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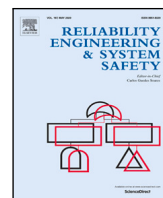
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## Joint condition-based maintenance and condition-based production optimization

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### ABSTRACT

Developments in sensor equipment and the Internet of Things increasingly allow production facilities to be monitored and controlled remotely and in real-time. Organizations can exploit these opportunities to reduce costs and improve reliability by employing condition-based maintenance (CBM) policies. Another recently proposed option is to adopt condition-based production (CBP) policies that control the deterioration of equipment by dynamically adapting the production rate. This study compares their performance and introduces a fully dynamic condition-based maintenance and production (CBMP) policy that integrates both policies. Numerical results show that their cost-effectiveness strongly depends on system characteristics such as the planning time for maintenance, the cost of corrective maintenance, and the rate and volatility of the deterioration process. Integrating condition-based production decisions into a condition-based maintenance policy substantially reduces the failure risk, while fewer maintenance actions are performed. Interestingly, in some situations, the combination of condition-dependent production and maintenance even yields higher cost savings than the sum of their separate cost savings. Moreover, particularly condition-based production is able to cope with incorrect specifications of the deterioration process. Overall, there is much to be gained by making the production rate condition dependent, also, and sometimes even more so, if maintenance is already condition-based.

### 1. Introduction

Maintenance activities are a major cost driver for modern production facilities. For instance, manufacturing firms typically face maintenance costs ranging between 15–40% of their total expenses [1], and for power plants and offshore wind farms maintenance costs constitute up to 30% of the total costs [2,3]. Consequently, efficient and effective operations and maintenance strategies are of crucial importance for the profitability and competitiveness of firms. Various developments that provide opportunities to improve operational decision making are decreasing prices of monitoring equipment, advances in the Internet of Things (IoT), and improved machine learning techniques to process large amounts of condition information. These developments enable operators to monitor and control production facilities remotely and in real-time.

In light of these developments, many studies aim to reduce maintenance costs and improve equipment reliability by implementing flexible maintenance policies that schedule maintenance based on condition information. Such policies try to schedule maintenance just before imminent failure, thereby avoiding wastage of remaining useful life

of equipment and lowering the number of unexpected failures. The effectiveness of such condition-based maintenance (CBM) policies, however, heavily depends on the characteristics of the deterioration process and on logistical planning times [4].

Another way in which the condition information can be used is by implementing condition-based production (CBP) policies, i.e., controlling deterioration of equipment by dynamically adapting the production rate based on condition information [5]. This approach exploits the fact that machines typically deteriorate faster at higher production rates.

The third option is the integration of both CBM and CBP into a fully dynamic policy with condition-based maintenance and condition-based production (CBMP). Surprisingly, despite the abundance of condition monitoring in practice and the fact that both CBM and CBP are studied, CBMP policies have not been considered in the literature to the best of our knowledge. An example of a real-life system where CBMP is expected to be valuable is an offshore wind farm. Turbine components such as gearboxes and generators deteriorate over time and their condition is closely monitored (e.g., by measuring noise, vibration, and

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temperature). As long as a turbine is in good condition, it can produce at a high rate while the condition information is used to schedule maintenance. However, initiating maintenance requires a considerable planning time to arrange spare parts, skilled technicians, and specialized vessels. During this planning time, the condition information cannot be used to improve maintenance decisions anymore, but it can still be used in real time to dynamically adjust the production rate of the turbine. Thus, once maintenance has been scheduled, the production rate can be used as a tool to enhance equipment reliability.

A main goal of our study is to compare the performance of the three policies (CBM, CBP, and CBMP), providing guidance on determining when the combination is particularly beneficial. Furthermore, we analyze the structure of the optimal CBMP policy, leading to insights on how maintenance and production flexibility can best be jointly exploited. As our study is exploratory, we consider a single piece of equipment with a single condition parameter. The deterioration rate of the system depends on the adjustable production rate and thus the deterioration process can be controlled by adapting the production speed. We obtain numerical results by formulating Markov decision processes.

Our study shows that the two dynamic decisions (partially) complement each other for a wide range of systems. Condition-based maintenance aims to schedule maintenance just-in-time whereas condition-based production extends the equipment lifetime and improves reliability by reducing the short-term failure risk if needed. CBMP combines both benefits by reducing the failure risk for high deterioration levels, thereby creating the opportunity to apply a less conservative maintenance policy. All policies that use condition information result in reduced failure risks, fewer maintenance interventions, and lower costs. Moreover, integrating the two dynamic decisions improves the concept of just-in-time maintenance, since CBMP performs maintenance at higher deterioration levels than CBM while realizing fewer failures. These results are practically relevant and of interest to reliability engineers because the policies we describe are useful to prevent equipment failures. More generally, we believe that it is important to study reliability engineering problems in the broader context of the underlying planning problems, and the logistical issues that potentially play a role. Our study serves that purpose.

The remainder of this study is organized as follows. In Section 2, we discuss the relevant literature on maintenance and production decisions. In Section 3, we formally introduce the problem that we consider. The Markov decision processes that we use to determine optimal policies are described in Section 4. In Section 5, we discuss the optimal policies and compare their effectiveness. We conclude and provide future research suggestions in Section 6.

## 2. Literature review

Despite the growing interest in the interaction between maintenance and production decisions, both fields are typically considered in isolation. De Jonge and Scarf [6] and Ding and Kamaruddin [7] provide general reviews on maintenance, whereas Alaswad and Xiang [8] focus on condition-based maintenance. The literature on production decisions under uncertainty, such as uncertain machine failures, is reviewed by Mula et al. [9]. The most recent review that focuses on the interaction between production and maintenance is conducted by Sethi et al. [10]. Glock and Grosse [11] wrote a review on the effect of controllable production rates in inventory systems. Interestingly, the authors mention that the production rate may affect the machine lifetime and the risk that a system goes out-of-control. However, the option to use condition information for making production decisions is not mentioned. In the remainder of this section, we first discuss studies on maintenance and particularly address studies that compare the performance of time-based maintenance with that of condition-based maintenance. Thereafter we discuss studies on production decisions

that either affect the failure behavior of equipment or that include condition monitoring.

An abundance of research in the area of maintenance optimization exists. Within this field a distinction can be made between static time-based maintenance policies, and dynamic condition-based maintenance policies. Examples of recent studies that consider time-based maintenance are Chaabane et al. [12] and Hashemi et al. [13], and examples of recent studies on condition-based maintenance are Shahraki et al. [14] and Wang et al. [15]. Studies that compare the two types of maintenance policies are of interest when a decision between the two needs to be made. Some existing comparative studies consider only a single signal of potential failure [16], or a limited number of condition states [17,18], whereas others consider deterioration on a continuous scale. Pandey et al. [19] do so, but mainly consider the threshold for scheduling condition-based maintenance as fixed. De Jonge et al. [4] include the effect of various practical factors, whereas Xiang et al. [20] include the effect of the environment. Zio and Compare [21] and Crowder and Lawless [22] only consider a single example and do not provide general insights. Finally, Huynh et al. [23] include random shocks, and Bouvard et al. [24] consider multi-component systems. Thus, many studies consider the benefit of using condition information for maintenance decisions, but these do not use this information to actively control the deterioration process.

The production literature shows a growing interest in the interaction between production decisions and the failure behavior of the system. We remark that there is also a vast amount of studies on the joint optimization of production and maintenance decisions that assume that the production decisions do not affect the failure behavior of the system [e.g.,25,26] or that focus on scheduling a set of production tasks [e.g.,27,28]. However, in the remainder of this section, we only address studies on production planning decisions that directly affect the failure behavior of the system, and thereby also its reliability.

