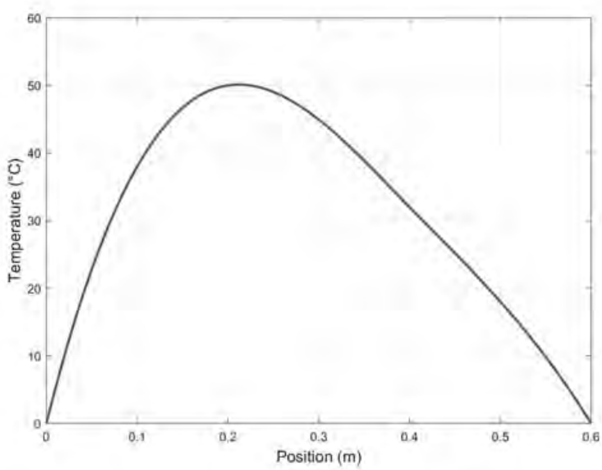


Figure C.1: Initial temperature distribution in the physical system (exercise 1).

b) Now, we put both ends of the system in contact with an infinitely large reservoir at $T = 0$. Express this information about the physical system in a mathematical way.

c) Use your physics knowledge to predict what will happen to the temperature distribution over time. What is the difference between the situation in question 1a and in this question?

d) Roughly sketch the temperature distribution in the physical system at different times ($t = 0$, $t \to \infty$, and two intermediate times). Clearly label the axes. Do these sketches agree with your answer to question 1c?

e) The PDE describes the temperature as a function of position and time. However, in the graph of the initial temperature distribution and your sketches in question 1d there is no 'time'. Explain.

f) Roughly sketch the evolution of the temperature in the physical system at different positions. More specifically, sketch three graphs: for positions $x = 0$, $x = 0.2$ and $x = 0.4$. Do these sketches agree with your answer to question 1c?

g) Do the sketches in 1d and 1f agree with your answer to question 1b? If yes, continue. If not, go back and find out what went wrong.

h) The 3D plot in Figure C.2 shows the temperature as function of position and time. How does this 3D plot relate to your sketches from questions 1d and 1f? Check for consistency!

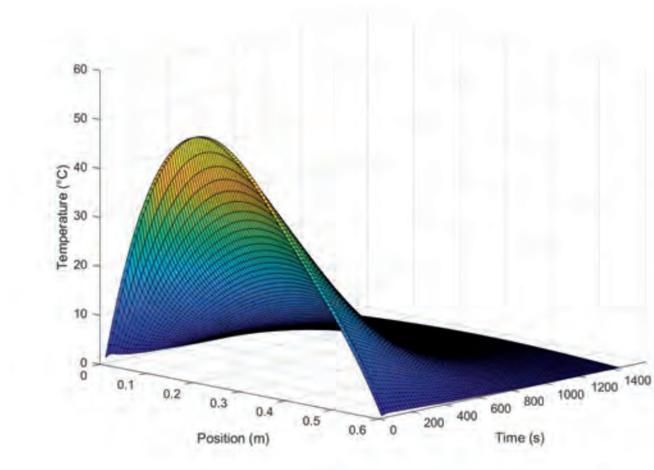


Figure C.2: Three-dimensional plot of the temperature at every point in the physical system and its evolution over time (exercise 1).

i) Discuss $\frac{\partial T}{\partial t}(0.4, t)$. Give a physical and a mathematical interpretation. Use the 3D plot and your own sketches.

j) Use the 3D plot and your own sketches to look at the initial temperatures at $x_1 = 0.1$ and $x_2 = 0.2$. Do these temperatures differ? What will happen in the system because of this?

k) Do the same for the initial temperatures at $x_1 = 0.1$ and $x_2 = 0.1+h$, with $0 < h < 0.1$. Let h become smaller and smaller. Do these temperatures differ? What will happen in the system because of this temperature difference?

l) Use the 3D plot and your own sketches to discuss the following partial derivative: $\frac{\partial T}{\partial x}(0.1, t)$. Connect your answer from question 1k to this partial derivative.

m) Explain in words how the following concepts are related: heat flow, slope, temperature gradient, direction of the heat flow, $\frac{\partial T}{\partial x}$ and sign of the partial derivative.

n) Under which physical and mathematical conditions will there be no heat flow? Connect your insights from the previous questions and the graphs, and explain your reasoning.

