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## The impact of individual differences on network relations

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*Document Version*

Publisher's PDF, also known as Version of record

*Publication date:*

2015

[Link to publication in University of Groningen/UMCG research database](#)

*Citation for published version (APA):*

Muñoz Herrera, M. (2015). *The impact of individual differences on network relations*. [S.n.].

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# Heterogeneous network games: Conflicting preferences

## 4.1 Introduction

In multiple social and economic interactions, actors aim to coordinate their choices to improve their well-being. For instance, individuals purchase products, choose schools or attend social events influenced by the decisions of those around them, be it their friends, family or colleagues. As this environment generally takes the form of a social network, it is clear that the study of coordination problems in networks is key to understand social outcomes and welfare through strategic behavior. In this regard, the literature on coordination games assumes these interactions to be anonymous, so there is no intrinsic difference between actors involved in them (Galeotti et al., 2010). What mainly determines a player's choices is her position in the network. Nonetheless, individual preferences are fundamental to place in context the strength of social influence over such decisions (Bojanowski and Buskens, 2011). For example, when choosing between two products, a potential purchaser requires less influence from her neighbors to acquire the product she likes than to purchase the product she dislike, in case she wants also to choose alike with her neighbors.

We here propose a model in which actors, embedded in a network of social relationships, can choose how to behave from a set of two possible choices. Clearly, there are two general classes of stable outcomes. Either all actors coordinate in the same choice, so that behavior in the social network is specialized in a single action, or some actors choose one action and others choose the other, so that behavior is hybrid. As actors differ in their preferences, they do not attain the same payoffs depending on the action chosen by them and by those around them in the social network. Thus, the interplay, between the gains an actor obtains

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<sup>0</sup>This chapter is co-authored with Penelope Hernandez and Angel Sanchez and is published in *Games and Economic Behavior*.

by coordinating with others and her individual preference, can lead to inequalities in the benefits actors can achieve.

In this direction, our model provides a very simple framework to understand how inequalities between actors depend on how heterogeneous in preferences the network is. Interactions in this study portray situations in which actors have individual preferences that influence their inclination to adopt one behavior or another, but aim to behave alike with those around them. Thus, if two actors are interacting and their preferences are in conflict, each has incentives to choose the same as the counterpart, for this makes them better off, but they have a conflict over which behavior is the one they want to coordinate on.

Within this framework, we develop a complete characterization of the network configurations in equilibrium for two classes of games: strategic complements and strategic substitutes. The equilibrium characterization is made for two conditions: complete and incomplete information. Furthermore, we compare two classes of stable outcomes in which actors do not have incentives to unilaterally change their behavior: equilibria. In the first case, actors are informed about the identities of all others around them, so that they choose how to behave while knowing what those around them prefer (i.e. Nash equilibria). This first consideration is fundamental to understand the effect of conflicting preferences on network relationships. The second case is such that actors know their individual preference and have an idea of how the population is composed, but cannot directly observe the identity of those around her (i.e. Bayesian Nash equilibria). This assumption is more realistic than that of complete information, especially for larger sizes in the population.

With such a simple model, we are able to pinpoint the effect of heterogeneity on preferences and to identify the impact of individual differences when actors have incomplete information, as compared to the framework introduced by Galeotti et al. (2010), where information is incomplete but actors are assumed homogeneous. These authors analyzed two well-known and important classes of games, namely strategic complements and strategic substitutes, finding that on a network, local information arising from the network structure leads to a set of (symmetric) equilibria. When information is incomplete, equilibria are refined and only certain types of them are allowed. Particularly, those with specific monotonicity properties of the actions with respect to the number of connections of the player.

However, compared to Galeotti et al. (2010) we show here that introducing heterogeneity in this framework leads, first, to a very large set of Nash equilibria under complete information, which can be classified in different types according to the satisfaction of the players and the diversity in their actions; and, second, the refinement attained under incomplete information is much more stringent, in so far as only a restricted class of equilibria survives. This is particularly relevant for coordination games, because as presented before in Chapter 1, even the simplest coordination games played on networks have multiple equilibria, which display a wide range of possible outcomes. For this reason, it is important to stress that we are able to obtain these results in a very simple model, which by virtue of its simplicity allows us to highlight all these differences and provides an illustration of

what could take place in more complicated situations.

