

University of Groningen

On approximations, complexity, and applications for copositive programming

Gijben, Luuk

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version

Publisher's PDF, also known as Version of record

Publication date:

2015

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Gijben, L. (2015). *On approximations, complexity, and applications for copositive programming*. [Thesis fully internal (DIV), University of Groningen]. [S.n.].

Copyright

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license. More information can be found on the University of Groningen website: <https://www.rug.nl/library/open-access/self-archiving-pure/taverne-amendment>.

Take-down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.

Bibliography

- [ACE95] Lars-Erik Andersson, Gengzhe Chang, and Tommy Elfving. Criteria for copositive matrices using simplices and barycentric coordinates. *Linear Algebra and its Applications*, 220:9–30, 1995.
- [AK06] Vikraman Arvind and Piyush P. Kurur. Graph isomorphism is in SPP. *Information and Computation*, 204(5):835–852, 2006.
- [AM14] Amir Ali Ahmadi and Anirudha Majumdar. DSOS and SDSOS optimization: LP and SOCP-based alternatives to sum of squares optimization. *48th Annual Conference on Information Sciences and Systems (CISS)*, pages 1–5, 2014.
- [BAD09] Samuel A. Burer, Kurt M. Anstreicher, and Mirjam Dür. The difference between 5×5 doubly nonnegative and completely positive matrices. *Linear Algebra and its Applications*, 431:1539–1552, 2009.
- [Bas69] Victor J.D. Baston. Extreme copositive quadratic forms. *Acta Arithmetica*, XV:319–327, 1969.
- [Bau65] Leonard D. Baumert. *Extreme Copositive Quadratic Forms*. PhD thesis, California Institute of Technology, Pasadena, California, 1965.
- [Bau66] Leonard D. Baumert. Extreme copositive quadratic forms. *Pacific Journal of Mathematics*, 19(2):197–204, 1966.
- [Bau67] Leonard D. Baumert. Extreme copositive quadratic forms. II. *Pacific Journal of Mathematics*, 20(1):1–20, 1967.
- [BD08] Stefan Bundfuss and Mirjam Dür. Algorithmic copositivity detection by simplicial partition. *Linear Algebra and its Applications*, 428:1511–1523, 2008.

- [BD09] Stefan Bundfuss and Mirjam Dür. An adaptive linear approximation algorithm for copositive programs. *SIAM Journal on Optimization*, 20(1):30–53, 2009.
- [BDdK⁺00] Immanuel M. Bomze, Mirjam Dür, Etienne de Klerk, Cornelis Roos, Arie J. Quist, and Tamás Terlaky. On copositive programming and standard quadratic optimization problems. *Journal of Global Optimization*, 18:301–320, 2000.
- [BdK02] Immanuel M. Bomze and Etienne de Klerk. Solving standard quadratic optimization problems via linear, semidefinite and copositive programming. *Journal of Global Optimization*, 24(2):163–185, 2002.
- [BE12] Immanuel M. Bomze and Gabriele Eichfelder. Copositivity detection by difference-of-convex decomposition and ω -subdivision. *Mathematical Programming*, 138:365–400, 2012.
- [Ber09] Dennis S. Bernstein. *Matrix Mathematics: Theory, Facts and Formulas (second edition)*. Princeton University Press, 2009.
- [BGM82] László Babai, D. Yu. Grigoryev, and David M. Mount. Isomorphism of graphs with bounded eigenvalue multiplicity. In *Proceedings of the Fourteenth Annual ACM Symposium on Theory of Computing*, STOC '82, pages 310–324, New York, NY, USA, 1982. ACM.
- [BJ10] Immanuel M. Bomze and Florian Jarre. A note on Burer’s copositive representation of mixed-binary QPs. *Optimization Letters*, 4(3):465–472, 2010.
- [BJR11] Immanuel M. Bomze, Florian Jarre, and Franz Rendl. Quadratic factorization heuristics for copositive programming. *Mathematical Programming Computation*, 3(1):37–57, 2011.
- [BM76] John A. Bondy and Uppaluri S.R. Murty. *Graph Theory with Applications*. The Macmillan Press Ltd., 1976.
- [Bom87] Immanuel M. Bomze. Remarks on the recursive structure of copositivity. *Journal of Information and Optimization Sciences*, 8(3):243–260, 1987.
- [Bom89] Immanuel M. Bomze. Copositivity and optimization. *Methods in Operations Research*, 58:27–35, 1989.