Within this area, we can distinguish three main research streams. The first and largest stream assumes that failure risks depend on the current production rate and possibly on the age of the system. This stream assumes that the production rate does not affect the deterioration rate and that no condition information is available. For instance, Martinelli [29,30,31] studies optimal production policies for systems with production-dependent failures under multiple failure modes. Tan [32] optimize the production rate such that inventory holding costs are minimized in a setting with stochastic demand. Shen et al. [33] study optimal switching policies for warm standby systems in which idle units have a lower failure risk than the active ones, which can be interpreted as switching between two production rates. The studies in this stream assume that producing at higher rates increases the current failure risk but does not result in permanent deterioration of the system, as in our study.

The second stream takes into account production-dependent deterioration, which is particularly encountered for rotating and moving equipment such as conveyor belts, cutting tools, and wind turbine gearboxes [34,35]. Cheng et al. [36] jointly optimize the production rate and the scheduling of preventive maintenance actions for cutting machines. They take into account that higher cutting speeds reduce both the reliability and the lifetime of the machine. However, no condition information is available and only a single fixed production rate is selected. Zied et al. [37] consider inventory, backlog, and maintenance costs, and dynamically optimize the production rate to balance inventory holding costs with backlog costs. The relation between production and deterioration is modeled by a virtual age (also used by others, e.g., Ouaret et al. [38] and Polotski et al. [39]), in which aging is proportional to the production rate. Rivera-Gómez et al. [40] study a similar setting and incorporate the possibility that defective products are produced. The above studies do consider production-dependent deterioration, but use virtual age models rather than monitoring actual deterioration levels. Uit het Broek et al. [5] do

address the value of monitoring the deterioration process for a single-unit system with production-dependent deterioration, but restrict their attention to condition-based production decisions for a predetermined block-based maintenance policy. Although the studies in this stream consider production-dependent deterioration, they do not connect it to both condition monitoring and optimizing maintenance decisions.

The third stream incorporates condition information into policies that optimize both maintenance and production decisions. However, in this stream the deterioration rate cannot be controlled by adjusting the production rate. For instance, Boukas and Liu [41] and Irvani and Duenyas [42] study a single-unit system with inventory, backlog, and maintenance costs. Both optimize the production and maintenance rate, but the production rate does not affect the deterioration rate. Within this stream, there are also studies that optimize production lot-sizes while the production rate is either fixed or does again not affect the deterioration process. For instance, Jafari and Makis [43], Peng and Van Houtum [44], Cheng et al. [45,46], and Liu and Zhao [47] consider systems with a constant production rate and determine optimal production lot-sizes. Condition information is used to determine whether maintenance is initiated or a new lot is produced. In a broader sense, also the study of Sun et al. [48] belongs to this stream. They use condition information to optimize the reallocation of units within a series system where the position determines the load and thereby the deterioration rate of the unit. Although the studies in this stream optimize production and maintenance decisions under the presence of condition information, they do not control the deterioration rate of the system by dynamically adjusting the production rate.

Summarizing, although there is a growing interest in the interaction between production and maintenance, production rate decisions based on condition information are rarely considered. Existing studies have mainly focused on the effect of production decisions on the failure rate, and not on how they affect deterioration. The few studies that do take the permanent effect of the production rate into account, generally model this by a virtual age and do not consider monitoring of the actual deterioration level. Studies that use condition information for production decisions either ignore the relation between the production rate and the deterioration rate or focus on optimizing the production batch size. To the best of our knowledge, no existing study compares condition-based maintenance to condition-based production for systems with production-dependent deterioration. Furthermore, there is no study that jointly optimizes condition-based maintenance and condition-based production rate decisions for such systems.

### 3. Problem description

We study a single-unit system whose condition can be described by a single deterioration parameter. The production rate of the system is adjustable over time, and the deterioration rate (i.e., the average amount of additional deterioration per time period) depends on the production rate. The deterioration process is described by a nondecreasing continuous-time continuous-state stochastic process  $X = \{X(t) \mid t \geq 0\}$ . Deterioration level 0 indicates that the unit is as-good-as-new, whereas deterioration levels exceeding  $L$  indicate that the unit has failed.

Maintenance interventions restore the system to the as-good-as-new state and require a fixed and given planning time  $s$ . If the system is still functioning at the end of this planning time, preventive maintenance will be performed at a cost  $c_{pm}$ . If, on the other hand, the system has failed during the planning time, a more expensive corrective maintenance action at a cost  $c_{cm}$  will be performed after the planning time. We assume that the time that is needed to carry out maintenance is negligible.

We let  $\theta(t, x) \in \{0, 1\}$  be the decision variable that denotes whether maintenance is initiated at time  $t$  (since the last system renewal) and condition  $x$ . The complete maintenance policy is denoted as  $\theta =$

$\{\theta(t, x) \mid t \geq 0, x \geq 0\}$ . We let  $c(X(t))$  be the maintenance cost as a function of deterioration level  $X(t)$ , that is,

$$c(X(t)) = \begin{cases} c_{pm} & \text{if } X(t) < L, \\ c_{cm} & \text{otherwise.} \end{cases}$$

The system can produce at different production rates that range from 0 (idle) to 1 (maximum rate). We denote the set of possible production rates as  $U = [0, 1]$ . When the system has failed, it cannot produce and the production rate is fixed at 0. We let  $u = \{u(t, x) \mid t \geq 0, x \geq 0\}$  denote the production policy. Furthermore, at the maximum production rate, the system produces a revenue of  $\pi$  per period. When the system does not produce at its maximum rate, there is a revenue loss that is proportional to the production rate  $u \in U$ , which equals  $(1 - u)\pi$  per time period.

The deterioration rate depends on the production rate and is denoted by  $g(u)$ . We refer to this function as the production–deterioration relation (pd-relation for short). The pd-relation  $g$  is assumed to be increasing because the system is assumed to deteriorate faster for higher production rates. We let  $\mu_{min} = g(0)$  and  $\mu_{max} = g(1)$  denote the minimum and maximum deterioration rate, respectively. Moreover, for a given production rate  $u$ , we consider a stationary deterioration process, i.e., the deterioration increments do not depend on the current deterioration level.

Our aim is to minimize the long-run average cost. This is a suitable optimality criterion in settings where decisions need to be made very frequently and where the process repeats itself time and again [49], which is also the case for our problem. Discounting has a negligible effect in such settings. Examples of other recent studies in the area of maintenance that also consider this criterion are Havinga and De Jonge [50] and Hu and Chen [51]. We continue to express the costs as a function of a given maintenance policy  $\theta$  and a given production policy  $u$ . For a given maintenance policy  $\theta$  and a given realization of the deterioration process  $X$ , we can derive all moments  $\tau_i, i \in \mathbb{N}$ , at which maintenance is performed. Recall that maintenance requires a fixed planning time  $s$ , and thus the first maintenance action is performed  $s$  time units after maintenance is initiated for the first time, that is,  $\tau_0 = \inf\{t \geq 0 \mid \theta(t, X(t)) = 1\} + s$ . We find the subsequent maintenance moments by  $\tau_i = \inf\{t > \tau_{i-1} \mid \theta(t, X(t)) = 1\} + s, i \in \mathbb{N} \setminus \{0\}$ .

For a given  $\theta$  and  $u$ , the total expected cost up to time  $t$  equals

$$J(\theta, u, t) = E \left[ \sum_{i=0}^{\infty} I_{\{\tau_i \leq t\}} \cdot c(X(\tau_i)) \right] + \pi \cdot E \left[ \int_0^t (1 - u(s, X(s))) ds \right],$$

where  $I_{\{\tau_i \leq t\}}$  is the indicator function that equals one if  $\tau_i \leq t$  and zero otherwise. The first term represents the expected maintenance costs and the second term the expected revenue losses. We define the long-run average cost as

$$J(\theta, u) = \limsup_{t \rightarrow \infty} \frac{J(\theta, u, t)}{t}.$$

The minimal long-run average cost equals  $J^* = \inf_{\theta, u} J(\theta, u)$ . Our aim is to determine a joint maintenance and production policy  $(\theta^*, u^*)$  that minimizes the long-run average costs, that is, to determine  $\theta^*$  and  $u^*$  such that  $J(\theta^*, u^*) = J^*$ .