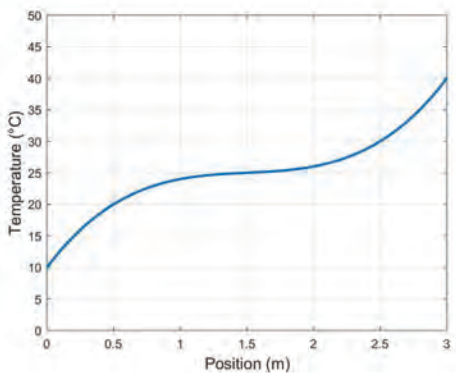


Figure C.3: Initial temperature distribution in the physical system (exercise 2).

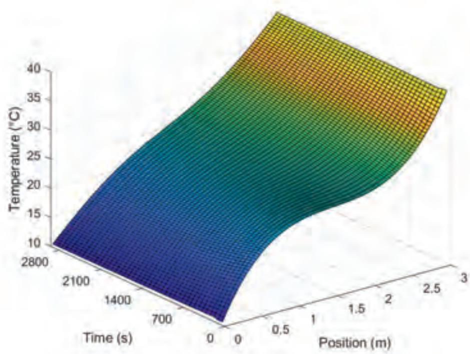


Figure C.4: Three-dimensional plot of the temperature at every point in the physical system and its evolution over time (exercise 2).

Exercise 2

In another physical system that we can describe by the heat equation (see introduction), we have the following boundary conditions: $T(0, t) = 10$ and $T(3, t) = 40$ for $0 < t < \infty$. The graph in Figure C.3 below shows the initial temperature distribution in the system.

a) What is the physical meaning of these boundary conditions?

b) Use your physics knowledge to explain what will happen to the temperature distribution over time. Roughly sketch the temperature distribution at different times ($t = 0$, $t \to \infty$, and one intermediate time) and connect these to your explanation.

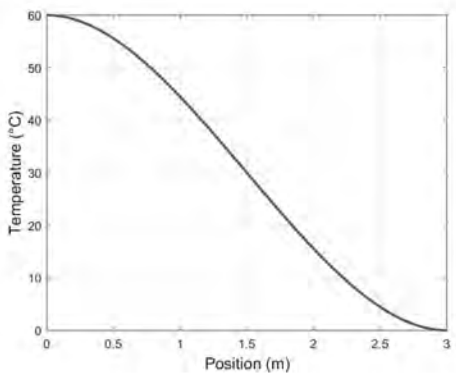


Figure C.5: Initial temperature distribution in the physical system (exercise 3).

Check if your sketches are consistent with the boundary conditions.

c) The 3D plot in Figure C.4 shows the temperature as a function of position and time. Do your sketches in question 2b agree with this 3D plot?

d) Discuss the heat flow in the system after a short time and after a very long time. Specify the direction of the heat flow and explain how this relates to the 3D plot and your own sketches.

Exercise 3

A third physical system that we can describe by the heat equation (see introduction) is an isolated system. The graph in Figure C.5 shows the initial temperature distribution in the system.

a) Explain what an isolated system is.

b) Use your physics knowledge to explain what will happen to the temperature distribution over time. Roughly sketch the temperature distribution at different times ($t = 0$, $t \to \infty$, and one intermediate time) and connect these to your explanation.

c) Express the boundary conditions mathematically.

d) The 3D plot in Figure C.6 shows the temperature as a function of position and time. Do your sketches in question 3b agree with this 3D plot?

e) Discuss the heat flow in the system after a short time and after a very long time. Connect your explanation to the 3D plot.

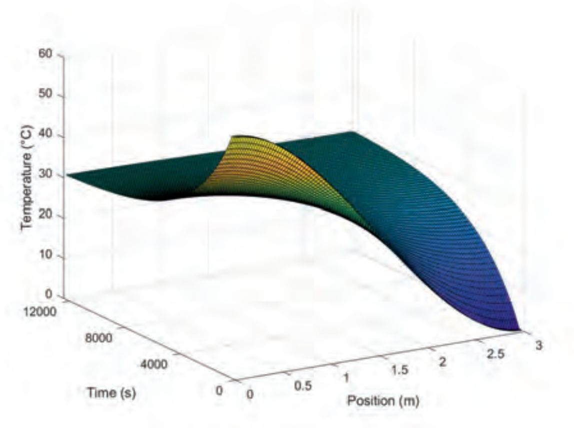


Figure C.6: Three-dimensional plot of the temperature at every point in the physical system and its evolution over time (exercise 3).