- [Bom12] Immanuel M. Bomze. Copositive programming - recent developments and applications. *European Journal of Operations Research*, 216:509–520, 2012.
- [BSM03] Abraham Berman and Naomi Shaked-Monderer. *Completely Positive Matrices*. World Scientific Pub. Co. Pte. Inc., 2003.
- [BSU12] Immanuel M. Bomze, Werner Schachinger, and Gabriele Uchida. Think co(mpletely)positive ! matrix properties, examples and a clustered bibliography on copositive optimization. *Journal of Global Optimization*, 52(3):423–445, 2012.
- [BSU14a] Immanuel M. Bomze, Werner Schachinger, and Reinhard Ullrich. From seven to eleven: completely positive matrices with high CP-rank. *Linear Algebra and its Applications*, 459(15):208–221, 2014.
- [BSU14b] Immanuel M. Bomze, Werner Schachinger, and Reinhard Ullrich. New lower bounds and asymptotics for the CP-rank. <http://www.newton.ac.uk/preprints/NI14048.pdf>, 2014. Preprint.
- [Bur09] Samuel A. Burer. On the copositive representation of binary and continuous nonconvex quadratic programs. *Mathematical Programming*, 120:479–495, 2009.
- [Bur12] Samuel A. Burer. Copositive programming. In M.F. Anjos and J.B. Lasserre, editors, *Handbook of Semidefinite, Conic and Polynomial Optimization: Theory, Algorithms, Software and Applications*, pages 201–218. Springer, New York, 2012.
- [CFI92] Jin-Yi Cai, Martin Fürer, and Neil Immerman. An optimal lower bound on the number of variables for graph identification. *Combinatorica*, 12(4):389–410, 1992.
- [CHL70] Richard W. Cottle, G.J. Habetler, and C.E. Lemke. On classes of copositive matrices. *Linear Algebra and its Applications*, 3:295–310, 1970.
- [CS94] Gengzhe Chang and Thomas W. Sederberg. Nonnegative quadratic Bézier triangular patches. *Computer Aided Geometric Design*, 11:113–116, 1994.
- [DA13] Hongbo Dong and Kurt Anstreicher. Separating doubly nonnegative and completely positive matrices. *Mathematical Programming*, 137(1):131–153, 2013.

- [DD12] Peter J.C. Dickinson and Mirjam Dür. Linear-time complete positivity detection and decomposition of sparse matrices. *SIAM Journal on Matrix Analysis and Applications*, 33(3):701–720, 2012.
- [DDGH13a] Peter J.C. Dickinson, Mirjam Dür, Luuk Gijben, and Roland Hildebrand. Irreducible elements of the copositive cone. *Linear Algebra and its Applications*, 439(6):1605–1626, 2013.
- [DDGH13b] Peter J.C. Dickinson, Mirjam Dür, Luuk Gijben, and Roland Hildebrand. Scaling relationship between the copositive cone and Parrilo’s first level approximation. *Optimization Letters*, 7(8):1669–1679, 2013.
- [DDV14] Christian Dobre, Mirjam Dür, and Frank Vallentin. A copositive formulation for the stability number of infinite graphs. <http://arxiv.org/pdf/1305.1819v1.pdf>, 2014. Preprint.
- [DG14] Peter J.C. Dickinson and Luuk Gijben. On the computational complexity of membership problems for the completely positive cone and its dual. *Computational Optimization and Applications*, 57(2):403–415, 2014.
- [Dia62] Palahenedi H. Diananda. On nonnegative forma in real variables some or all of which are nonnegative. *Mathematical Proceedings of the Cambridge Philosophical Society*, 58(1):17–25, 1962.
- [Dic10] Peter J. C. Dickinson. An improved characterisation of the interior of the completely positive cone. *Electronic Journal of Linear Algebra*, 20:723–729, 2010.
- [Dic11] Peter J.C. Dickinson. Geometry of the copositive and completely positive cone. *Journal of Mathematical Analysis and Applications*, 380(1):377–395, 2011.
- [Dic13] Peter J.C. Dickinson. *The Copositive Cone, the Completely Positive Cone and their Generalisations*. PhD thesis, University of Groningen, 2013.
- [DJL94] John H Drew, Charles R. Johnson, and Raphael Loewy. Completely positive matrices associated with M-matrices. *Linear and Multilinear Algebra*, 37(4):303–310, 1994.