#### 3.1. Control strategies

We define a strategy as a combination of a production policy and a maintenance policy. The various strategies that we consider differ in their flexibility regarding the maintenance and production decisions. We consider two maintenance policies (referred to as *block-based* and *condition-based*) and two production policies (referred to as *max-rate* and *condition-based rate*).

In the remainder of this study, we let the *fixed maintenance and production* (FMP) strategy refer to a block-based maintenance policy combined with the max-rate production policy. This strategy has no flexibility and does not use condition information. The *condition-based*

production (CBP) strategy employs a block-based maintenance policy combined with an adjustable production rate. The *condition-based maintenance* (CBM) strategy refers to a condition-based maintenance policy combined with the max-rate production policy. The *condition-based maintenance and production* (CBMP) strategy combines condition-based maintenance with condition-based production.

**Maintenance policies**

The block-based maintenance policy is a static maintenance policy that fixes all maintenance actions in advance. Under this policy, the decision maker selects a block length  $T$  and maintenance is performed every  $T$  time units. We refer to such a time interval of  $T$  time units as a block. If the unit is functioning at the end of a block, preventive maintenance is performed; otherwise more expensive corrective maintenance is required. Additional maintenance actions during a block are not possible. Thus, a system failure results in production losses until the end of the current block. This maintenance policy is not affected by the planning time since it fixes all maintenance actions in advance.

We remark that, in theory, this policy can lead to very long downtimes in case of an early failure during a block. However, for the optimal block-based maintenance policy, any failures typically occur towards the end of the block (see also Section 5). In this case, generally either (i) the required planning time does not allow for an additional maintenance action before the scheduled maintenance action anymore, or (ii) it is preferred to have a short amount of downtime instead of additional maintenance just before the planned action. Furthermore, adding the possibility to schedule additional maintenance upon failure would complicate our block-based maintenance model, and thereby distract from our main message.

The condition-based maintenance policy is flexible and allows the decision maker to plan maintenance interventions based on the condition information. Thus, given the current deterioration level  $x$  it is determined whether maintenance will be scheduled, independent of the current time. Recall that there is a maintenance planning time  $s$  between initiating and performing maintenance. The case  $s = 0$  implies that maintenance can be carried out instantaneously. At the end of the planning time, preventive maintenance is carried out if the system is still functioning, whereas corrective maintenance is performed if the system has failed. System failure before or during the planning time results in production losses until the end of the planning time.

**Production policies**

The production policies define a set of admissible production rates, denoted by  $\mathcal{A}(x)$ , as a function of the current deterioration level  $x \in X$ . Under the max-rate policy, the system produces at its maximum rate as long as it is functioning. Thus the set of admissible production rates equals

$$\mathcal{A}(x) = \begin{cases} \{1\} & \text{if } x < L, \\ \{0\} & \text{if } x \geq L. \end{cases}$$

The condition-based production policy is fully flexible and allows the decision maker to control the production rate at any time, and so

$$\mathcal{A}(x) = \begin{cases} U & \text{if } x < L, \\ \{0\} & \text{if } x \geq L. \end{cases}$$

**4. Markov decision process formulation**

The solution procedure that we use to determine optimal policies is the formulation and analysis of Markov decision processes (MDPs). This framework is applicable to sequential decision making problems with outcomes that are partially uncertain, which is also the case for our problem. Markov decision processes are typically used in discrete-time settings with a finite number of states [49]. Therefore, in Section 4.1, we first discuss how we discretize the state space, the time horizon, and the set of admissible production rates. The structure of the Markov

decision process depends on the maintenance policy that is used. In Section 4.2, we formulate an MDP with a finite time horizon for the block-based maintenance policy. Thereafter, in Section 4.3, we formulate an MDP with an infinite time horizon for the condition-based maintenance policy.

**4.1. Discretization**

We partition the continuous deterioration interval  $[0, L]$  into  $m$  equally sized intervals of length  $\Delta X = L/m$ , and then discretize this to the ordered set of midpoints  $\bar{X} = \{(i + 0.5)\Delta X \mid i = 0, \dots, m - 1\}$ . All deterioration levels above  $L$  are merged into a single state with index  $m$  that indicates system failure. The time horizon is discretized into periods with length  $\Delta t$ , and there is a decision epoch at the start of each period. The continuous set of production rates is discretized into  $n + 1$  production rates that are uniformly distributed between the minimum and maximum production rate, that is,  $\bar{U} = \{i/n \mid i = 0, \dots, n\}$ . Note that we can approximate a continuous system arbitrarily close by setting the step sizes sufficiently small, although smaller step sizes also result in increased computation times.

We let  $F_{\Delta t, u}$  denote the distribution function of the additional amount of deterioration during a time period  $\Delta t$  when producing at rate  $u$ . For ease of notation, we drop the subscript  $\Delta t$  in the remainder of this study. To obtain the transition probabilities in the discretized process, we set the probability of staying in the same deterioration state to  $F_u(0.5\Delta X)$ ; the probability of moving from state  $k$  to state  $k+i$ , where  $i \geq 1$  and  $k+i < m$ , to  $F_u((i+0.5)\Delta X) - F_u((i-0.5)\Delta X)$ ; and the probability of moving from state  $k$  to the failed state  $m$  to  $1 - F_u((i-0.5)\Delta X)$  where  $i = m - k$ . Summarizing, the transition probabilities of the discrete deterioration process are

$$P_u(k, k+i) = \begin{cases} 0 & \text{if } i < 0, \\ F_u(0.5\Delta X) & \text{if } i = 0, \\ F_u((i+0.5)\Delta X) - F_u((i-0.5)\Delta X) & \text{if } 0 < i < m - k, \\ 1 - F_u((i-0.5)\Delta X) & \text{if } i = m - k. \end{cases}$$

**4.2. MDP for block-based maintenance**

The block-based maintenance policy results in a renewal process in which the maintenance actions are the renewal points. Each block starts at the as-good-as-new deterioration level, regardless of the actions and deterioration realizations in previous blocks. As a result, for a given block length  $T$ , it is sufficient to minimize the total expected costs during a single block. Thus, systems with a block-based maintenance policy can be formulated as an MDP with a finite time horizon. Below we formulate such an MDP, and in our numerical analysis we use the backward induction algorithm [49, ch. 4.5] to determine corresponding optimal policies.

We let  $\tau \in \{0, \Delta t, 2\Delta t, \dots, T\}$  denote the remaining time until the end of the current block, i.e., maintenance is performed when  $\tau = 0$ . Let  $V(x, \tau)$  denote the total expected cost in the remainder of the current block given that the current deterioration level of the system is  $x \in \bar{X}$ . At the end of the block, maintenance is performed and thus

$$V(x, 0) = \begin{cases} c_{pm} & \text{if } x < m, \\ c_{cm} & \text{if } x = m. \end{cases}$$

In all other periods, a production rate  $u$  can be selected. This affects the production loss  $(1 - u)\pi\Delta t$  and the expected future costs. Thus, for  $\tau > 0$  we get

$$V(x, \tau) = \min_{u \in \bar{U}(x)} \left\{ (1 - u)\pi\Delta t + \sum_{i=0}^{m-x} P_u(x, x+i) V(x+i, \tau - \Delta t) \right\},$$

where  $\bar{U}(x)$  is the discretized set of admissible production rates for a given production policy, as described in Sections 3.1 and 4.1.

The expected total costs during a single block of length  $T$  equals  $V(0, T)$ , and the long-run average cost per time unit equals  $V(0, T)/T$ . We find the optimal block length  $T^*$  by solving the system up to a sufficiently large value  $\tau_{max}$  and selecting  $T^* \in \arg \min_{0 < T \leq \tau_{max}} \{V(0, T)/T\}$ .

