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## Blending of mathematics and physics

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# Blending of mathematics and physics

Undergraduate students' reasoning in the context of  
the heat equation

Sofie Van den Eynde

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**KU LEUVEN**

## **Blending of mathematics and physics**

Undergraduate students' reasoning in the context of the heat equation

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