Summary

Summarize what you know about $\frac{\partial T}{\partial x}$ and $\frac{\partial T}{\partial t}$. Specify what these partial derivatives mean physically and mathematically and use the graphs in your explanation.

EXTRA: Exercise 4 [1]

Another physical system that we can describe by the heat equation (see introduction), has the following boundary conditions: $\frac{\partial T}{\partial x}(0, t) = -4$ and $T(4, t) = 8$ for $0 < t < \infty$. The graph in Figure C.7 shows the initial temperature distribution in the system:

a) What is the physical meaning of these boundary conditions?

b) Use your physics knowledge to explain what will happen to the temperature distribution over time. Roughly sketch the temperature distribution at different times ($t = 0$, $t \to \infty$, and one intermediate time) and connect these to your explanation. Check if your sketches are consistent with the boundary conditions.

c) The 3D plot in Figure C.8 shows the temperature as a function of position and time. Do your sketches in question 4b agree with this 3D plot?

[1] This exercise was not tested in the teaching-learning interviews due to time constraints.

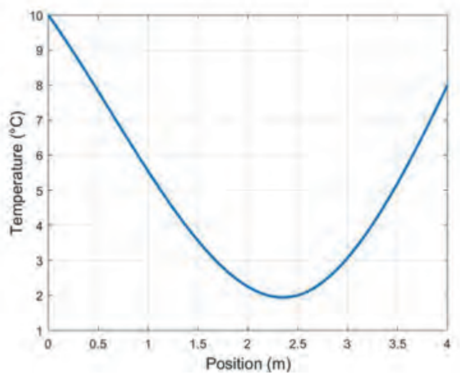


Figure C.7: Initial temperature distribution in the physical system (exercise 4).

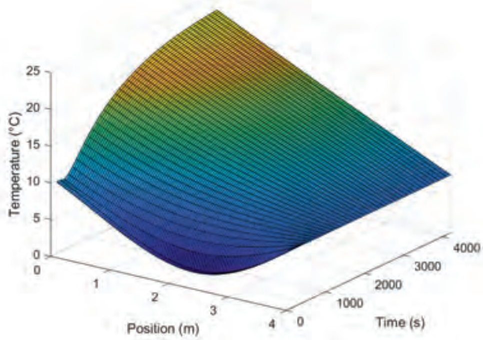


Figure C.8: Three-dimensional plot of the temperature at every point in the physical system and its evolution over time (exercise 4).

d) Discuss the heat flow in the system after a short time and after a very long time. Specify the direction of the heat flow and explain how this relates to the graph.

Summary

The role of mathematics in physics is multifaceted: for example, mathematics can function as a ’toolbox’ (technical function), as a language (communicative function) or for building logical-deductive reasoning (structural function). Moreover, mathematics has not only been essential for the development of physics, but conversely, many mathematical concepts arose from a desire to describe nature. The intertwining of mathematics and physics is so strong that it is sometimes difficult to separate the two. Yet this strong connection is not always reflected in education. In physics education, mathematics is often reduced to its technical function of describing relationships and making calculations. In mathematics education, physics is often reduced to a context in which abstract mathematical ideas are illustrated and applied. This dichotomy makes it difficult for pupils and students to integrate their knowledge of both disciplines. The question of how best to bring the two together to support learners’ learning remains largely unanswered, despite the amount of discipline specific educational research on learning and teaching mathematics and physics separately. Recent research on the interplay of mathematics and physics in students’ reasoning shows that this often remains a challenge even for more advanced students (further into the bachelor’s and master’s degrees). These students are usually proficient in performing calculations (which refers to the technical function of mathematics), but even for them the structural role of mathematics in physics proves difficult to grasp. How education can be designed to better support this is not yet very clear.

In this thesis, for a specific topic (partial differential equations), we examine how students bring mathematics and physics together in their reasoning, what the specific difficulties they face in doing so are, and ultimately how we can respond to them in designing educational learning activities.