### 4.3. MDP for condition-based maintenance

We formulate systems with a condition-based maintenance policy as an MDP with an infinite time horizon. In our numerical examples we use the value iteration algorithm [49, ch. 8.5] to determine  $\epsilon$ -optimal policies. We let  $\tau \in S := \{0, \Delta t, 2\Delta t, \dots, s, ns\}$  denote the remaining planning time until the next scheduled maintenance intervention, in which ‘ns’ indicates that maintenance is *not scheduled* yet. The state of the system is described by the current deterioration level  $x \in \bar{X}$  and the remaining planning time  $\tau$ . Let  $V_n(x, \tau)$  denote the value function after  $n$  iterations of the value iteration algorithm. At the end of the planning time (i.e.,  $\tau = 0$ ), maintenance is performed and the deterioration level immediately jumps to the as-good-as-new level. After maintenance is performed, the production decision is the same as if we started the time period at deterioration level  $x = 0$ , and thus

$$V_n(x, 0) = \begin{cases} c_{pm} + V_n(0, ns) & \text{if } x < m, \\ c_{cm} + V_n(0, ns) & \text{if } x = m. \end{cases}$$

During the planning time (i.e.,  $0 < \tau \leq s$ ), the decision maker can only decide on the production rate  $u$ . The remaining planning time is reduced by  $\Delta t$ , regardless of the production decision. The value function during the planning time equals

$$V_n(x, \tau) = \min_{u \in \bar{U}(x)} \left\{ (1-u)\pi\Delta t + \sum_{i=0}^{m-x} P_u(x, x+i) V_{n-1}(x+i, \tau - \Delta t) \right\}.$$

Before maintenance is scheduled (i.e.,  $\tau = ns$ ), the decision maker can decide on the production rate and whether or not to schedule maintenance. If maintenance is scheduled, the remaining planning time is set to  $\tau = s$ . Otherwise, the remaining planning time remains  $\tau = ns$ . The value function before maintenance is scheduled optimizes over these two options by choosing  $\tau \in \{ns, s\}$ , and equals

$$V_n(x, ns) = \min_{\tau \in \{ns, s\}, u \in \bar{U}(x)} \left\{ (1-u)\pi\Delta t + \sum_{i=0}^{m-x} P_u(x, x+i) V_{n-1}(x+i, \tau) \right\}.$$

We initialize the value iteration algorithm by setting  $V_0(x, \tau) = 0$  for all  $x \in \bar{X}$  and  $\tau \in S$ .

## 5. Numerical analysis

We continue by analyzing the effectiveness of the various strategies. Our approach is to first consider a base case and then present a sensitivity analysis where we vary the different system parameters. The base case is chosen somewhat arbitrarily, but in such a way that the effects are representative for a large set of instances that we initially considered. Furthermore, we believe that the chosen values are practically realistic and in accordance with other studies. Using the base case, we describe through what mechanisms CBP reduces cost, how the performance of CBM compares to that of CBP, and how the two interact with each other. A sensitivity analysis then continues to explore when CBP and CBM are particularly effective in isolation or in combination (i.e., CBMP). Before discussing the results, we first introduce the deterioration process and the base case parameter values that we consider.

### 5.1. Deterioration process

Various stochastic processes have been suggested in the literature to model deterioration, including compound Poisson processes, Brownian motions with drift, and gamma processes. In this study, we use stationary gamma processes since these are the most appropriate to model monotonically increasing deterioration such as erosion, wear, and fatigue [8,52]. The stationary gamma process is a flexible process for which the deterioration rate and volatility can be controlled by two parameters. This enables us to study a wide scope of systems with different deterioration characteristics. The gamma process is a

continuous-time continuous-state process, and we will discretize it as described in Section 4.1.

The increments of a gamma process are independently gamma distributed. Denoting the shape parameter by  $\alpha > 0$  and the scale parameter by  $\beta > 0$ , the gamma density function for the increment per time unit is given by

$$f_{\alpha\beta}(x) = \frac{x^{\alpha-1} \exp(-x/\beta)}{\Gamma(\alpha) \beta^\alpha},$$

where  $\Gamma(\alpha) = \int_0^\infty z^{\alpha-1} \exp(-z) dz$  is the gamma function. The deterioration increment  $Y$  per time unit has mean  $E[Y] = \alpha\beta$  and variance  $\text{Var}(Y) = \alpha\beta^2$ . The density function corresponding to increments per period with length  $\Delta t$  is obtained by scaling the shape parameter to  $\alpha\Delta t$ .

We relate the shape and scale parameters to the pd-relation  $g$  such that the deterioration increment per time unit has the following three properties. First, the deterioration increments have mean  $E[Y | u] = g(u)$ . Second, if the system produces at the maximum rate, the variance of the deterioration increments equals  $\text{Var}(Y | u = 1) = \sigma_{\max}^2$ . Third, the coefficient of variation (i.e., the standard deviation divided by the mean) of the deterioration increments is not affected by the production rate. It can easily be verified that this is accomplished by setting the shape parameter equal to  $\alpha = \mu_{\max}^2 / \sigma_{\max}^2$  and the scale parameter, as a function of the production rate, equal to  $\beta(u) = g(u) \cdot \sigma_{\max}^2 / \mu_{\max}^2$  (recall that  $g(1) = \mu_{\max}$ ).

### 5.2. Base case system

The base case parameter values are listed in Table 1. The preventive maintenance cost is  $c_{pm} = 20$  and the corrective maintenance cost is  $c_{cm} = 100$ . For real-life systems, for instance wind turbines, corrective maintenance is often much more costly than preventive maintenance, because of collateral damage and the lower salvage value of failed components. The revenue when producing at the maximum rate is normalized to  $\pi = 1$  per time unit. The system fails at deterioration level  $L = 100$ , and maintenance requires a planning time of  $s = 5$  time units.

For the pd-relation we consider the same parametric form as Uit het Broek et al. [5], that is,  $g(u) = \mu_{\min} + (\mu_{\max} - \mu_{\min})u^\gamma$ . This parametric form with three parameters is very flexible and can be used to model a wide variety of relations between the production speed and the deterioration speed. When the approach is applied in practice, condition and production speed data can be used to estimate the values of the parameters. The pd-relation is concave for  $0 < \gamma < 1$ , linear for  $\gamma = 1$ , and convex for  $\gamma > 1$ . The parameter  $\mu_{\min} = 0.1$  describes the deterioration rate when the system is idle, and  $\mu_{\max} = 1.5$  is the deterioration rate when the system produces at the maximum rate. For the base case we set  $\gamma = 1.5$ , and also in the sensitivity study we mainly focus on convex pd-relations as those are conceivably most likely to be encountered in real-life systems. For instance, blades of gas turbines rotate at varying speeds, and the resulting stress increases quadratically in this speed [53]. Murthy and Jack [54] also elaborate on the relation between usage intensity and degradation.

The expected time until failure when producing at the maximum rate equals approximately  $L/\mu_{\max} \approx 67$  time units. The system only has to produce  $c_{pm}/\pi = 20$  time units at the maximum rate to compensate for the preventive maintenance cost and thus the system is expected to be profitable. However, an expensive failure is expected to result in a loss-making cycle because producing at the maximum rate for  $c_{cm}/\pi = 100$  time units is required to compensate for this.

To closely approximate the continuous deterioration process, we partition the time horizon and the deterioration levels into small intervals with respective lengths  $\Delta t = 1.0$  and  $\Delta X = 0.05$ . Note that  $\Delta X$  should be small compared to the expected deterioration increment per time period, that is,  $\Delta X \ll \Delta t \cdot \mu_{\max}$ . The continuous action space is discretized into  $n = 50$  non-idle production rates. We note in passing that we also considered other base cases. The results for the other base cases were comparable to the current one and did not provide new insights, we therefore did not include those.

