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Time-varying wage Phillips curves in the euro area with a new measure for labor market slack

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**Abstract**

In recent years, the relationship between wage growth and the unemployment gap, known as the wage Phillips curve, has been puzzlingly weak: whereas the unemployment gap was low, wage growth was low as well. We consider two possible explanations for this ‘low wage growth puzzle’: (i) a structural change in the relationship between wage growth and labor market slack, and (ii) a failure of the unemployment gap to adequately capture labor demand conditions. We propose a new measure for labor market slack based on a survey among firms asking whether the shortage of labor is limiting production. This labor shortage indicator points to hidden slack not captured by the unemployment gap, which resolves the low wage growth puzzle. Our estimates of the wage Phillips curve for the five biggest euro area countries also suggest that the wage Phillips curve has changed over time, but not uniformly across countries.

1. Introduction

The wage Phillips curve, which relates nominal wage growth to labor market slack, has regained attention in recent years, especially within the policy domain. This was triggered by the apparent disconnect in some advanced economies between labor market conditions, which improved markedly, and wage growth dynamics, which remained subdued despite the sustained rebound in economic activity at the time. In fact, whereas the unemployment rate in the euro area moved from 12% in 2013 to 10.3% in 2016, wage growth fell from 1.8% to 1.4% within the same time period. Fig. 1 shows that wage growth has been stagnant after the 2008 crisis in the five major euro area countries as well. Moreover, a study by the European Central Bank (ECB) shows that wage growth projections were often too optimistic (ECB, 2016).\textsuperscript{1} In this paper, we aim to shed light on the dwindling performance of the wage Phillips curve in the euro area. We consider two possible (and not mutually exclusive) explanations: (i) a change in the relationship between wage growth and labor market slack, and (ii) a failure of commonly used measures for labor market slack to adequately capture labor demand conditions.

The literature suggests several explanations for why the Phillips curve relationship might change over time. Daly and Hobijn (2014), for instance, show that wage pressures arising from labor market slack weaken in times of persistently low inflation due to downward nominal wage rigidity. The Phillips curve may also have become flatter in recent years due to increased central bank credibility and more firmly anchored inflation expectations (Ball and Mazumder, 2011; Blanchard, 2016), the increasing role of globalization and external supply shocks (Stock, 2011; Gordon, 2013; Albuquerque and Baumann, 2017), information frictions (Colibion et al., 2018; Okuday et al., 2020) and changes in labor market institutions (Thomas and Zanetti, 2009; Zanetti, 2011). However, some recent studies suggest that the Philips curve has steepened. For instance, Bulligan and Viviano (2017) report a steepening of the wage Phillips curve in Italy, Spain and France where the sensitivity of hourly wage changes to labor market slack is found to have increased after the global financial crisis. Similarly, Skarica and Nobile (2016) report a steepening of the Phillips curve in Italy and Spain after the crisis, which according to

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\footnote{The views expressed do not necessarily reflect the official position of De Nederlandsche Bank, the Eurosystem and the European Commission. We like to thank two reviewers for their helpful comments on a previous version of the paper.}

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\footnote{Similarly, inflation remained quite low, despite the marked closing of the output gap in the euro area, casting doubt on the stability and reliability of the price Phillips curve (see, among others, Riggi and Venditti, 2015). This has led to a discussion about the usefulness of the inflation Phillips curve to forecast inflation; see McKnight et al. (2020) and references cited therein.}

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**ABSTRACT**

In recent years, the relationship between wage growth and the unemployment gap, known as the wage Phillips curve, has been puzzlingly weak: whereas the unemployment gap was low, wage growth was low as well. We consider two possible explanations for this ‘low wage growth puzzle’: (i) a structural change in the relationship between wage growth and labor market slack, and (ii) a failure of the unemployment gap to adequately capture labor demand conditions. We propose a new measure for labor market slack based on a survey among firms asking whether the shortage of labor is limiting production. This labor shortage indicator points to hidden slack not captured by the unemployment gap, which resolves the low wage growth puzzle. Our estimates of the wage Phillips curve for the five biggest euro area countries also suggest that the wage Phillips curve has changed over time, but not uniformly across countries.
the authors might reflect the impact of structural reforms. Likewise, Albuquerque and Baumann (2017) show that US inflation has become more responsive to economic slack since 2013.2

Alternatively, some authors have questioned the appropriateness of traditional measures for labor market slack in Phillips curve estimations. The unemployment gap, for instance, might not adequately capture changes in labor market conditions due to biased estimates of the natural rate of unemployment. In addition, changes in the unemployment gap reflect both supply and demand shocks which can be hard to disentangle. Some studies therefore suggest using alternative slack measures (see e.g. Brandolini et al., 2006). For instance, according to Ball and Mankuz (2015), a significant Phillips curve relationship between inflation and unemployment in the US is obtained if short-term unemployment, i.e. those unemployed for less than 26 weeks, is used as a proxy for slack. According to the authors, it is the short-term unemployed that put downward pressure on wages rather than the long-term unemployed, since the attachment of the latter to the labor force is relatively weak (see also Krueger et al., 2014). Likewise, Hornstein et al. (2014) construct an alternative labor market slack measure that accounts for differences in labor market attachment among non-employed individuals. They report that their non-employment index and broader measures of unemployment are moving closely together until the Great Recession, but after the Great Recession there appears to be a break in the relationship between unemployment and the broader measures of resource underutilization.1

Similarly, Blanchflower and Levin (2015) find that underemployment, i.e. involuntary part-time employment, and hidden unemployment (discouraged workers) appear crucial to understand the recent sluggish growth in wages in the US. Also the IMF (2017) has proposed an indicator of labor market slack which takes involuntary part-time employment into account. Finally, Krause et al. (2008), Trigari (2009), Faccini et al. (2013) and Zanetti (2014), among others, point out that the vacancies-to-employment ratio, which captures the degree of labor market tightness, is a key measure in the relationship between slack and inflation in theoretical models based on search and matching frictions in the labor market. In a similar vein as this literature, we propose a new slack measure based on a survey among firms asking whether the shortage of labor is limiting production. As we will argue below, this measure does not suffer from the drawbacks of the unemployment gap as indicator of labor market slack.