- [dK08] Etienne de Klerk. The complexity of optimizing over a simplex, hypercube or sphere: a short survey. *Central European Journal of Operations Research*, 16(2):111–125, 2008.
- [dKP02] Etienne de Klerk and Dmitrii V. Pasechnik. Approximation of the stability number of a graph via copositive programming. *SIAM Journal on Optimization*, 12(4):875–892, 2002.
- [DLC14] Bo Dong, Matthew M. Lin, and Moody T. Chu. Nonnegative rank factorization - a heuristic approach via rank reduction. *Numerical Algorithms*, 65(2):251–274, 2014.
- [Don13] Hongbo Dong. Symmetric tensor approximation hierarchies for the completely positive cone. *SIAM Journal on Optimization*, 23(3):1850–1866, 2013.
- [DR10] Igor Dukanovic and Franz Rendl. Copositive programming motivated bounds on the stability and the chromatic numbers. *Mathematical Programming*, 121(2):249–268, 2010.
- [DS08] Mirjam Dür and Georg Still. Interior points of the completely positive cone. *Electronic Journal of Linear Algebra*, 17:48–53, 2008.
- [Dür10] Mirjam Dür. Copositive programming - a survey. In M. Diehl, F. Glineur, E. Jarlebring, and W. Michiels, editors, *Recent Advances in Optimization and its Applications in Engineering*, pages 3–20. Springer-Verlag Berlin Heidelberg, 2010.
- [DV13] Christian Dobre and Juan Vera. Exploiting symmetry in copositive programs via semidefinite hierarchies. http://www.optimization-online.org/DB_FILE/2013/12/4181.pdf, 2013. Preprint.
- [EJ08] Gabriele Eichfelder and Johannes Jahn. Set-semidefinite optimization. *Journal of Convex Analysis*, 15:767–801, 2008.
- [Gad58] Jerry W. Gaddum. Linear inequalities and quadratic forms. *Pacific Journal of Mathematics*, 8(3):411–414, 1958.
- [GJ79] Michael R. Garey and David S. Johnson. *Computers and Intractability: A Guide to the Theory of NP-Completeness*. W.H. Freeman and co., 1979.

- [GL07] Nebojsa Gvozdenović and Monique Laurent. Semidefinite bounds for the stability number of a graph via sums of squares of polynomials. *Mathematical Programming*, 110(1):145–173, 2007.
- [GL08] Nebojsa Gvozdenović and Monique Laurent. The operator Ψ for the chromatic number of a graph. *SIAM Journal on Optimization*, 19(2):572–591, 2008.
- [GLS88] Martin Grötschel, László Lovász, and Alexander Schrijver. *Geometric Algorithms and Combinatorial Optimization*. Springer-Verlag (Berlin), 1988.
- [HH69] Emilie Haynsworth and Alan J. Hoffman. Two remarks on copositive matrices. *Linear Algebra and its Applications*, 2:387–392, 1969.
- [Hil12] Roland Hildebrand. The extreme rays of the 5×5 copositive cone. *Linear Algebra and its Applications*, 437(7):1538–1547, 2012.
- [HJ62] Marshall Hall Jr. Discrete problems. In *Survey of Numerical Analysis John Todd (Ed.)*, pages 518–542. McGraw-Hill Book Company Inc., 1962.
- [HJR05] Leslie Hogben, Charles R. Johnson, and Robert Reams. The copositive completion problem. *Linear Algebra and its Applications*, 408:207–211, Oct 2005.
- [HN63] Marshall Hall, Jr. and Morris Newman. Copositive and completely positive quadratic forms. *Mathematical Proceedings of the Cambridge Philosophical Society*, 59:329–339, 1963.
- [Hor76] Reiner Horst. An algorithm for nonconvex programming problems. *Mathematical Programming*, 10(1):312–321, 1976.
- [Hor97] Reiner Horst. On generalized bisection of n -simplices. *Mathematics of Computation*, 66(218):691–698, 1997.
- [HPT95] Reiner Horst, Panos M. Pardalos, and Nguyen Van Thoai. *Introduction to Global Optimization*. Nonconvex Optimization and Its Applications Vol. 3. Springer, 1995.
- [HRMP95] Christoph Helmberg, Franz Rendl, Bojan Mohar, and Svatopluk Poljak. A spectral approach to bandwidth and separator problems in graphs. *Linear and Multilinear Algebra*, 39:73–90, 1995.