Partial differential equations are typically covered in undergraduate courses in physics and mathematics, and both disciplines play an important role in this course. For example, the heat equation describes heat transport in a rod and thus provides a prime example of the interplay between the two disciplines. To fully describe the physical system, an initial condition and boundary conditions are required in addition

to the differential equation. The initial condition establishes the initial temperature distribution and the boundary conditions impose conditions on the temperature or heat flow at the two ends of the rod. For example, the rod may be kept at a constant temperature at the edges or it may be isolated there. When solving a problem, students must bring together knowledge from physics and mathematics to set up or interpret the boundary conditions. Therefore, these boundary conditions became the focus of our research.

To analyze students' reasoning processes, we use the framework of *conceptual blending*. This framework has its origins in linguistics, where Fauconnier and Turner (1998) developed it to describe how people make meaning by selectively bringing together information from two or more previous experiences/other domains. The framework has been used previously within physics education research to describe the bringing together, or *blending*, of mathematics and physics.

As a first step (**Chapter 2**), we studied what difficulties students have in mathematically describing the boundary conditions of a physical system. We conducted in-depth interviews with twelve students in which we asked them to solve some problems while thinking aloud. The goal was to identify difficulties and characterize them in terms of their position in the conceptual blending framework, i.e., difficulties of a physical or mathematical nature, or difficulties related to blending both. Our analysis shows that conceptual blending provides a good framework for describing students' difficulties: we were able to identify both difficulties in mathematics and physics separately, but also found that blending mathematics and physics can go wrong in different ways. This supports the idea that knowledge of mathematics and physics separately is insufficient to bring both disciplines together when describing a physical phenomenon mathematically.

In addition to understanding the difficulties, we also wanted to gain a better understanding of students' reasoning process when they combine mathematical and physical knowledge so that we could develop better educational learning activities that support students in that blending process. Therefore, in a second study, we developed a new method of analysis: the *dynamic blending diagram* (DBD) (**Chapter 3**). When constructing a DBD, we first categorize the elements of the student's reasoning as physics, mathematics, or blended. On top of that, we represent the order of those elements by numbering them and we connect elements that were related by the student. In this way, students' reasoning and blending process is visually represented.
We conducted in-depth interviews with four pairs of students. The data was analyzed using DBDs, which showed that graphs can be important to support the blending of mathematics and physics (**Chapter 4**). Students who analyzed graphs from the problem statement and/or constructed their own graphs were often better at formulating the correct boundary condition. We found that constructing graphs can help make the connection between the physical phenomenon of heat flow and its mathematical description as a partial derivative of temperature with respect to position, a connection

that appeared to be not obvious to many students. However, we also found that graphical reasoning alone was often not enough. Based on graphs, students were able to eliminate incorrect options for the mathematical description, but only one pair managed to provide a proper and complete justification for the correct relationship.

In a final study (**Chapter 5**), we combined the findings from the previous studies with findings from the research literature to develop educational materials that encourage and support the blending of mathematics and physics. We developed a tutorial to help students make the connection between the physical phenomenon of heat flow and its mathematical description as a partial derivative of temperature with respect to position, since the previous interview series showed that this is difficult for most students. The design of the tutorial is based on three design principles: (1) explicitly focusing on both the mathematical and physical aspects of the intended reasoning, (2) encouraging graphical reasoning to stimulate blending of mathematics and physics, and (3) the principle of blended encapsulation, developing the concept of derivative step by step while linking it to physical meaning.
The concept of (partial) derivative is crucial for understanding the heat equation, its boundary conditions, and the relationship between the partial derivative of temperature with respect to position and heat flow. Therefore, we start from the layered model for the concept of derivative as developed in mathematics didactics by Zandieh (2000). We extend this framework using the conceptual blending framework and our knowledge of the beneficial effect of graphical reasoning to represent the relationship between the mathematical concept of the partial derivative $\partial T/\partial x$ and the physical concept of heat flow. This results in our so-called *blended partial derivative framework*, which provides the theoretical basis for the approach in the tutorial. This framework consists of several layers and columns, which provides opportunities to encourage the blend between mathematics and physics in smaller steps. The step-by-step construction of the layers was named *encapsulation* in the context of mathematics education research. Because of the explicit link to physics that we make at each layer, we speak here of *blended encapsulation*.
The developed tutorial was tested with three groups of three students in teaching-learning interviews. From the results, we conclude that the blended encapsulation approach can help students in recognizing how temperature differences lead to heat flow and how this can be formulated mathematically. We also make some recommendations to optimize the current design, such as encouraging students to reason about heat flow through a specific point in the system rather than the system as a whole.