**Table 1**

Base case system used in the numerical analysis.

Parameter	Value	Interpretation
$c_{pm}$	20.00	Preventive maintenance cost
$c_{cm}$	100.00	Corrective maintenance cost
$\pi$	1.00	Production revenue at maximum rate
$L$	100	Failure deterioration level
$s$	5	Planning time for maintenance
$\gamma$	1.50	Shape of pd-relation
$\mu_{min}$	0.10	Mean of deterioration increments when idle
$\mu_{max}$	1.50	Mean of deterioration increments at maximum rate
$\sigma_{max}$	3.00	Standard deviation of deterioration increments
$n$	50	Number of non-idle production rates
$\Delta t$	1.00	Length of time periods
$\Delta X$	0.05	Discretization size deterioration

**Table 2**

Performance statistics for the four strategies when applied to the base case.

	Fixed	CBP	CBM	CBMP
PM threshold $T$ or $M$	$T = 42$	$T = 60$	$M = 70.20$	$M = 78.80$
Mean cost per time unit	0.562	0.424	0.409	0.379
St.dev. cost per time unit	4.343	2.834	3.353	2.957
Mean production per time unit	0.995	0.922	0.999	0.977
Mean time to maintenance	42.00	60.00	53.31	59.19
Mean time between failures	995.12	6365.37	2456.39	4525.96
Mean deterioration at maintenance	62.46	81.43	79.75	86.11

**5.3. Cost savings for the base case system**

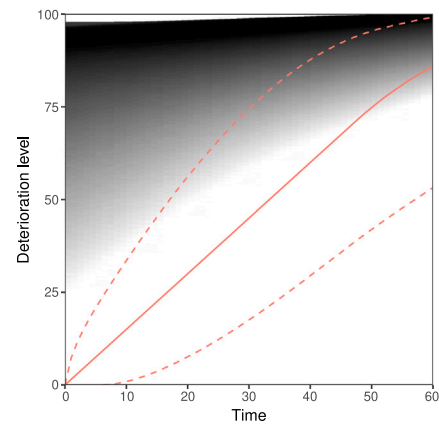
We examine the effectiveness of flexible production and flexible maintenance decisions by comparing the four control strategies introduced in Section 3.1. Table 2 compares the performance of the four considered strategies when applied to the base case. For block-based maintenance with production at the maximum rate, the optimal block length is 42 periods and the mean time between failures (MTBF) is 995 periods, implying that failure occurs only during 4% of the blocks. Noting that the expected deterioration level at the end of the block is 63 and the failure deterioration level is 100, these failures typically occur towards the end of the block. This means that periods of downtime are short and generally overlap with the planning time. Other purely time-based maintenance policies would therefore result in preventive maintenance that is carried out at a similar frequency. For block-based maintenance with condition-based production (CBP) the chance of failure during a block is further reduced to less than 1%.

Overall observations are that the fixed strategy is by far the worst, that CBM is slightly more effective than CBP considering expected costs, and that introducing a flexible production policy has the positive side-effect of lowering the cost variance. We also see that having both a flexible maintenance and a flexible production policy reduces the expected cost even further. Next, we will explain these results by studying the optimal policies more closely.

**Condition-based production**

Because CBM has been studied much more extensively than CBP, we start by discussing the optimal structure of the CBP strategy. With an adjustable production rate, the optimal block length turns out to be  $T = 60$ . The results for this policy are largely in line with the findings of Uit het Broek et al. [5], although they predetermined rather than optimized the block length, and we refer to their study for a detailed discussion of the CBP policy structure in this case. Fig. 1 shows the production rate in gray scale, ranging from black (no production) to white (maximum rate). The solid line indicates the expected deterioration trajectory, whereas the dashed lines indicate a region that contains the deterioration level with 95% certainty for a given point in time.

Three areas can be distinguished in Fig. 1. Firstly, for low deterioration levels compared to the remaining time until maintenance (white lower triangular area), a failure is unlikely and the system produces



**Fig. 1.** The optimal production rate for the CBP strategy in gray scale, ranging from black (no production) to white (maximum rate). The solid line indicates the expected deterioration trajectory, and the dashed lines indicate a region that contains the deterioration level with 95% certainty at a given point in time.

at the maximum rate. Secondly, for intermediate to high deterioration levels (gray area), the production rate is gradually reduced in order to reduce the deterioration rate, resulting in a lower risk of failure and improved reliability. Thirdly, for extremely high deterioration levels compared to the remaining time to maintenance (small white upper triangular area), failure is almost certain, and production is maximized by producing at the maximum rate. It has to be noted, however, that the effect of this third area on the long-run average cost is negligible as its states are almost never reached.

We also note from the upper dashed line in Fig. 1 that CBP does not immediately slow down production when the system deteriorates faster than expected. As long as there is sufficient time to prevent failure at a later stage, it is better to continue producing at the maximum rate, since it is possible that deterioration in the remainder of the block is lower than expected. If this does not happen, then production can still be slowed down, whereas lost production cannot be made up for. Another observation from Fig. 1 is that, even under the expected deterioration path (solid line), production is slowed down towards the end of the block. This reduces the volatility of the deterioration process, and thereby reducing the risk of failure due to a large deterioration increment.

Fig. 2 shows the effect of the block length on the average cost per time period, the probability that the system fails during a block, and the mean production per period for both the fixed strategy (dashed) and the CBP strategy (solid). For block lengths exceeding 40, the failure risk under the fixed strategy rapidly increases and therefore this strategy cannot permit to schedule longer blocks. The CBP strategy reduces the failure risk by reducing the production rate when needed. This allows CBP to be less conservative and schedule considerably longer blocks ( $T^* = 60$  instead of  $T^* = 42$ ), while simultaneously increasing the MTBF from 995 to 6365 periods, and reducing the average cost per time period by 25% compared to the fixed strategy.

Besides the cost reduction, CBP also achieves a significantly lower cost standard deviation (2.83 instead of 4.34) by reducing the number of expensive failures. Note that in terms of cost, the CBP strategy is also less sensitive to small changes in the block length, which offers a practical advantage as in real-life systems the exact maintenance moment may be uncertain.

**Condition-based maintenance and production**

We continue with the two strategies with condition-based maintenance (i.e., CBM and CBMP) and compare their performance to the strategies with a static maintenance policy (i.e., FMP and CBP). Introducing only a flexible maintenance policy (CBM) reduces the cost

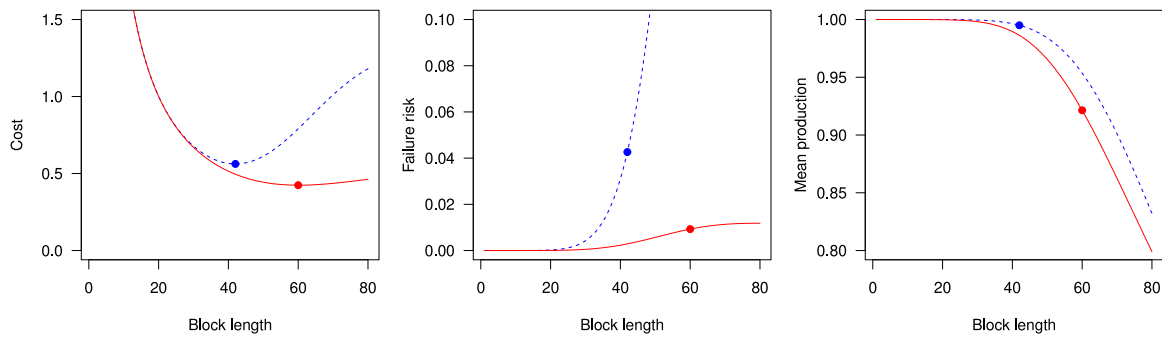


Fig. 2. Effect of block length  $T$  on the costs per period, the failure risk per block, and the mean production per period for the fixed strategy (dashed blue) and the CBP strategy (solid red). The dots indicate the optimal block length for both strategies.

by 27% compared to the fixed strategy, and is thus slightly more cost effective than CBP for the base case. The main advantage of CBM over CBP is its higher production output, as it does not slow down production to control deterioration. On the other hand, CBM performs more maintenance actions than CBP (mean time to maintenance of 53 instead of 60), and still fails around 2.6 times as often. This is partially caused by the inability of CBM to respond to large deterioration increments during the planning time, whereas CBP can lower the production speed at any time. Failures imply high maintenance cost and thus the higher number of failures also explains the higher standard deviation of the cost for CBM compared to CBP.