We re-examine the wage Phillips curve for the five biggest euro area countries, i.e. Germany, France, Italy, Spain and the Netherlands, by considering alternative measures for labor market slack. In particular, for each country we estimate the wage Phillips curve, augmented by both backward- and forward-looking explanatory variables. Our benchmark Phillips curve specification features the unemployment gap as a proxy for labor market slack. The results for this benchmark specification are compared to an alternative specification in which the labor shortage indicator from the European Commission (EC) is used to capture labor market slack. This indicator is based on a survey conducted by the EC in which firms are asked to what extent labor shortage is considered an important factor hampering production. In contrast to the unemployment gap, the labor shortage indicator is not prone to estimation bias and might therefore better capture changes in labor market conditions. Furthermore, it better captures changes in demand for labor than the unemployment gap which is the equilibrium outcome of both demand and supply shocks. However, even if our proposed measure better reflects slack in the labor market than the unemployment gap, supply shocks (like structural reforms) may change the relationship between our labor shortage indicator and wage growth. We therefore explicitly take potential time variation in the Phillips curve relationship into account, using Bayesian methods. We deliberately focus on estimates at the country level rather than at the aggregate euro area level, so as to account for potential heterogeneity across countries in, for instance, labor market institutions and regulations (Skarica and Nobile, 2016; IMF, 2017; Gross and Semmler, 2019).4

We find that the wage Phillips curve flattened in Germany and steepened in Spain in the years after the financial crisis, regardless of which measure for labor market slack is used. A change in the Phillips curve slope is also detected for France, Italy and the Netherlands, yet the nature of this change differs across the benchmark and alternative Phillips curve specifications. When using the unemployment gap as a proxy for labor market slack, our results suggest that the wage Phillips curve flattened in Italy, the Netherlands and, to a somewhat lesser extent, France. In contrast, when using the labor shortage indicator, the results indicate a steepening of the wage Phillips curve in Italy and France, and a stable Phillips curve relationship in the Netherlands after the crisis.

By comparing both measures for slack to broader measures of unemployment (i.e. underemployment), we conclude that the recent dwindling performance of the benchmark wage Phillips curve is to a significant degree due to ‘hidden slack’ that is not captured by the unemployment gap and which continues to weigh on wage growth dynamics. Our results therefore suggest that policymakers ought to consider broader indicators of labor market slack, such as the labor shortage indicator, when assessing wage growth dynamics, especially following severe economic crises.

The remainder of the paper is organized as follows. The following section provides a simple theoretical model to explain that both demand and supply shocks affect the Phillips curve relationship. The third section describes our proposed labor shortage indicator in more detail. Section 4 discusses the empirical model and data used, while Section 5 presents the

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1 There is a related discussion about the question of whether the inflation Phillips curve is time-varying. A good example is the recent study by Fu (2020) who finds strong evidence in support of the time-varying slopes of the Phillips curve in the US with different measures of inflation expectations.

2 As the authors point out, whether this implies that the standard unemployment rate understates or overstates the true degree of resource underutilization in the labor market after the Great Recession depends on the true resource underutilization. If one believes that the alternative measures best reflect the true state of the labor market, then the standard unemployment rate understates how much labor is idle after 2007. If, however, one believes that the non-employed should be weighted by their workforce attachment, then the standard unemployment rate overstated true resource underutilization for most of the post-2007 period.

3 The study by Bulligan and Viviano (2017) comes closest to our work, as it also focuses on wage inflation in several euro area countries and examines changes in the Phillips curve over time, using a different method than the one employed here. The most important difference is that Bulligan and Viviano (2017) use the unemployment rate as slack indicator.
main estimation results and robustness checks. Finally, Section 6 concludes.

2. The Phillips curve and correlated shocks

Estimates of the slope of the Phillips curve hinge on the identification of shocks to labor demand. When labor demand shocks are correlated with labor supply shocks, slope estimates may be biased due to simultaneity issues. Indeed, shifts in labor market outcomes may be due to changes in labor demand, labor supply or both. In the latter case, the labor and demand curves shift outward or inward simultaneously, resulting in an apparent insignificant relationship between labor market conditions and wage dynamics.

This is illustrated in Fig. 2. The figure shows a simple model of the labor market, with the horizontal axis representing the degree of labor market slack and the vertical axis representing wage inflation. In the left-hand side panel, the labor market faces only demand shocks, which cause movements in the (red-colored) demand curve. The equilibrium outcomes of wage inflation and labor market slack, indicated by the intersections of the supply and demand curves, are observed by the econometrician and suggest a significant and negative slope of the labor supply curve and hence the Phillips curve. In the right-hand side panel, the labor market faces positively correlated labor demand and labor supply shocks, implying that the demand and supply curves move in the same direction. In this case, the equilibrium outcome suggests an insignificant relationship between wage inflation and labor market slack, even though the actual shape of the labor supply curve is not flat. Hence, in order to successfully infer the slope of the Phillips curve, one needs to be able to isolate labor demand shocks from labor supply shocks.

We can also illustrate this point using a standard New Keynesian model. The model consists of forward-looking, infinitely-lived households, who consume goods and services, supply labor and invest in physical capital and one-period bonds with the aim of maximizing some utility function. The markets for labor, goods and services are monopolistically competitive, which allows for a wedge between wages and the marginal rate of substitution and between prices and marginal costs. Following Galí (2011), we can gauge the level of unemployment by taking the difference between the desired level of hours worked, which equates the marginal rate of substitution to the real wage, and the actual level of hours worked. Wage- and price adjustments are infrequent and modeled according to a Calvo-type contract. An inflation- and output-gap-targeting central bank that sets the nominal interest rate closes the model. We focus on two shocks in the model: a labor demand shock, which is modeled as a labor-augmenting productivity shock, and a labor supply shock, that affects the disutility from labor.

We simulate the model twice, each time assuming the economy faces shocks to labor demand. In the first simulation round, we assume that labor demand shocks are uncorrelated with labor supply shocks. In the second simulation round, instead, the two shocks are perfectly negatively correlated. Fig. 3 plots the simulated series for unemployment and wage inflation from both simulation rounds. The left-hand side panel shows the series from the simulation with only labor demand shocks. The series suggest a significant, downward-sloping wage Phillips curve. The series plotted in the right-hand side panel, however, suggest a much weaker Phillips curve relationship, even though the parameters that govern the actual Phillips curve slope are the same in both simulations. As in Fig. 2, this exercise makes clear that it is important to isolate demand shocks from supply shocks in order to correctly infer the relationship between labor market slack and wage inflation. Unfortunately, this is more easily said than done. Our proposed way forward is to use a measure for labor market slack that we feel better reflects demand for labor than the unemployment gap, which, by definition, reflects changes in both labor demand and supply.

3. An alternative measure for labor market slack

The traditional wage Phillips curve relates nominal wage growth, $w_t$, to a measure for labor market slack, $s_t$, i.e.

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5 Here, we closely follow Hobijn (2018).

6 A full description of the model, and the benchmark calibration of its structural parameters, is provided in the Appendix.

7 Note that a positive shock to the disutility from labor causes labor supply to fall. Hence, a negative correlation between labor demand and labor supply shocks implies a shift of the labor demand and labor supply curves in the same direction.
with \( \omega \) a constant that can be interpreted as the long-run wage growth or labor productivity, and \( \epsilon_t \) the error term. As long as workers are able to bargain for a stable share of the economy’s value added, wage growth is generally in line with trend labor productivity growth (IMF, 2017). The parameter \( \gamma \) measures the slope of the wage Phillips curve. The sign of \( \gamma \) depends on the measure used to capture labor market slack. The simple representation of the wage Phillips curve in (1) predicts that additional labor market slack, i.e. an increase in \( s_t \), puts downward pressure on wages as more workers compete for the same number of jobs.