-
- [HT71] John E. Hopcroft and Robert E. Tarjan. A v^2 algorithm for determining isomorphism of planar graphs. *Information Processing Letters*, 1(1):32–34, 1971.
- [HUS10] Jean-Baptiste Hiriart-Urruty and Alberto Seeger. A variational approach to copositive matrices. *SIAM Review*, 52(4):593–629, 2010.
- [HW74] John E. Hopcroft and J. K. Wong. Linear time algorithm for isomorphism of planar graphs (preliminary report). In *Proceedings of the Sixth Annual ACM Symposium on Theory of Computing*, STOC '74, pages 172–184, New York, NY, USA, 1974. ACM.
- [JR08] Charles R. Johnson and Robert Reams. Constructing copositive matrices from interior matrices. *Electronic Journal of Linear Algebra*, 17:9–20, 2008.
- [JS09] Florian Jarre and Katrin Schmallowsky. On the computation of C^* certificates. *Journal of Global Optimization*, 45(2):281–296, 2009.
- [Kay87] Mohammad Kaykobad. On nonnegative factorization of matrices. *Linear Algebra and its Applications*, 96:27–33, 1987.
- [Kea78] Baker Kearfott. A proof of convergence and an error bound for the method of bisection in \mathbb{R}^n . *Mathematics of Computation*, 32(144):1147–1153, 1978.
- [KG12] Vassilis Kalofolias and Efstratios Gallopoulos. Computing symmetric nonnegative rank factorizations. *Linear Algebra and its Applications*, 436:421–435, 2012.
- [Kha79] Leonid Khachiyan. A polynomial algorithm in linear programming (in russian). *Doklady Akademii Nauk SSSR*, 244:1093–1096, 1979.
- [Las11] Jean-Bernard Lasserre. A new look at nonnegativity on closed sets and polynomial optimization. *SIAM Journal on Optimization*, 21(3):864–885, 2011.
- [Las14] Jean-Bernard Lasserre. New approximations for the cone of copositive matrices and its dual. *Mathematical Programming*, 144(1):265–276, 2014.

- [Lau08] Monique Laurent. Semidefinite programming in combinatorial and polynomial optimization. *Nieuw Archief voor Wiskunde (NAW)*, 5(4), 2008.
- [Lau09] Monique Laurent. Sums of squares, moment matrices and optimization over polynomials. In *Emerging applications of algebraic geometry*, volume 149 of *IMA Vol. Math. Appl.*, pages 157–270. Springer, New York, 2009.
- [Luk82] Eugene M. Luks. Isomorphism of graphs of bounded valence can be tested in polynomial time. *Journal of Computer and System Sciences*, 25(1):42–65, 1982.
- [MA13] Shashank K. Mehta and Pawan Aurora. Completely positive formulation of the graph isomorphism problem. <http://arxiv.org/pdf/1301.2390v1.pdf>, 2013. Preprint.
- [McK81] Brendan D. McKay. Practical graph isomorphism. *Congressus Numerantium*, 30:45–87, 1981.
- [Miy95] Takunari Miyazaki. The complexity of McKay’s canonical labeling algorithm. In *Groups and Computation II*, volume 28 of *DIMACS Series in Discrete Mathematics and Theoretical Computer Science*, pages 239–256. American Mathematical Society, 1995.
- [MK87] Katta G. Murty and Santosh N. Kabadi. Some \mathcal{NP} -complete problems in quadratic and nonlinear programming. *Mathematical Programming*, 39(2):117–129, 1987.
- [Mot52] Theodore S. Motzkin. Copositive quadratic forms. *National Bureau of Standards Report*, 1818:11–22, 1952.
- [NTZ11] Karthik Natarajan, Chung-Piaw Teo, and Zhichao Zheng. Mixed 0-1 linear programs under objective uncertainty: a completely positive representation. *Operations Research*, 59:713–728, 2011.
- [Par00] Pablo A. Parrilo. *Structured Semidefinite Programs and Semialgebraic Geometry Methods in Robustness and Optimization*. PhD thesis, California Institute of Technology, 2000.
- [Pól28] György Pólya. Über Positive Darstellung von Polynomen. *Vierteljschr. Naturforsch. Ges. Zürich*, 73:141–145, 1928. Collected Papers, Vol. 2, MIT Press, Cambridge, MA, London, 1974, pp. 309-313.