The cost reduction by introducing both an adjustable production rate and a flexible maintenance policy (i.e., CBMP) is 33%, implying that condition-based production and condition-based maintenance partially complement each other. The adjustable production rate improves the condition-based maintenance policy in three ways. First, the failure risk during the planning time is lower because the operator can respond to the actual deterioration. This clearly reduces the expected maintenance cost per cycle. Second, exploiting this benefit, a less conservative preventive maintenance threshold is used, resulting in fewer maintenance actions. Third, if during the planning time the system deteriorates so fast that a failure becomes (almost) unavoidable, then the expected production losses are minimized by producing at a more efficient rate until failure occurs. The phenomenon that failures are sometimes unavoidable is an immediate consequence of the fact that deterioration is stochastic and that even idle systems may slowly deteriorate. However, although such states do exist, it is unlikely to reach these states under the optimal policy and the additional cost savings of optimizing the production rate for such states is only small. Furthermore, in practice, systems will generally be turned off in such states for safety reasons. Recall that CBP lowers the production rate towards the end of a block in order to reduce the risk of a failure even if deterioration is as expected. A similar observation applies to the CBMP strategy, but now production is slowed down towards the end of the planning period, unless deterioration is much lower than expected. Moreover, for various systems, production is even already slowed down before maintenance is scheduled. We conclude that the ability to vary the production rate is exploited in a wide range of scenarios.

#### 5.4. Parameter sensitivity

In the previous section, we have seen that both CBM and CBP realize considerable cost savings in the base case, and applying them together is even more effective. However, the joint cost savings are less than the sum of the separate savings, showing that CBM and CBP only partly complement each other for the base case. In this section, we perform a sensitivity study to obtain further insights into the comparative performance of CBM and CBP and into whether the two can enhance each other's performance. Besides cost savings, we also consider other performance measures such as expected production,

mean time between maintenance, and probability of a failure. The results are obtained by studying the base case system while deviating various parameters one by one.

#### Planning time

We first consider the planning time  $s$  required to carry out maintenance, see Fig. 3. The fixed strategy and the CBP strategy use block-based maintenance policies that are not affected by the planning time, and we indeed see that the costs for these strategies are independent of  $s$ . CBP realizes a cost saving of 25% compared to the fixed strategy, regardless of the planning time. CBM realizes a cost saving of 34% when there is no planning time, and its effectiveness obviously decreases in the length of the planning time. If the planning time equals the optimal block length under the fixed strategy (i.e.,  $T = 42$ ), then CBM immediately schedules maintenance upon each maintenance action and is not able to realize a cost saving compared to the fixed strategy.

CBMP utilizes the condition information to both schedule maintenance and adapt the production rate. However, similar to the CBM policy, the value of condition information for the maintenance planning decreases if the planning time increases. As a result, CBMP converges to CBP as  $s$  increases. For short planning times, the major part of the cost saving is due to the condition-based maintenance decisions and we indeed see that CBM and CBMP result in almost the same cost if there is no planning time. A marginal cost difference exists because CBMP can use a slightly higher maintenance threshold by reducing the production rate already before maintenance is scheduled, thereby reducing the volatility of the deterioration increments and thus reducing the risk of an instant failure.

We observe that for planning times of at least 37 periods, the cost saving of CBMP is larger than the sum of cost savings of CBM and CBP. Thus, the two dynamic decisions can enhance each other's performance.

We conclude that the planning time strongly affects the effectiveness of the various policies. For short planning times, CBM outperforms CBP, whereas for longer planning times CBP is preferred. Although CBM is preferred when the planning time is short, CBP realizes a considerable cost saving as well and can thus be a viable alternative for CBM based on factors not considered in this study. On the other hand, for long planning times, CBM does not reduce costs and is thus not a viable alternative for CBP in that case. Moreover, CBMP is by far the most beneficial for intermediate planning times.

#### Maintenance cost and revenue

Fig. 4 shows how the cost savings of the condition-based strategies are affected by the other system parameters. We first assess the effect of the corrective maintenance cost. If  $c_{cm}$  equals  $c_{pm}$ , CBP only marginally reduces cost compared to the fixed strategy by 1% while CBM realizes a considerable cost saving of 18%. The small cost saving realized by CBP is because the total production can be increased in the rare case a failure is virtually inevitable. Thus, if failures do not induce additional



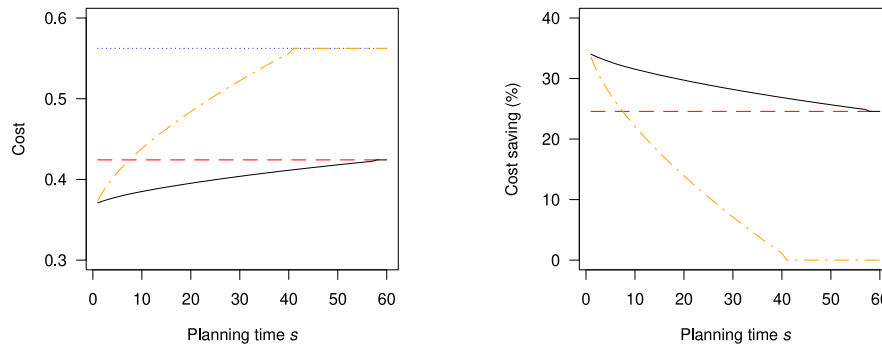


Fig. 3. Effect of planning time  $s$  on the total average cost and on the relative cost saving compared to the fixed strategy. The results are given for the fixed strategy (dotted blue), CBM (dot dashed orange), CBP (dashed red), and CBMP (solid black).

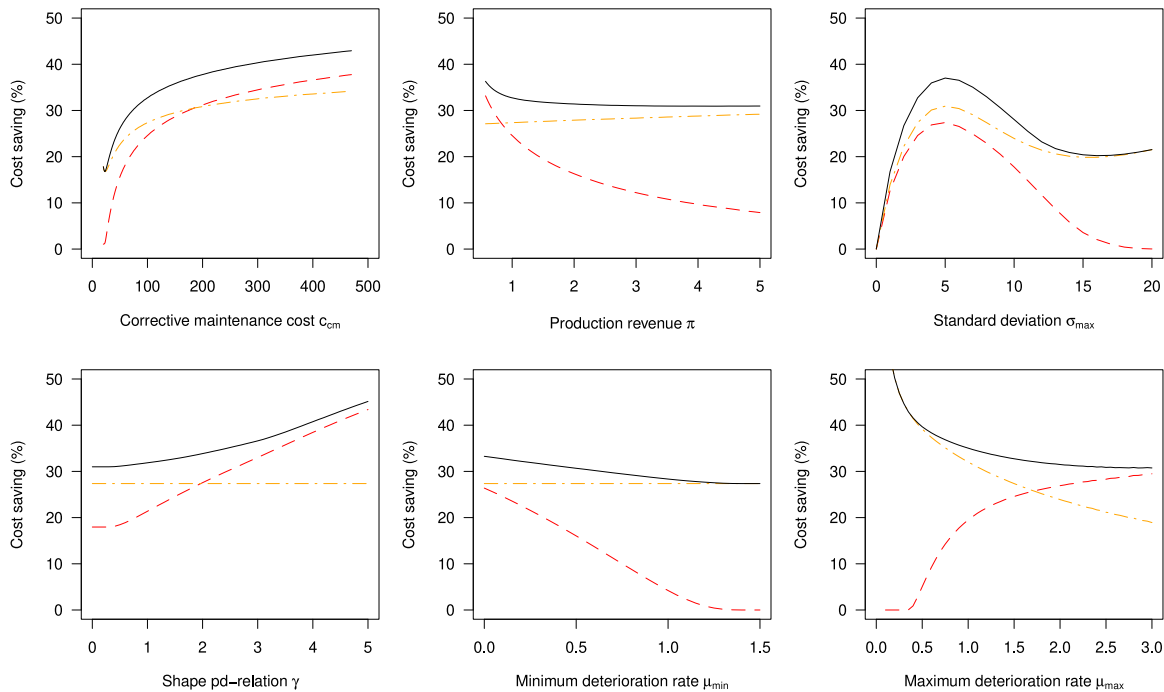


Fig. 4. Effect of various parameters on the relative cost savings of CBP (dashed red), CBM (dot dashed orange), and CBMP (solid black) compared to the fixed strategy.