A commonly used measure for \( s_t \) is the unemployment gap, i.e. the difference between the unemployment rate, that is, the incidence of active job-seekers who are out of work and available to start working within two weeks, and the natural rate of unemployment. An increase in the unemployment gap indicates greater labor market slack which, by the wage Phillips curve, leads to a decline in wages. There are, however, several reasons to suspect that the unemployment gap does not adequately capture labor market slack, which would thereby leave the econometrician with an incorrect representation of the Phillips curve relationship. First, the unemployment rate does not take into account measures of underemployment, causing the unemployment gap to potentially underestimate labor market slack. Second, the natural rate of unemployment, which is unobserved and must therefore be inferred from the data, might suffer from an estimation bias, causing the unemployment gap and estimates of the Phillips curve slope to be biased as well.

In response to the first issue, Blanchflower and Levin (2015) propose to consider wider definitions of unemployment when assessing the overall degree of labor market slack. Particularly, some people may not currently seek work, despite being available, i.e. discouraged workers, while others may be actively seeking work, yet are not immediately available to start working. Moreover, some workers may be employed on a part-time basis, but wish to work more hours. Part-time employment has been rising across most euro area economies for over a decade, mainly owing to structural factors such as the growth in the services industry and the rise in female participation in the labor force. However, a recent study by the ECB (2017) shows that a non-negligible share of part-time workers would like to work more hours. The representation of labor market slack by the unemployment rate might be especially biased in times when the economy is hit by a large adverse shock, such as the global financial crisis, during which more workers may become discouraged in their job search or start working fewer hours than desired (Coré, 2017).

While it is difficult to properly measure the unemployment rate, it may be even more challenging to obtain reliable estimates of the natural rate of unemployment (see, for instance, Staiger et al., 1997). At the root of the problem lies the uncertainty regarding the model specification used to estimate the natural rate of unemployment. According to a study by the European Commission (EC), models that are based on static or adaptive expectations yield more pro-cyclical estimates of the natural rate of unemployment than those based on rational expectations when there are large labor market swings (EC, 2014). One reason is that static/adaptive expectations models do not take full account of price rigidities that have been shown to play an important role in the adjustment process of the labor market. When based on static expectations, EC estimates of the natural rate of unemployment for Spain after the crisis are much higher (26.4% in 2015) than those based on rational expectations models (22%), thereby suggesting a much tighter labor market.

Finally, the unemployment gap measure reflects both labor supply and demand shocks which may cause identification problems as outlined in the previous section. Although we cannot be fully certain that our proposed alternative slack measure only reflects demand for labor, we argue that it comes closer to measuring labor demand than the unemployment gap.

In our analysis of the wage Phillips curve, we employ two measures for labor market slack. As a benchmark, and in line with traditional specifications of the wage Phillips curve, we use the unemployment gap taken from the EC. In view of the above mentioned concerns, we also employ an alternative slack measure. In particular, we use the aggregated response to a question regarding labor shortage in an EC survey among firms. The EC survey asks firms on “factors limiting production” and contains a question whether the “shortage of labor force” is a factor that hampers production, to be answered by either “yes” or “no”. The responses are compiled into an index that measures the difference between

\[ w_t = \omega + \gamma s_t + \epsilon_t, \]

Fig. 3. Simulated series for unemployment and wage inflation. Note: Units are expressed as percentage deviations from steady state. The straight solid lines represent a linear fitted curve.

\[ \text{Note: Units are expressed as percentage deviations from steady state. The straight solid lines represent a linear fitted curve.} \]
the number of “yes” and “no” answers as a percentage of total answers. A rise (fall) of the index points towards more (fewer) problems in attracting labor and thereby indicates less (more) labor market slack. A score of zero indicates neutrality. The survey is conducted on a quarterly basis in different sectors, e.g., industry and services, in all euro area countries, and is available as early as 1985Q1 for some countries. In what follows, we focus on the industry sector which covers 23,940 companies for the euro area as a whole. As the data is quite volatile, we use four-quarter averages of the labor shortage indicator.

Fig. 4 compares the evolution of the labor shortage indicator (left axis) with the unemployment gap (right axis) between 1999Q1 and 2016Q2 in Germany, France, Italy, Spain, and the Netherlands. To ease comparison, we multiplied the unemployment gap by minus one, such that higher (lower) values of both measures indicate less (more) labor market slack. As shown in the figure, the two labor market slack measures behaved quite differently during this period, especially in Italy where the labor shortage indicator showed signs of increased labor market slack already in 2001, whereas the inverted unemployment gap became negative only after the peak of the crisis. What is also apparent is that in most countries, the unemployment gap indicates a decline in slack at the end of our sample period, pointing towards an improvement in labor market conditions. In contrast, the labor shortage indicator shows no such improvement during this period, but suggests a strong persistence in labor market slack inherited from the crisis. For instance, the Spanish unemployment gap appears to be closing relatively rapidly since 2013 (partly due to a sharp increase in most estimates of the natural rate of unemployment after the crisis), whereas the labor shortage indicator shows a much more muted recovery of the labor market. The same is observed in France, Italy, and the Netherlands.

It thus seems that, at least in more recent years, the unemployment gap and labor shortage indicator tell different stories regarding the labor market, with the former suggesting a stronger improvement in labor market conditions than the latter. In order to gauge which of these stories is most plausible, we look at data on underemployment, which the EC also provides. This data, which is referred to as “supplementary indicators to unemployment”, quantifies the amount of (i) underemployed part-time workers, (ii) persons seeking work, but not immediately available, and (iii) persons available to work, but not seeking. These indicators thereby provide a broader measure of labor market slack. Unfortunately, these data are available only from 2008 onward and therefore not suited for our main empirical analysis.

Fig. 5 displays the number of underemployed (as a percentage of the active population) for three periods: (i) the pre-crisis, (ii) the peak of the crisis, and (iii) the end of our sample in 2016Q2. We approximate the pre-crisis period by 2008Q1, as no earlier data are available. The peak of the crisis refers to the period with the highest reported rate of underemployment since 2008Q1. According to the figure, there has been a marked increase in underemployment during the crisis, which persisted well into the post-crisis period. With the exception of Germany, the latest data in our sample shows that labor market slack has not dissipated or reverted to pre-crisis levels. This observation seems at odds with the recent development of the unemployment gap, which shows signs of normalization. On the other hand, the behavior of the underemployment rate is consistent with the labor shortage indicator, which also suggests a strong persistence in labor market slack in recent years.