In this thesis, for a specific topic (boundary conditions for the heat equation), we explored in detail how students bring mathematics and physics together in their reasoning, and the difficulties they face in doing so. We drew on the framework of conceptual blending and used this framework to describe the difficulties and to design materials that can encourage and support this blending process. The three studies in this dissertation use qualitative data and analysis methods, and examine the reasoning of a small number of students. In follow-up research, we recommend that this be expanded on a larger scale. Our findings are context-specific, but they also have implications for other topics within physics where mathematical reasoning plays an important role.

Samenvatting

De rol van wiskunde in natuurkunde is veelzijdig: wiskunde kan bijvoorbeeld fungeren als 'toolbox' (technische functie), als taal (communicatieve functie) of voor het opbouwen van logisch-deductieve redeneringen (structurele functie). Wiskunde is bovendien niet enkel essentieel geweest voor de ontwikkeling van natuurkunde, maar omgekeerd ontstonden ook veel wiskundige concepten vanuit een verlangen om de natuur te beschrijven. De verwevenheid tussen wiskunde en natuurkunde is zo sterk dat het soms moeilijk is beide uit elkaar te halen.
Toch komt deze sterke verbondenheid niet altijd tot uiting in het onderwijs. In het natuurkundeonderwijs wordt wiskunde vaak herleid tot zijn technische functie om verbanden te beschrijven en berekeningen te maken. In wiskundeonderwijs wordt natuurkunde vaak gereduceerd tot een context waarin abstracte wiskundige ideeën geïllustreerd en toegepast worden. Deze tweedeling maakt het voor leerlingen en studenten lastig om hun kennis van beide disciplines te integreren. De vraag hoe beiden best worden samengebracht om het leerproces van lerenden te ondersteunen, blijft grotendeels onbeantwoord, ondanks het vele vakdidactisch onderzoek naar het leren en onderwijzen van wiskunde en van natuurkunde afzonderlijk. Recent onderzoek naar het samenspel van wiskunde en natuurkunde in de redeneringen van studenten toont aan dat dit ook voor meer gevorderde studenten (verder in de bachelor en master) vaak een uitdaging blijft. Deze studenten zijn meestal wel vaardig in het uitvoeren van berekeningen (wat refereert naar de technische functie van wiskunde), maar ook voor hen blijkt de structurele rol van wiskunde in natuurkunde moeilijk te vatten. Hoe het onderwijs ingericht kan worden om dit beter te ondersteunen is nog niet zo duidelijk.

In dit proefschrift onderzoeken we voor een specifiek onderwerp (partiële differentiaalvergelijkingen) hoe studenten wiskunde en natuurkunde samenbrengen in hun redeneringen, wat de specifieke moeilijkheden zijn die ze hierbij ondervinden, en uiteindelijk hoe we daarop kunnen inspelen bij het ontwerpen van onderwijsleeractiviteiten.

Partiële differentiaalvergelijkingen worden typisch in de bacheloropleidingen natuurkunde en wiskunde behandeld en bij de bespreking ervan komen wiskunde en natuurkunde vaak samen aan bod. De warmtevergelijking beschrijft bv. warmtetransport in een staaf en vormt zo een uitgelezen voorbeeld van het samenspel tussen beide disciplines. Om het fysische systeem volledig te beschrijven zijn naast de differentiaalvergelijking nog een beginvoorwaarde en randvoorwaarden nodig. De beginvoorwaarde legt de initiële temperatuurverdeling vast en de randvoorwaarden leggen voorwaarden op aan de temperatuur of warmtestroom aan de twee uiteinden van de staaf. De staaf kan bijvoorbeeld aan de randen op een constante temperatuur gehouden worden of ze kan daar geïsoleerd zijn. Bij het oplossen van een probleem moeten studenten kennis uit natuurkunde en wiskunde samenbrengen om de randvoorwaarden op te stellen of te interpreteren. Daarom werden deze randvoorwaarden de focus van ons onderzoek.