costs on top of the preventive maintenance cost, then CBM is clearly more effective than CBP. This is intuitive as it is not beneficial to avoid a failure by slowing down production, which is the main benefit of an adjustable production rate.

For increasing corrective maintenance costs, avoiding failures becomes more important. CBM can only achieve this by being conservative and initiating maintenance at a lower deterioration threshold, i.e., the maintenance policy has to be robust for above average deterioration. CBP, on the other hand, can use a more optimistic maintenance policy and predominantly uses the adjustable production rate to avoid failure which only causes additional costs when above average deterioration is observed. For instance, if  $c_{cm}$  increases from 25 to 250, CBM decreases the maintenance threshold from 86.2 to 63.9 while CBP only reduces the block length from 62 to 60. Moreover, for extremely high corrective maintenance costs (say  $c_{cm} \geq 400$ ), CBP becomes insensitive to changes in the corrective maintenance costs as the failure risk becomes negligible while costs under CBM continue to increase. Finally, as long as corrective maintenance is more expensive than preventive maintenance, the adjustable production rate complements the condition-based maintenance policy by reducing the failure risk during the planning time.

Considering the effect of the production revenue  $\pi$ , we observe that CBM becomes slightly more cost-effective when the production revenue

increases, while CBP becomes significantly less effective. CBP reduces the maintenance cost by decelerating the production rate, which is more expensive for high values of  $\pi$ . For extremely high production revenues, it is optimal to only focus on maximizing production and avoid failures by scheduling very short blocks, thus CBP converges to the fixed strategy.

#### Volatility deterioration increments

The third graph shows the effect of the volatility of the deterioration increments. For standard deviations close to zero, the deterioration process is stable, implying that condition information has limited value. When the volatility starts to increase, all condition-based strategies effectively use the condition information and considerably outperform the fixed strategy. CBP reduces the failure risk during a block and improves the reliability by decreasing the production rate when the deterioration level is high relative to the remaining time to maintenance. Doing so allows this strategy to be less conservative and schedule longer blocks than the fixed strategy (e.g.,  $T = 62$  versus  $T = 47$  for  $\sigma_{max} = 2$ ). However, the flexible production rate comes with revenue losses, while the flexibility of CBM does not induce additional costs. As a result, CBP is slightly less effective than CBM for small volatilities.

For intermediate volatilities, the uncertainty during the planning time becomes considerable, and CBM has to significantly lower the threshold at which maintenance is initiated to cope with this uncertainty. However, the volatility is still low enough to be handled by adjusting the production rate. As a result, combining both dynamic decisions is most advantageous for an intermediate volatility.

For high volatilities, the system does not gradually deteriorate but failures are likely to be caused by a single extreme deterioration increment. Such shocks arrive suddenly and condition monitoring does not provide any information on the arrival of these shocks. CBP reduces their magnitude by producing at a lower rate, such that the system is not expected to fail from a single shock, and immediately switches to the idle mode after a shock occurred. For extremely high volatilities, reducing the size of the shocks does not outweigh the induced production losses and we indeed observe that the cost savings of CBP diminish if the volatility continues to increase. CBM and CBMP, on the other hand, produce at the maximum rate until the system fails and then schedule maintenance, thereby even reducing costs for extremely volatile processes.

#### Parameters $pd$ -relation

We continue with the effect of the  $pd$ -relation parameters  $\gamma$ ,  $\mu_{\min}$ , and  $\mu_{\max}$ , see the bottom three graphs of Fig. 4. Recall that  $\gamma$  describes whether the  $pd$ -relation is concave ( $0 < \gamma < 1$ ) or convex ( $\gamma > 1$ ). The minimum deterioration rate ranges from no deterioration when idle ( $\mu_{\min} = 0$ ) to maximum deterioration when idle ( $\mu_{\min} = \mu_{\max} = 1.5$ ), in which case the production rate does not affect the deterioration rate. The major insights are that (1) all strategies realize a significant cost saving for all values of  $\gamma$ , (2) for concave  $pd$ -relations CBM is preferred while CBP is better for 'more convex'  $pd$ -relations, (3) CBM and CBP complement each other for all values for  $\gamma$ , and (4) condition-based production decisions improve the CBM policy even if  $\mu_{\min}$  is high compared to  $\mu_{\max}$ . Finally, the cost savings of CBM decrease as  $\mu_{\max}$  increases, whereas the cost savings of CBP actually increase. Higher values for  $\mu_{\max}$  imply that the system deteriorates faster and, as a result, both the fixed strategy and CBM have to schedule more maintenance interventions, thereby considerably increasing costs. CBP overcomes this by slightly reducing the production rate. For very high values of  $\mu_{\max}$ , CBP already reduces the production rate for low deterioration levels in order to produce at a more efficient rate. Note that, similar to the planning time parameter  $s$ , CBM converges to the fixed strategy if  $\mu_{\max}$  continues to increase.

#### 5.5. Parameter estimation errors

So far we have assumed that all system parameters are known with certainty, whereas in practice various parameters must be estimated. In this section, we compare the robustness of the considered strategies with respect to incorrect estimations of the parameters of the deterioration process. Although all system parameters are uncertain to some extent, we focus on the mean and the standard deviation of the deterioration increments per time unit as they are crucial for maintenance and production decisions. We vary the estimated mean  $\hat{\mu}_{\max}$  from 0.2 (almost no deterioration) to 3.0 (twice as fast as the base system), while the coefficient of variation is kept constant. The estimated standard deviation  $\hat{\sigma}_{\max}$  ranges from 0.2 (very stable deterioration) to 6.0 (twice as volatile as the base system).

Fig. 5 shows the average costs of all four strategies when they are optimized based on the estimated values  $\hat{\mu}_{\max}$  (top) and  $\hat{\sigma}_{\max}$  (bottom) but are applied to our base system with  $\mu_{\max} = 1.5$  and  $\sigma_{\max} = 3.0$ . For the base system (left panels), the three main observations are that the fixed strategy is significantly more sensitive to estimation errors in the mean than the condition-based strategies, that the three condition-based strategies are similarly affected by estimation errors for both the mean and the standard deviation, and that underestimating parameters is often worse than overestimating them. Note that for

large underestimations of the deterioration rate, the cost of the fixed strategy actually decreases if the estimation error increases further. This is because for large underestimations, the fixed strategy schedules too long blocks and the system is likely to fail every block, resulting in very high maintenance costs. The production revenues do not outweigh the corrective maintenance costs and it is better to schedule even fewer maintenance interventions, which happens if the estimated value decreases further.