Although the labor shortage indicator seems to be a good measure for labor market slack, as it captures changes in labor market conditions...
across the business cycle quite well, it also has some limitations. Most importantly, the indicator only covers the industry sector and therefore does not contain information on labor market slack in other sectors. However, for a shorter sample period data are also available for the services sector. It turns out that the survey responses from the industry sector correlate quite strongly with those from the services sector in France (0.5), Germany (0.7), Italy (0.8) and the Netherlands (0.8), with only Spain exhibiting a weak correlation between the two series (−0.05). We therefore conclude that, for the majority of our sample, trends in the industry survey are sufficiently correlated with trends in other sectors.9

4. Empirical strategy

4.1. The model

We estimate an augmented version of the traditional wage Phillips curve, given by Equation (1), that is similar to the specification studied in theoretical models (e.g. Gali, 2011) and recent empirical work (e.g. Bulligan and Viviano, 2017). In particular, our main specification is a hybrid Phillips curve that includes lagged nominal wage growth, \( w_{t-1} \), and expected inflation, \( \pi_t^e \):

\[
w_t = \alpha_0 + \rho w_{t-1} + \gamma_1 s_t + \alpha_2 \pi_t^e + \epsilon_t.
\]

(2)

Whereas the lag in wage growth captures the observed persistence in wage dynamics, inflation expectations are aimed to capture potential forward-looking behavior of wage setters.10

In order to take into account potential changes in the relationship between wage growth and labor market slack, we allow the coefficients \( \alpha_0, \rho, \gamma \) and \( \alpha \) in (2) to vary over time.11 In particular, we estimate the following state-space model:

\[
w_t = \alpha_0 + \rho w_{t-1} + \gamma_1 s_t + \alpha_2 \pi_t^e + \epsilon_t = x_t \beta_t + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, \Sigma),
\]

(3)

\[
\beta_t = \beta_{t-1} + v_t, \quad v_t \sim \mathcal{N}(0, \Omega),
\]

(4)

with \( \text{cov}(\epsilon_t, \epsilon_s) = 0 \) and \( x_t \equiv [\beta_t' s_t, \pi_t^e]' \), and where \( \beta_t \equiv [\alpha_0, \rho, \gamma_1, \alpha_2]' \) are the time-varying parameters to be estimated. We use Bayesian estimation techniques (i.e. Gibbs sampling) to estimate the model (3)–(4).12 The first 10,000 samples in our training sample to initialize \( \beta_0, \Phi, \Omega \). The prior distribution for the variance \( \Phi \) is an Inverse Gamma distribution, i.e. \( \Phi \sim \mathcal{IG}(\tau_1, \tau_2) \), with the scaling parameter initialized at \( \tau_0 = 0.1 \). The prior for \( \Omega \) is an Inverse Wishart distribution, i.e. \( \Omega \sim \mathcal{IW}(\Phi, \tilde{T}_0) \), where \( \tilde{T}_0 = \Phi^{-1} \Sigma^{-1} \tilde{T}_0 \times \tilde{r} \) is the scaling matrix and where we set \( r = 0.35 \). The reason we choose a relatively high value for the scaling factor \( r \) is that several empirical studies suggest strong instability in Phillips curve relationships in the euro area (e.g. Oinonen and Paloviita, 2014; Riggi and Venditti, 2015; Bulligan and Viviano, 2017). In order to remain more consistent with recent empirical observations and allow for plausible behavior of the time-varying parameters, we opt for an informative prior for \( \Phi \). A total of 12,000 draws were used for the Gibbs-sampling algorithm, of which the first 10,000 draws were discarded.

4.2. Data description

As mentioned earlier, we use two measures for labor market slack, \( s_t \). As a benchmark, we use the unemployment gap taken from the European Commission. Our alternative measure is the labor shortage indicator, which we described in Section 3, obtained from the EC Industry Survey. For nominal wage growth, \( w_t \), we use the year-on-year growth rate of negotiated wages taken from the ECB and shown in Fig. 1.14 Finally, for inflation expectations, \( \pi_t^e \), we use one-year-ahead inflation expectations from Consensus Forecasts.15

Our data runs from 1999Q1 to 2016Q2 and estimations are performed for the five biggest euro area countries, i.e. Germany, France, Italy, Spain and the Netherlands, which together comprise about 80% of euro area GDP. As it is likely that the policies of the ECB ushered in a new monetary regime, we deliberately exclude observations prior to the inception of the euro. According to Benati (2008), a change in the monetary regime might significantly alter the statistical properties of the inflation process. Insofar as such a change affects inflation expectations, the Phillips curve relationship could change as well, making it more difficult to interpret the potential time variation in the Phillips curve parameters.

5. Estimation results

5.1. Constant parameter estimates

Before we discuss the results for the time-varying parameter model (3)–(4), we first estimate a Phillips curve with constant parameters, i.e. Equation (2), using ordinary least squares. Table 1 shows the results for the benchmark specification in which the unemployment gap is used as labor market slack measure, whereas Table 2 shows the results for the alternative specification that uses the labor shortage indicator as slack measure.

The results show that the Phillips curve slope has, for both specifications and for all countries, the expected sign and suggests a negative relationship between wage growth and the unemployment gap, and a positive relationship between wage growth and the labor shortage indicator. There are, however, notable differences across countries with regards the estimation results. The results for the benchmark specification suggest that the Phillips curve is steepest in Germany, with an estimated slope of around −0.4, and flattest in Spain, with a slope of around −0.04. In contrast, according to the alternative specification, the strongest wage/slack relationship is found in Spain, whereas the results for France point to a relatively weak Phillips curve relationship. For Italy, no significant Phillips curve slope is found, regardless of which measure for slack is used.

14 We consider negotiated wages rather than compensation per employees or compensation per hour since the latter are subject to one-offs (e.g. bonuses), changes in social contributions, fiscal policy shocks, wage drift and compositional effects, and other idiosyncrasies not linked to collective bargaining.

15 Tables 3–7 in the Appendix provide descriptive statistics of the main variables considered.
The estimated parameters for the remaining explanatory variables do not vary much across the two specifications, and also the performance of the two specifications in terms of explaining the variation in wage growth is comparable, as confirmed by the small differences in the adjusted $R^2$. Lastly, the alternative specification points (at least for most countries) to a somewhat stronger persistence in wage growth dynamics and a greater contribution of inflation expectations than the benchmark specification.

Notes: Standard errors in parentheses; ***, **, and * indicate significance levels of 1%, 5%, and 10%, respectively. Estimation performed using OLS.

5.2. Time-varying estimates of the wage Phillips curve slope

The estimates for the time-varying parameter model are shown in Fig. 6, which shows the estimated evolution of the slope parameter, $\gamma_t$, between 2001Q4 and 2016Q2 (solid blue lines). For comparison purposes, we also added the estimation results of the Phillips curve with constant parameters from Tables 1 and 2, as indicated by the horizontal dashed lines.