-
- [PR01] Victoria Powers and Bruce Reznick. A new bound for Pólya’s theorem with applications to polynomials positive on polyhedra. *Journal of Pure and Applied Algebra*, 164:221–229, 2001.
- [PR07] Janez Povh and Franz Rendl. A copositive programming approach to graph partitioning. *SIAM Journal on Optimization*, 18(1):223–241, 2007.
- [PR09] Janez Povh and Franz Rendl. Copositive and semidefinite relaxations of the quadratic assignment problem. *Discrete Optimization*, 6:231–241, 2009.
- [PVZ06] Javier Peña, Juan Vera, and Luis F. Zuluaga. LMI approximations for cones of positive semidefinite forms. *SIAM Journal on Optimization*, 16:1076–1091, 2006.
- [PVZ07] Javier Peña, Juan Vera, and Luis F. Zuluaga. Computing the stability number of a graph via linear and semidefinite programming. *SIAM Journal on Optimization*, 18(1):87–105, 2007.
- [PY93] Li Ping and Feng Yu Yu. Criteria for copositive matrices of order four. *Linear Algebra and its Applications*, 194:109–124, 1993.
- [Rob73] Raphael M. Robinson. Some definite polynomials which are not sums of squares of real polynomials. In *Selected questions of algebra and logic (collection dedicated to the memory of A. I. Mal’cev) (Russian)*, pages 264–282. Izdat. “Nauka” Sibirsk. Otdel., Novosibirsk, 1973.
- [Sch88] Uwe Schöning. Graph isomorphism is in the low hierarchy. *Journal of Computer and System Sciences*, 37(3):312–323, 1988.
- [Sch03] Alexander Schrijver. *Combinatorial optimization: polyhedra and efficiency*, volume 24 of *Algorithms and Combinatorics*. Springer, 2003.
- [Sch09] Pascal Schweitzer. *Problems of Unknown Complexity, Graph Isomorphism and Ramsey Theoretic Numbers*. PhD thesis, Universität des Saarlandes, 2009.
- [Sho77] Naum Z. Shor. Cut-off method with space extension in convex programming problems. *Cybernetics*, 13:94–96, 1977.
- [SM09] Naomi Shaked-Monderer. A note on upper bounds on the CP-rank. *Linear Algebra and its Applications*, 431:2407–2413, 2009.

- [SMBJS13] Naomi Shaked-Monderer, Immanuel M. Bomze, Florian Jarre, and Werner Schachinger. On the CP-rank and the minimal cp factorizations of a completely positive matrix. *SIAM Journal on Matrix Analysis and Applications*, 34(2):355–368, 2013.
- [TH88] Hoang Tuy and Reiner Horst. Convergence and restart in branch-and-bound algorithms for global optimization. Application to concave minimization and d.c. optimization problems. *Mathematical Programming*, 41(1):161–183, 1988.
- [Tuy88] Hoang Tuy. Effect of the subdivision strategy on convergence and efficiency of some global optimization algorithms. *Journal of Global Optimization*, 1(1):23–36, 1988.
- [WL68] Boris Weisfeiler and A. A. Lehman. A reduction of a graph to a canonical form and an algebra arising during this reduction (in Russian). *Nauchno-Technicheskaya Informatsia*, 2(9):12–16, 1968.
- [Yca82] Bernard Ycart. Extrémales du cône des matrices de type non négatif, à coefficients positifs ou nuls. *Linear Algebra and its Applications*, 48:317–330, 1982.
- [YN76] D.B. Yudin and A.S. Nemirovskii. Informational complexity and efficient methods for the solution of convex extremal problems. *Ekonomika i Matematicheskie Metody*, 12(2):357–369, 1976. English translation: *Matekon* 13 (3) (1977) 25-45.