Om de redeneerprocessen van studenten te analyseren, maken we gebruik van het kader van *conceptual blending*. Dit kader vindt zijn oorsprong in de linguïstiek, waar Fauconnier en Turner (1998) het ontwikkelden om te beschrijven hoe mensen betekenis geven door het selectief samenbrengen van informatie uit twee of meer eerdere ervaringen/andere domeinen. Het kader werd binnen vakdidactisch onderzoek natuurkunde al eerder gebruikt om het samenbrengen, oftewel *blenden*, van wiskunde en natuurkunde te beschrijven.

Als eerste stap (**hoofdstuk 2**) bestudeerden we welke moeilijkheden studenten ondervinden bij het wiskundig beschrijven van de randvoorwaarden van een fysisch systeem. We hebben diepte-interviews afgenomen bij twaalf studenten waarin we ze vroegen om hardop denkend enkele problemen op te lossen. Het doel was om moeilijkheden te identificeren en deze te karakteriseren in termen van hun positie in het conceptual blending kader, d.w.z. moeilijkheden van natuurkundige of wiskundige aard, of moeilijkheden die gerelateerd zijn aan het blenden van beide. Onze analyse toont aan dat conceptual blending een goed kader biedt om de moeilijkheden van studenten te beschrijven: we konden zowel moeilijkheden in wiskunde als natuurkunde apart identificeren, maar zagen ook dat het blenden van wiskunde en natuurkunde op verschillende manieren fout kan lopen. Dit ondersteunt het idee dat kennis van wiskunde en natuurkunde afzonderlijk onvoldoende is om beide domeinen te kunnen samenbrengen bij het wiskundig beschrijven van een fysisch fenomeen.

Naast inzicht in de moeilijkheden wilden we ook meer inzicht krijgen in het redeneerproces van studenten wanneer zij wiskundige en natuurkundige kennis combineren zodat we betere onderwijsleeractiviteiten kunnen ontwikkelen die de studenten ondersteunen in dat blending proces. Daarom ontwikkelden we in een tweede studie een nieuwe analysemethode: het *dynamic blending diagram* (DBD) (**hoofdstuk 3**). Bij het opstellen van een DBD categoriseren we eerst de elementen uit de redenering van de student als natuurkunde, wiskunde of blended. Daarbovenop geven we de volgorde van die elementen weer door ze te nummeren en verbinden we elementen die door de student met elkaar in verband werden gebracht. Op die manier wordt de redenering en het blending process van studenten visueel voorgesteld.

We namen diepte-interviews af met vier duo's studenten. De data werd geanalyseerd met behulp van DBDs. Deze analyse leerde dat grafieken belangrijk kunnen zijn om het blenden van wiskunde en natuurkunde te ondersteunen (**hoofdstuk 4**). Studenten die aangeboden grafieken analyseerden en/of zelf nieuwe grafieken construeerden waren vaak beter in het opstellen van de juiste randvoorwaarde. We vonden dat het construeren van grafieken kan helpen om het verband te leggen tussen het fysische fenomeen warmtestroom en zijn wiskundige beschrijving als partiële afgeleide van de temperatuur naar de positie, een verband dat voor veel studenten niet evident bleek te zijn. We zagen echter ook dat grafisch redeneren alleen vaak niet volstond. Op basis van grafieken waren studenten in staat om verkeerde opties voor de wiskundige beschrijving te elimineren, maar slechts één duo slaagde erin om een goede en volledige verantwoording te geven voor het correcte verband.