For Germany, both the benchmark specification (left column) and alternative specification (right column) point to an initial steepening of the Phillips curve, followed by a flattening of the Phillips curve that persists during the crisis period. Towards the end of the sample, the wage/slack relationship in Germany becomes insignificant (in Bayesian terms). As discussed in Section 3, and evidenced by data on underemployment, Germany exhibited a marked improvement in labor market conditions after the crisis, with broad measures of unemployment dipping below pre-crisis levels. However, wage growth has remained subdued: whereas wages grew, on average, by about 3% in 2008, in 2016 wage growth was only 2%. Therefore, whereas the constant parameter estimates point towards a strong and significant relationship between wages and labor market slack in Germany, the time-varying parameter estimates suggest that this relationship has weakened in recent years.

For Italy, we find strong time variation in the Phillips curve slope, which seems to underlie the insignificance of the wage/slack relationship inferred from the constant parameter estimates. Moreover, the results for the benchmark specification are strikingly different from those for the alternative specification. When using the unemployment gap to proxy labor market slack, the results suggest that the wage Phillips curve in Italy has flattened in more recent years of our sample. However, according to the alternative specification, the Phillips curve has steepened since around 2010. Recall from our discussion in Section 3 that, at least for the most recent years, the labor shortage indicator seems to better gauge labor market conditions than the unemployment rate, as its dynamics have been more consistent with broader measures of unemployment. Therefore, together with the fact that wage growth in Italy has remained stubbornly low (0.7% in 2016 compared to 3.5% in 2008, on average), we consider the results from the alternative specification more convincing and conclude that the Phillips curve in Italy has steepened in the aftermath of the crisis.

Similar results are found for the Netherlands and France: whereas the benchmark specification points to a flattening of the wage Phillips curve, the alternative specification indicates a stable or even stronger wage/slack relationship since 2010. In both countries, the labor shortage indicator shows greater persistence in labor market slack after the crisis than the unemployment gap, which is again more consistent with broader measures of unemployment. Also, wage growth has been weak in these countries compared to the period preceding the crisis. Together, these observations favor the results from the alternative specification that a significant wage Phillips curve emerged following the crisis.

In the case of Spain, both specifications find a strengthening of the wage/slack relationship after 2008. This finding is in line with the observed surge in unemployment during the crisis that was accompanied by suppressed wage dynamics. The steepening of the Spanish wage Phillips curve is most pronounced for the alternative specification. When using the unemployment gap to proxy labor market slack, the results suggest that the wage Phillips curve in Spain has steepened in recent years of our sample. However, according to the alternative specification, the Phillips curve has steepened since around 2010. Recall from our discussion in Section 3 that, at least for the most recent years, the labor shortage indicator seems to better gauge labor market conditions than the unemployment rate, as its dynamics have been more consistent with broader measures of unemployment. Therefore, together with the fact that wage growth in Spain has remained stubbornly low (1.1% in 2016 compared to 3.5% in 2008, on average), we consider the results from the alternative specification more convincing and conclude that the Phillips curve in Spain has steepened in the aftermath of the crisis.

In sum, for those countries where the crisis led to a relatively strong surge in unemployment (France, Italy, the Netherlands, and Spain), and where the unemployment gap has been unable to fully capture the

---

16 The estimated parameters for the remaining explanatory variables do not vary much across the two specifications, and also the performance of the two specifications in terms of explaining the variation in wage growth is comparable, as confirmed by the small differences in the adjusted $R^2$. Lastly, the alternative specification points (at least for most countries) to a somewhat stronger persistence in wage growth dynamics and a greater contribution of inflation expectations than the benchmark specification.

Notes: Standard errors in parentheses; ***, **, and * indicate significance levels of 1%, 5%, and 10%, respectively. Estimation performed using OLS.

5.2. Time-varying estimates of the wage Phillips curve slope

The estimates for the time-varying parameter model are shown in Fig. 6, which shows the estimated evolution of the slope parameter, $\gamma_t$, between 2001Q4 and 2016Q2 (solid blue lines). For comparison purposes, we also added the estimation results of the Phillips curve with constant parameters from Tables 1 and 2, as indicated by the horizontal dashed lines.

For Germany, both the benchmark specification (left column) and alternative specification (right column) point to an initial steepening of the Phillips curve, followed by a flattening of the Phillips curve that persists during the crisis period. Towards the end of the sample, the wage/slack relationship in Germany becomes insignificant (in Bayesian terms). As discussed in Section 3, and evidenced by data on underemployment shown in Fig. 5, Germany exhibited a marked improvement in labor market conditions after the crisis, with broad measures of unemployment dipping below pre-crisis levels. However, wage growth has remained subdued: whereas wages grew, on average, by about 3% in 2008, in 2016 wage growth was only 2%. Therefore, whereas the constant parameter estimates point towards a strong and significant relationship between wages and labor market slack in Germany, the time-varying parameter estimates suggest that this relationship has weakened in recent years.

For Italy, we find strong time variation in the Phillips curve slope, which seems to underlie the insignificance of the wage/slack relationship inferred from the constant parameter estimates. Moreover, the results for the benchmark specification are strikingly different from those for the alternative specification. When using the unemployment gap to proxy labor market slack, the results suggest that the wage Phillips curve in Italy has flattened in more recent years of our sample. However, according to the alternative specification, the Phillips curve has steepened since around 2010. Recall from our discussion in Section 3 that, at least for the most recent years, the labor shortage indicator seems to better gauge labor market conditions than the unemployment rate, as its dynamics have been more consistent with broader measures of unemployment. Therefore, together with the fact that wage growth in Italy has remained stubbornly low (0.7% in 2016 compared to 3.5% in 2008, on average), we consider the results from the alternative specification more convincing and conclude that the Phillips curve in Italy has steepened in the aftermath of the crisis.

Similar results are found for the Netherlands and France: whereas the benchmark specification points to a flattening of the wage Phillips curve, the alternative specification indicates a stable or even stronger wage/slack relationship since 2010. In both countries, the labor shortage indicator shows greater persistence in labor market slack after the crisis than the unemployment gap, which is again more consistent with broader measures of unemployment. Also, wage growth has been weak in these countries compared to the period preceding the crisis. Together, these observations favor the results from the alternative specification that a significant wage Phillips curve emerged following the crisis.

In the case of Spain, both specifications find a strengthening of the wage/slack relationship after 2008. This finding is in line with the observed surge in unemployment during the crisis that was accompanied by suppressed wage dynamics. The steepening of the Spanish wage Phillips curve is most pronounced for the alternative specification. When using the unemployment gap to proxy labor market slack, the results suggest that the wage Phillips curve in Spain has steepened in recent years of our sample. However, according to the alternative specification, the Phillips curve has steepened since around 2010. Recall from our discussion in Section 3 that, at least for the most recent years, the labor shortage indicator seems to better gauge labor market conditions than the unemployment rate, as its dynamics have been more consistent with broader measures of unemployment. Therefore, together with the fact that wage growth in Spain has remained stubbornly low (1.1% in 2016 compared to 3.5% in 2008, on average), a strong relationship between wage growth and the labor shortage indicator ensues.

