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On approximations, complexity, and applications for copositive programming

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Nomenclature

Functions and Operators

$\text{Aut}(\mathbb{R}_+^n)$	The automorphism group of the nonnegative orthant, page 40
δ_{ij}	The Kronecker delta, page 86
Diag	Operator that turns a vector \mathbf{d} into a diagonal matrix D such that $D_{ii} = d_i$ for all i , page 3
diag	Operator that creates a vector \mathbf{a} from a matrix A such that $a_i = A_{ii}$ for all i , page 3
$\langle A, B \rangle$	Inner product of A and B , page 3
\mathbf{x}^β	For $\mathbf{x} \in \mathbb{R}^n$ and $\beta \in \mathbb{Z}_+^n$, we have $\mathbf{x}^\beta = \prod_{i=1}^n x_i^{\beta_i}$, page 16
$\mathcal{O}(\bullet)$	The big-O notation describing limit behavior of functions, page 3
$\mathbb{R}[\mathbf{x}]$	Ring of polynomials in \mathbf{x} with coefficients in \mathbb{R} , page 3
$\text{Tr}(A)$	Trace of A , page 3
Vec	operator that creates a vector from a matrix by stacking its columns, page 3
$ \mathbf{a} $	Sum of the absolute values of the elements of \mathbf{a} , page 3
$A \circ B$	Hadamard product of the matrices A and B , page 3
$A \otimes B$	Kronecker product of the matrices A and B , page 3
$r_{\mathbb{Y}_n^r}^*(A)$	For a hierarchy \mathbb{Y}_n^r approximating the copositive cone, $r_{\mathbb{Y}_n^r}^*(A)$ is the smallest \bar{r} such that $A \in \mathbb{Y}_n^{\bar{r}}$, page 20
\mathcal{G}_n	Group of automorphisms w.r.t. scalings and permutations, page 40
$\text{CP-rank}(A)$	The CP-rank of a matrix A , page 8

SOS sum of squares, page 15

Graph notation

α_G The stability number of a graph G , page 23

$\bar{G} = (V, \bar{E})$ The complement graph of $G = (V, E)$, page 3

A_G The adjacency matrix of a graph G , page 23

$G = (V, E)$ Graph with vertex set V and edge set E , page 3

Matrix notation

$(\mathbb{Y}_n^r)_{r \in \mathbb{Z}_+}$ A placeholder for any hierarchy of cones approximating the copositive cone from within, page 20

\mathcal{CP}^n The completely positive cone of order n , page 8

\mathcal{COP}^n The copositive cone of order n , page 4

\mathcal{C}_n^r Approximation hierarchy for the copositive cone, page 14

$\mathbf{A}_{\bullet i}$ The i^{th} column of the matrix A , page 86

$\mathbf{A}_{i \bullet}$ The i^{th} row of the matrix A , page 86

\mathcal{D} The set of $n \times n$ positive diagonal matrices, page 67

$\tilde{\mathcal{N}}^n$ Set of $n \times n$ nonnegative real matrices with zero diagonal, page 41

\mathcal{N}^n Set of $n \times n$ nonnegative real matrices, page 2

\mathcal{K}_n^r The Parrilo r cone of order n , page 15

\mathcal{P}_n the set of $n \times n$ permutation matrices, page 3

\mathcal{S}_+^n The semidefinite cone of order n , page 4

$\mathbb{R}^{n \times m}$ Set of real $n \times m$ matrices, page 2

\mathbb{S}^n Set of $n \times n$ symmetric real matrices, page 2

\mathcal{Q}_n^r Approximation hierarchy for the copositive cone, page 16

$A_{\mathcal{I}}$ Principle submatrix of A containing elements whose row and column indices are in the set \mathcal{I} , page 40

E_n^{ij}	The all zero matrix of order n except for the $(i, j)^{th}$ position which is equal to 1, page 25
$E_{\{i,j\}}$	The all zero matrix except for the $(i, j)^{th}$ and $(j, i)^{th}$ position, page 40
E_n	n -dimensional all ones matrix, page 3
I_n	n -dimensional identity matrix, page 3
$S(\theta)$	The Hildebrand matrices, page 46
dd	diagonally dominant, page 9
H	The Horn matrix, page 12
LMI	Linear matrix inequality, page 16
sdd	scaled diagonally dominant, page 9

Set notation

$\text{cl}(K)$	Closure of the set K , page 3
$\text{conv}(K)$	Convex hull of the set K , page 3
$\text{cone}(K)$	Conic hull of the set K , page 3
$\text{dim}(K)$	Dimension of the set K , page 3
$\text{Ext}(K)$	The set of extreme rays of a set K , page 11
$\text{int}(K)$	Interior of the set K , page 3
\mathcal{V}^A	Set of zeros of a matrix A , page 40
$\text{supp}(\mathcal{M})$	The support of a set \mathcal{M} , page 40

Vector notation

$(\mathbf{x}^{[\leq d]})$	Vector containing all monomials of degree at most d , page 15
$(\mathbf{x}^{[d]})$	Vector containing all monomials of degree exactly d , page 17
$S(K, -\varepsilon)$	ε inner approximation of the set K for some $\varepsilon \in \mathbb{R}_{++}$, page 28
$S(K, \varepsilon)$	ε outer approximation of the set K for some $\varepsilon \in \mathbb{R}_{++}$, page 28
Δ_n	The n dimensional standard simplex, page 2

\mathbb{Z}^n	Set of integer n -vectors, page 2
\mathbf{e}_n	n -dimensional vector of all ones, page 3
\mathcal{Q}	Either \mathbb{Q}^n or $(\mathbb{Q}^{n \times n} \cap \mathbb{S}^n)$, page 28
\mathbb{Q}^n	Set of rational n -vectors, page 2
\mathbb{R}^n	Set of real n -vectors, page 2
\mathbb{R}_+^n	Set of nonnegative real n -vectors, page 2
\mathbb{R}_{++}^n	Set of positive real n -vectors, page 2
$\text{supp}(\mathbf{u})$	The support of \mathbf{u} , page 40