In een laatste studie (hoofdstuk 5) combineerden we de bevindingen uit de vorige studies met bevindingen uit de onderzoeksliteratuur om onderwijsleermateriaal te ontwikkelen dat het blenden van wiskunde en natuurkunde stimuleert en ondersteunt. We ontwikkelden een tutorial die studenten moet helpen om het verband te leggen tussen het fysische fenomeen warmtestroom en zijn wiskundige beschrijving als partiële afgeleide van de temperatuur naar de positie, aangezien uit de vorige interviewreeks is gebleken dat dit moeilijk is voor de meeste studenten. Het ontwerp van de tutorial is gebaseerd op drie ontwerpprincipes: (1) expliciet aandacht besteden aan zowel de wiskundige als de natuurkundige aspecten van de beoogde redenering, (2) stimuleren van grafisch redeneren om het blenden van wiskunde en natuurkunde te stimuleren, en (3) het principe van *blended encapsulation*, het stapsgewijs ontwikkelen van het begrip afgeleide en het tegelijkertijd koppelen aan fysische betekenis.
Het begrip (partiële) afgeleide is van cruciaal belang voor het begrijpen van de warmtevergelijking, de randvoorwaarden ervan en de relatie tussen de partiële afgeleide van de temperatuur naar de positie en warmtestroom. Daarom starten we vanuit het gelaagde model voor het begrip afgeleide zoals dat in de wiskundedidactiek is ontwikkeld door Zandieh (2000). We breiden dit kader uit met behulp van het conceptual blending kader en onze kennis over het gunstige effect van grafisch redeneren om de relatie tussen het wiskundige concept van de partiële afgeleide $\partial T/\partial x$ en het fysische concept van warmtestroom weer te geven. Dit resulteert in ons zogenaamde *blended partial derivative framework*, wat de theoretische basis vormt voor de aanpak in de tutorial. Dit kader bestaat uit verschillende lagen en kolommen waardoor het kansen biedt om de blend tussen wiskunde en natuurkunde te stimuleren in kleinere stappen. Het stapsgewijs opbouwen van de lagen kreeg in de context van wiskundedidactisch onderzoek de naam *encapsulation*, omwille van de expliciete koppeling met de natuurkunde die we op elke laag maken, spreken we hier van *blended encapsulation*.
De ontwikkelde tutorial werd uitgetest met drie groepjes van drie studenten in onderwijsleerinterviews. Uit de resultaten concluderen we dat de blended encapsulation aanpak studenten kan helpen in het herkennen van de manier waarop

temperatuurverschillen leiden tot warmtestroom en hoe dit wiskundig kan worden geformuleerd. We doen ook enkele aanbevelingen om het huidige ontwerp te optimaliseren, zoals bijvoorbeeld studenten meer stimuleren om te redeneren over warmtestroom door een specifiek punt in het systeem in plaats van in het systeem als geheel.

In dit proefschrift gingen we voor een specifiek onderwerp (randvoorwaarden voor de warmtevergelijking) in detail na hoe studenten wiskunde en natuurkunde samenbrengen in hun redeneringen, en wat de moeilijkheden zijn die ze hierbij ondervinden. We hebben daarbij gebruik gemaakt van het kader van conceptual blending en gebruikte dit kader om de moeilijkheden te beschrijven en om materiaal te ontwerpen dat dit blending proces kan stimuleren en ondersteunen. De drie studies in dit proefschrift hebben een kwalitatief karakter en bestuderen de redeneringen van een klein aantal studenten. In vervolgonderzoek adviseren we om dit uit te breiden op grotere schaal. Onze bevindingen zijn contextspecifiek, maar ze hebben ook implicaties voor andere onderwerpen binnen de natuurkunde waar wiskundig redeneren een belangrijke rol speelt.

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Curriculum vitae

Sofie Van den Eynde was born in Asse, Belgium, on the sixth of december 1993. After completing her secondary education, she moved to Leuven to study Physics at KU Leuven with the intention to become a secondary school teacher. She combined the academic Bachelor and Master programmes in Physics with the Teacher Training. During her Master studies, she did an Erasmus exchange of one semester at DCU in Dublin, Ireland. For her Master thesis, she set up a physics education research study for which she collected data both in Ireland and Flanders and which resulted in her first publication in a peer-reviewed journal.

This first research experience sparked her interest and made her apply for a PhD position. The PhD position was a collaboration between the University of Groningen, where she spent the first two years, and KU Leuven, where she spent the other two years. During her PhD, she was also a visiting scholar for one month at the University of Maine, at the Physics Education Research Laboratory. Sofie presented her work at several international conferences and seminars at different research institutes.

As of September 2020, she works as a curriculum developer and teacher trainer at the PIE group (Professionalization and Innovation in Education) at the Faculty of Science and Engineering of the University of Groningen. In this role, she teaches in the University Teaching Qualification (UTQ) and supervises lecturers in writing their teaching portfolio. Moreover she support several clusters with implementing educational innovations in their degree programmes.

Academic output

Journal publications

Van den Eynde, S., Deprez, J., Goedhart, M., & De Cock, M. (2020). Undergraduate student's difficulties with boundary conditions for the diffusion equation. *International Journal of Mathematical Education in Science and Technology*.

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