In sum, for those countries where the crisis led to a relatively strong surge in unemployment (France, Italy, the Netherlands, and Spain), and where the unemployment gap has been unable to fully capture the

---

16 Recall that we remove a training sample of 10 quarters and use 1 lag of the dependent variable.
persistence in labor market slack during the aftermath of the crisis, the wage Phillips curve relationship is alive and well and can explain current subdued wage growth, provided the labor shortage indicator is used as a measure for labor market slack. It follows that a good understanding of past and current wage dynamics, and forecasts of future wage pressures, requires a broad assessment of labor market conditions, especially following severe economic crises. Furthermore, imposing a time-invariant Phillips curve slope may cause one to mistakenly conclude that the wage/slack relationship is significant, even though it is not (as we find for Germany), or the other way around (in the case of Italy). Lastly, our results reveal important heterogeneities within the euro area, with wage growth in some countries responding much stronger to labor market improvements than in other countries.

5.3. Robustness checks

We test the robustness of our main results in two ways. First, we use the unemployment rate rather than the unemployment gap as a measure for labor market slack in the benchmark Phillips curve specification. Second, for both specifications, we add labor productivity growth per employee as an explanatory variable to the wage Phillips curve.17

One reason to use the unemployment rate as a measure for labor market slack rather than the unemployment gap, is that the latter is an unobserved variable, which must be estimated and therefore is prone to estimation uncertainty. In our first robustness exercise, we replace the unemployment gap in the benchmark specification with

\[ \text{slack} = \text{unemployment gap} \]

\[ \text{slack} = \text{labor shortage indicator} \]

Fig. 6. Estimates of the wage Phillips curve slope, $\gamma_t$. Note: The figure shows estimates for $\gamma_t$ in Equation (3). The blue solid (dotted) lines reflect the 50th (16th and 84th) percentiles from the posterior distribution. The red horizontal dashed and dashed-dotted lines are the estimates, and corresponding 95% interval, from a time-invariant version of (3), estimated using OLS. (For interpretation of the references to color in this figure legend, the reader is referred to the Web version of this article.)

17 We also considered other alternative specifications of the wage Phillips curve, e.g. by including lagged HICP inflation as an explanatory variable, using productivity growth per hour rather than per employee, and using different lag structures. Also, we used different priors and training samples to initialize the Gibbs-sampling algorithm. The results of these robustness checks, which are available upon request, do not differ much from our main results. Moreover, we performed the Bai-Perron test for the Phillips curve with constant parameters and found that the identified breaks coincide with the movements in the time-varying Phillips curve slope.
unemployment rate. Data on the unemployment rate (in percentages of the active population) is seasonally adjusted and taken from Eurostat.

Estimates for the time-varying Phillips curve slope are shown in Fig. 7 (left column). For comparison purposes, we also report the results for the alternative specification in which the labor shortage indicator is used as slack measure (right column). The results are in line with our main results shown in the left column of Fig. 6. What is notable is that the flattening of the German Phillips curve in more recent years of our sample period seems more pronounced when the unemployment rate is used as slack measure, with the results exhibiting much less uncertainty surrounding the median estimates. Moreover, for France and Italy, a slight steepening of the Phillips curve is now observed at the end of the sample, which is more in line with the alternative Phillips curve specification.

Next, we add to both specifications of the wage Phillips curve labor productivity growth as explanatory variable, such that Equation (3) is replaced by

$$w_t = \omega_t + \rho_t w_{t-1} + \gamma_t s_t + \alpha_t \Delta_t + \mu_t A_t + \varepsilon_t,$$

with $A_t$ denoting labor productivity growth per employee. According to theory, an increase in labor productivity raises demand for labor, which in turn puts upward pressure on wage growth, above and beyond the effects of labor market tightness. As pointed out by the IMF (2017), as long as workers are able to bargain for a stable share of the economy’s value added, wage growth is generally in line with trend labor productivity growth.\[18\] Data on productivity growth are collected from Eurostat.

The time-varying estimates of the Phillips curve slope based on this augmented Phillips curve specification are shown in Fig. 8. Interestingly,
the results are strikingly similar to our main results shown in Fig. 6. In fact, the same conclusions can be drawn from these results: the Phillips curve has flattened in more recent years of our sample period in Germany, yet steepened in Spain; for France, Italy and the Netherlands, we find a flattening of the Phillips curve after 2010 when using the unemployment gap as slack measure, and a steepening of the Phillips curve when using the labor shortage indicator.

6. Conclusion

Before the COVID-19 pandemic, the unemployment gap in the euro area fell markedly. However, wages remained low and increased less than predicted by traditional Phillips curves. We have therefore re-examined the wage Phillips curve for the five biggest euro area countries, i.e. Germany, France, Italy, Spain and the Netherlands, by explicitly taking into account potential time variation in the Phillips curve relationship and by considering an alternative measure for labor market slack. In particular, for each country we estimated the wage Phillips curve, augmented by both backward- and forward-looking explanatory variables, with time-varying parameters using Bayesian methods. As a benchmark, we use the unemployment gap as a measure for labor market slack. The results for the benchmark specification are compared to those for an alternative specification using the survey-based labor shortage indicator to capture labor market slack. We argue that this alternative measure may better assess slack in the labor market than the unemployment gap.

When using the unemployment gap as a proxy for labor market slack, our results suggest that the wage Phillips curve flattened in Italy, the Netherlands and, to a somewhat lesser extent, France. In contrast, when using the labor shortage indicator as slack measure, the results suggest a steepening of the wage Phillips curve in Italy and France, and a stable Phillips curve relationship in the Netherlands after the financial crisis. We relate these conflicting results to the observation that, unlike the labor shortage indicator, the unemployment gap has been unable to adequately capture the persistence in additional labor market slack (based on measures of underemployment). In fact, the unemployment gap points

Fig. 8. Estimates of the wage Phillips curve slope, $\gamma_t$, when controlling for labor productivity. Note: The figure shows estimates for $\gamma_t$ in Equation (5). The blue solid (dotted) lines reflect the 50th (16th and 84th) percentiles from the posterior distribution. The red horizontal dashed and dashed-dotted lines are the estimates, and corresponding 95% interval, from a time-invariant version of (5). (For interpretation of the references to color in this figure legend, the reader is referred to the Web version of this article.)
to a much stronger improvement in labor market conditions in these countries after the financial crisis than suggested by broader measures of unemployment. Since wage growth has remained stagnant, its relationship with the unemployment gap weakened, implying a flattening of the Phillips curve. In contrast, changes in the labor shortage indicator have been more consistent with broader measures of unemployment, suggesting more labor market slack than the unemployment gap. Consequently, the link between the labor shortage indicator and wage growth remained strong or even strengthened after the unemployment gap. This last result implies that it is highly problematic to estimate Phillips curve models for the euro area as a whole as is commonly done. Moreover, it is imperative to consider broad indicators of labor market slack in order to properly assess wage growth dynamics at the country level, especially following severe crises when different slack measures point towards different speeds of economic recovery. Our analysis shows that, at least in some countries, the wage Phillips curve relationship has not weakened, implying that wage and price pressures may re-emerge in the euro area once spare capacity is sufficiently absorbed.