The agility of CBP compared to that of CBM explains its more robust performance with regard to estimations errors. CBP can quickly react to the actual deterioration level if this differs from what was expected. This particularly allows, in case of underestimation of the deterioration rate, to better prevent expensive corrective maintenance actions by continuously adjusting the production rate based on the actual deterioration level. Furthermore, overestimating the mean deterioration rate leads to unnecessarily early maintenance interventions under CBM, which are relatively costly compared to the slightly reduced production rates under CBP.

Summarizing, the fixed strategy is very sensitive to estimation errors, as it plans maintenance without condition information and is not able to adapt. CBM does adapt by planning maintenance based on the observed level of deterioration, but can no longer react after maintenance has been planned. As a result, CBM is more robust than the fixed strategy but still quite sensitive to estimation errors if the planning time is long and/or corrective maintenance is expensive. Under such conditions, CBP and CBMP achieve a much lower cost when the deterioration process is mis-estimated, by adjusting the production rate according the actual deterioration level.

## 6. Conclusion

Ongoing developments in the fields of online condition monitoring and real-time decision making create opportunities to operate industrial systems more efficiently and reliably by implementing condition-based maintenance (CBM) or condition-based production (CBP). Although these policies have both been studied in isolation, we are the first to compare them and to consider their combination into a fully flexible condition-based maintenance and production (CBMP) policy. Our analysis and results have practical value for equipment for which conditions can be monitored, and for which a relation exists between production speed and degradation speed. This relation is particularly encountered for rotating and moving equipment such as conveyor belts, cutting tools, and wind turbine gearboxes [34,35].

Our study focuses on a single piece of equipment that gradually deteriorates and that is continuously monitored. An extensive numerical analysis based on Markov decision process formulations has revealed a number of valuable insights. First, for almost all considered settings, all three dynamic strategies clearly outperform the simple strategy with static maintenance and a fixed production rate. Second, the cost reduction mechanisms of CBM and CBP are quite different. CBP typically reduces the failure risk significantly at the expense of lower expected production, whereas CBM policies are characterized by higher expected production but substantially more failures too. Thus, CBM is mainly recommended in cases with high production revenues, whereas CBP is more effective when failures are more severe. The comparative performance of CBM and CBP depends on what effect dominates. Moreover, CBMP improves the trade-off between production and maintenance costs further, resulting in more production than CBP and fewer failures and improved reliability compared to CBM. In practical settings where both production and maintenance decisions can be made dynamically, it is therefore often best to use condition information for both of them. In fact, sometimes the savings of the combined strategy are more than the sum of the savings of CBM and CBP.

We also examined how the various policies perform under wrongly estimated parameter values for the deterioration process, as these are

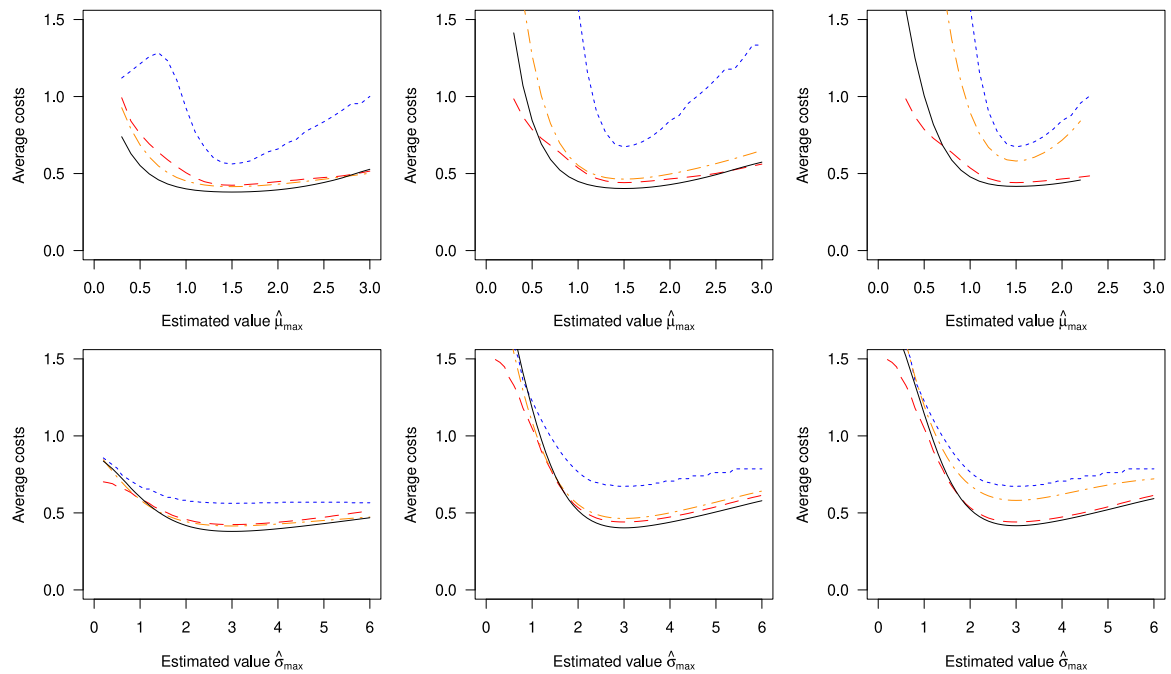


Fig. 5. Effect of mis-estimating the deterioration rate  $\mu_{\max} = 1.5$  (top) and the standard deviation  $\sigma_{\max} = 3.0$  (bottom) on the costs realized by FMP (dotted blue), CBM (dot dashed orange), CBP (dashed red), and CBMP (solid black). The left panels show the results for the base system, the middle panels increase the corrective maintenance cost from 100 to 300, and the right panels also increase the planning time from 5 to 20.

often uncertain for real-life systems. This showed once more that condition information should be taken into account when available, since all dynamic strategies were considerably more robust to estimation errors of the deterioration rate than the fixed strategy. We also found that CBM is much less robust for large estimation errors than CBP if the planning time is long or corrective maintenance is expensive, as the latter can dynamically correct for wrong decisions. It follows that it is also beneficial to adopt the dynamic strategies if there is uncertainty of the deterioration parameters.

There are numerous research opportunities within this direction. One direction is to include unreliable condition information or uncertain parameter values for the deterioration process directly into the optimization. It is expected that condition-based production decisions can better cope with such uncertainties compared to condition-based maintenance as the effect of a wrong production decision is less severe than a wrong maintenance decision. After a production rate is set, this can be revised when more condition information becomes available whereas an expensive maintenance action cannot be made undone.

Moreover, incorporating condition-based production rates into multi-unit systems seems a promising direction. Production facilities often face contracts with considerable penalties if a minimum total production target is not met. For systems with many units (e.g., offshore wind farms), some units may be idle, but if too many fail simultaneously then penalties are incurred. For such systems, condition-based production decisions can be used to desynchronize the deterioration levels of various units in order to minimize the risk that multiple units fail shortly after each other. Similarly, adjustable production rates can also be applied to do the opposite, namely synchronizing the deterioration level of (a subset of) the units such that their maintenance can be clustered, thereby reducing setup costs. Clearly, the optimal policy is situation dependent and deserves attention.

Another direction is to study multi-unit systems with limited maintenance capacity, for instance, due to the need for specialized equipment or the limited availability of skilled technicians. When multiple units require maintenance around the same time this can result in long downtimes (and thus revenue losses) due to the limited maintenance capacity. With a dynamic production planning, units can produce at

different rates such that their preferred maintenance moments can be spread, thereby reducing peak demand for scarce maintenance equipment.

#### CRediT authorship contribution statement

**Michiel A.J. uit het Broek:** Conceptualization, Methodology, Writing - original draft, Writing - review & editing, Software, Validation, Investigation, Visualization. **Ruud H. Teunter:** Conceptualization, Methodology, Writing - original draft, Writing - review & editing. **Bram de Jonge:** Conceptualization, Methodology, Writing - original draft, Writing - review & editing. **Jasper Veldman:** Conceptualization, Methodology, Writing - original draft, Writing - review & editing.

#### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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