Declaration of competing interest

I declare that I have no relevant or material financial interests that relate to the research described in the paper with the title “Time-varying wage Phillips curves in the euro area with a new measure for labor market slack”.

A. Descriptive statistics

Table 3
Descriptive statistics: Germany.

<table>
<thead>
<tr>
<th></th>
<th>Wage growth</th>
<th>Unemployment gap</th>
<th>Labor shortage indicator</th>
<th>Inflation expectations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>2.03</td>
<td>0.28</td>
<td>5.14</td>
<td>1.58</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.71</td>
<td>0.59</td>
<td>3.12</td>
<td>0.34</td>
</tr>
<tr>
<td>Number of obs.</td>
<td>70</td>
<td>70</td>
<td>70</td>
<td>70</td>
</tr>
<tr>
<td>Maximum</td>
<td>3.77</td>
<td>1.88</td>
<td>10.58</td>
<td>2.38</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.09</td>
<td>-0.58</td>
<td>0.83</td>
<td>0.89</td>
</tr>
</tbody>
</table>

Table 4
Descriptive statistics: France.

<table>
<thead>
<tr>
<th></th>
<th>Wage growth</th>
<th>Unemployment gap</th>
<th>Labor shortage indicator</th>
<th>Inflation expectations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>2.16</td>
<td>-0.07</td>
<td>7.26</td>
<td>1.47</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.53</td>
<td>0.66</td>
<td>3.50</td>
<td>0.27</td>
</tr>
<tr>
<td>Number of obs.</td>
<td>70</td>
<td>70</td>
<td>70</td>
<td>70</td>
</tr>
<tr>
<td>Maximum</td>
<td>3.10</td>
<td>1.53</td>
<td>18.93</td>
<td>2.10</td>
</tr>
<tr>
<td>Minimum</td>
<td>1.20</td>
<td>-1.74</td>
<td>1.70</td>
<td>0.62</td>
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</tbody>
</table>

Table 5
Descriptive statistics: Italy.

<table>
<thead>
<tr>
<th></th>
<th>Wage growth</th>
<th>Unemployment gap</th>
<th>Labor shortage indicator</th>
<th>Inflation expectations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>2.16</td>
<td>0.27</td>
<td>2.84</td>
<td>1.76</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.78</td>
<td>1.08</td>
<td>2.33</td>
<td>0.43</td>
</tr>
<tr>
<td>Number of obs.</td>
<td>70</td>
<td>70</td>
<td>70</td>
<td>70</td>
</tr>
<tr>
<td>Maximum</td>
<td>4.20</td>
<td>2.26</td>
<td>8.45</td>
<td>2.69</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.59</td>
<td>-1.66</td>
<td>0.18</td>
<td>0.43</td>
</tr>
</tbody>
</table>

Table 6
Descriptive statistics: Spain.

<table>
<thead>
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<th></th>
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<th>Unemployment gap</th>
<th>Labor shortage indicator</th>
<th>Inflation expectations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>2.35</td>
<td>0.76</td>
<td>1.88</td>
<td>2.11</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>1.00</td>
<td>3.96</td>
<td>1.05</td>
<td>0.72</td>
</tr>
<tr>
<td>Number of obs.</td>
<td>70</td>
<td>70</td>
<td>70</td>
<td>70</td>
</tr>
<tr>
<td>Maximum</td>
<td>3.54</td>
<td>8.02</td>
<td>4.18</td>
<td>3.20</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.54</td>
<td>-5.46</td>
<td>0.23</td>
<td>0.38</td>
</tr>
</tbody>
</table>
Table 7
Descriptive statistics: Netherlands.

<table>
<thead>
<tr>
<th></th>
<th>Wage growth</th>
<th>Unemployment gap</th>
<th>Labor shortage indicator</th>
<th>Inflation expectations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>2.13</td>
<td>0.36</td>
<td>4.60</td>
<td>1.84</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>1.10</td>
<td>0.87</td>
<td>2.80</td>
<td>0.58</td>
</tr>
<tr>
<td>Number of obs.</td>
<td>70</td>
<td>70</td>
<td>70</td>
<td>70</td>
</tr>
<tr>
<td>Maximum</td>
<td>4.53</td>
<td>1.91</td>
<td>10.95</td>
<td>3.54</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.50</td>
<td>−0.99</td>
<td>1.23</td>
<td>0.77</td>
</tr>
</tbody>
</table>

B. Description of the New Keynesian model

B.1. Household consumption and savings

In every period \( t \), a household of type \( i \in [0, 1] \) chooses consumption, \( c_t \), labor supply, \( n_t(i) \), investment \( i_t \) in physical capital \( k_t \) that earns a nominal rental rate \( R_k(t) \), and nominal holdings of one-period bonds, \( B_t \), which earn a nominal gross risk-free interest rate of \( R_t \), in order to maximize expected lifetime utility

\[
E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left( c_{t+\tau}^{1-\sigma} - \frac{\pi_{t+\tau}}{\pi_t} n_{t+\tau}^{1+\phi} \right),
\]

subject to the period budget constraint:

\[
P_t c_t + P_t k_t + B_t = R_t B_{t-1} + R_{t+k_t} n_t(i) + P_t \pi_t,
\]

where \( W_t(i) \) denotes the nominal wage rate, \( P_t \) the consumer price index (CPI), and \( \pi_t \) firm profits. The variable \( \pi_{t+\tau} \) is a shock to the disutility of labor that affects labor supply and which evolves according to a stationary AR(1) process. The parameter \( \sigma > 0 \) determines the elasticity of intertemporal substitution, whereas \( \phi > 0 \) determines the inverse Frisch elasticity of labor supply. Capital evolves according to

\[
k_t = (1 - \delta) k_{t-1} + i_t,
\]

with \( \delta \in [0, 1] \) the depreciation rate of capital.

The first-order conditions that pin down the household’s consumption and savings decisions are given by

\[
c_t^{1-\sigma} = \beta E_t \left[ c_{t+\tau}^{1-\sigma} \frac{R_t}{\pi_{t+\tau}} \right],
\]

\[
c_t^{1-\sigma} = \beta E_t \left[ c_{t+\tau}^{1-\sigma} (R_{t+\tau} + 1 - \delta) \right],
\]

with \( \pi_t \equiv P_t / P_{t-1} \) and \( R_t \equiv R_{kt+1} / P_t \).

B.2. Labor supply and wage setting

Labor of household type \( i \) is bundled by a representative employment agency according to the following CES aggregator:

\[
n_t = \left( \int_0^1 n_t(i)^{-\epsilon_w} \, di \right)^{-\epsilon_w}.
\]

The parameter \( \epsilon_w > 0 \) measures the elasticity of substitution between differentiated labor inputs. Subject to an appropriate expenditure constraint, \( w_t n_t = \int_0^1 w_t(i)n_t(i) \, di \), the employment agency maximizes \( n_t \). This yields the following demand function for \( n_t(i) \):

\[
n_t(i) = \left( \frac{w_t(i)}{w_t} \right)^{-\epsilon_w} n_t,
\]

and aggregate wage index:

\[
w_t = \left( \int_0^1 w_t(i)^{1-\epsilon_w} \, di \right)^{\epsilon_w}.
\]

Households face a wage-setting constraint that prohibits a fraction \( \theta_w \in (0, 1) \) of households from adjusting their wage in a given period. These households keep wages fixed at the aggregate wage index from the previous period. Hence, the aggregate wage index can be written as

---

19 State-contingent securities ensure that household consumption is the same across all households of the same type.
\[ w_1^{t-s} = (1 - \theta_s)w_1^{t-s} + \theta_sw_{t-1}^{t-s} \]

where \( w_1 \) denotes the optimal reset wage.

Households choose \( w_t \) to maximize

\[
E_t \sum_{k=0}^{\infty} (\beta \theta)^k \left( \lambda_{t+k} + n_{t+k}(i) - \frac{n_t(i)}{1 + \phi} \right)
\]

with \( \lambda_t \) the marginal utility of consumption, subject to the demand function for \( n_t(i) \). The first-order condition that determines the optimal reset wage is given by

\[
\frac{w_1^{t-s} + \epsilon}{w_1^{t-s} + \epsilon} = \frac{E_t \sum_{k=0}^{\infty} (\beta \theta)^k \left( \lambda_{t+k} + n_{t+k}(i) - \frac{n_t(i)}{1 + \phi} \right)}{E_t \sum_{k=0}^{\infty} (\beta \theta)^k \lambda_{t+k} + n_{t+k}(i) + \frac{n_t(i)}{1 + \phi}}
\]

(14)

Unemployment, \( u_t \), is given by the difference between the desired number of hours worked, \( n_d(i) \), and the actual number of hours worked, \( n_t(i) \):

\[
u_t = n_d - n_t
\]

(15)

where the desired number of hours worked is determined by the condition that equates the wage rate with the marginal rate of substitution:

\[
w_t = \frac{z_{nd}^\phi}{\lambda_t^\phi}
\]

(16)

**B.3. Firms**

**B.3.1. Final goods firms**

Differentiated intermediate goods \( y_t(i) \), with \( i \in [0, 1] \), are assembled into the final good \( y_t \) by final goods firms according to

\[
y_t = \left( \int_0^1 y_t(i)^{\epsilon} \, di \right)^{\frac{1}{\epsilon}}
\]

where \( \epsilon > 1 \) measures the elasticity of substitution between intermediate goods. Maximizing the final good, subject to an appropriate expenditure constraint, yields the following demand schedule:

\[
y_t(i) = \left( P_t(i) \right)^{-\epsilon} y_t,
\]

(17)

and consumer price index:

\[
P_t = \left( \int_0^1 P_t(i)^{1-\epsilon} \, di \right)^{\frac{1}{1-\epsilon}}.
\]

(18)

**B.3.2. Intermediate goods firms**

Intermediate goods \( y_{t(i)} \) are produced using the following constant returns to scale production function:

\[
y_{t(i)} = k_{t-1}(i)^{\psi} (z_{ndt}n_t(i))^{1-\psi},
\]

(19)

with \( \psi \in [0, 1] \) and where \( z_{ndt} \) is a labor-augmenting productivity shock, which evolves according to a stationary AR(1) process. Cost-minimization implies the following demand conditions:

\[
r_{t(i)} = m_c \psi \frac{y_{t(i)}}{k_{t-1}}
\]

(20)

\[
w_t = m_c(1-\psi) \frac{y_{t(i)}}{n_t(i)}
\]

(21)

with \( m_c \), real marginal costs, given by
with $\Phi \equiv \varphi - \varphi(1 - \varphi)^{-1}$. Firms face a Calvo-type price-setting constraint and maximize profits while discounting future profits by the probability of non-price adjustment, denoted by $\theta \in (0, 1)$:

$$
\max_{P_t} \sum_{k=0}^{\infty} \theta^k \left( \Pi_{t+k}^{\text{y}}(i) - P_{t+k} m_{c,i+k} Y_{t+i+k}(i) \right)
$$

subject to the optimal demand schedule for $y_t(i)$ and the production technology, and where $\Pi_{t+k}$ satisfies $\Pi_{t+k} = 1/R_t$. The first-order condition that determines the optimal reset price $P_t$ is given by

$$
P_t = \frac{\epsilon}{\epsilon - 1} \left( \frac{\Pi_{t}}{P_t} \right)^{\frac{1}{\sigma}} \left( \frac{\Pi_{t}}{P_t} \right)^{\frac{\phi}{\epsilon}} \left( \frac{\Pi_{t}}{P_t} \right)^{\frac{\theta}{\epsilon - 1}}
$$

(23)

B.4. Public sector

The monetary authority (or ‘central bank’) conducts monetary policy according to the following Taylor-type rule:

$$
\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\beta} \left( \frac{y_t}{y_t^*} \right)^{\frac{1}{\epsilon}} \left( \frac{y_t}{y_t^*} \right)^{\frac{1}{\phi}}
$$

(24)

where $y_t^*$ denotes the level of output consistent with the flexible-price equilibrium.

B.5. Market clearing

Goods market clearing implies

$$
y_t = c_t + i_t,
$$

(25)

while labor market clearing implies

$$
y_t = k_t^{\sigma} (z_{nd} n_t) ^{1-\nu} \mathcal{D}_t^{-1}.
$$

(26)

where $\mathcal{D}_t \equiv \int_0^1 (P_t(i)/P_t)^{-1} \text{ measures price dispersion.}$

B.6. Benchmark calibration

The benchmark calibration of the model’s structural parameters is mostly based on estimates from Smets and Wouters (2003), and is given in Table 8.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>Depreciation rate of capital</td>
<td>0.025</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Output elasticity w.r.t. capital</td>
<td>$1/3$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Risk aversion coefficient</td>
<td>$1.6$</td>
</tr>
<tr>
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<td>Inverse Frisch elasticity</td>
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<td>Non-wage adjustment probability</td>
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<td>Non-price adjustment probability</td>
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<td>$\rho_{ns}$</td>
<td>Monetary policy parameters</td>
<td>$0.93$, $1.66$, $0.14$</td>
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<td>$\rho_{ns}$</td>
<td>Persistence of labor supply and demand shocks</td>
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References