The chapter is organized in six sections including this introduction. Section 4.2 presents our theoretical framework. The model is described in Section 4.3. Section 4.4 contains the characterization of equilibria with complete information. Section 4.5 presents the analysis with incomplete information. Section 4.6 concludes.

## 4.2 Framework

Our model studies a broad set of coordination games with strategic complements (substitutes). Actors interact in a fixed network by choosing an action from a binary choice set. Each individual has an order for the available choices so that one is preferred over the other. This order represents a player's preference. Coordinating in the liked option gives greater payoffs than in the disliked one. Independently of the chosen action, the more neighbors a player coordinates (anti-coordinates) with, the greater her utility in games with strategic complements (substitutes). This allows an analysis of heterogeneity in which players are endowed with different preferences so that their incentives to choose one action or another vary, even with the same number of neighbors. In addition, we consider heterogeneity in the level of connectivity each player has, so that two players with the same preference need not have equal number of neighbors. The linear payoff structure of our model allows to differentiate the behavior in the two classes of games for different distributions of preferences and connectivities in the network.

Our work is part of the research program on strategic behavior in social and economic networks by considering a rich set of coordination games with heterogeneous players, both in complete and incomplete information. The equilibrium characterization of our network games is carried in two directions. First we provide a complete characterization when players are informed about the size and shape of the network and the distribution of preferences between the players. This leads to the Nash equilibria of the network games. The natural relaxation of the informational assumptions allows for asymmetries where a player knows her preference but not that of others in the network. Individuals know the number of their contacts and the distribution of connections in the population, but are informed only of the probability of the identity of their contacts at the moment of making a choice (note that if there is no information at all on the neighbors' preferences, we would be in a setup of homogeneous networks, similar to that studied by Galeotti et al. 2010). Subsequently, we characterize Bayesian-Nash equilibria for such games.

### 4.2.1 Individual preferences

When people do things together (i.e. cooperate) they frequently need to adopt a certain technology or a set of behaviors or conventions. The problem is that on the one hand choosing the same behavior is better than choosing dissimilar ones, but on the other

people may differ in their preferences over different behaviors. Differences in individual preferences, which result in conflict about what behavior to adopt, are an essential class of heterogeneity that helps understand what the micro-processes are that lead to different patterns of behaviors in coordination games. By means of this, we can study how and under what conditions the differences in individual preferences make actors choose the behavior they dislike. And therefore, whether it is possible that in settings with conflicting preferences, actors can choose in ways that result in outcomes where all individuals behave in the same way. That is, we want to understand the conditions that can lead to social exclusion in the choice between different conventions and its effect of the benefits actors gain from their choices when actors differ in their individual preferences of the different choices.

A person's preference can have either a purely individual basis, but also a social one, such as norms and identities. The way we study heterogeneity in coordination games is by considering preferences based on a person's identity (Akerlof and Kranton, 2000; Tajfel and Turner, 1979): who she is. That is, depending on who a person is, she obtains more benefit from one behavior or another, when coordinating in her choice with those around her. The behavior that gives the highest benefit is denoted as the behavior she prefers. That is, a person's identity can generate her preferences over the behaviors she can choose from. For instance, which social setting a person chooses or which products to purchase. As a consequence, a determinant element of how inequalities and exclusion can arise in coordination games in networks is expressed by the interplay between the benefits of coordinating with others and a person's individual preference. If a person is surrounded by people choosing the behavior she does not prefer, it might be better for her to choose against her preference so that she can gain more benefits from the relationships with others. Otherwise, if choosing in accordance with one's identity but staying isolated, a person can end up with a worse outcome.

There are conflicting preferences in the desired outcome when two players of different (the same) identity interact in games with strategic complements (substitutes). Given their best responses we observe multiple Nash equilibria expressed by the distribution of choices in the network. We denote as *specialized* the cases where the entire set of players coordinates on the same action. Depending on the distribution of identities in the network, in this action profile there are players who choose what they like (*satisfactory*) and others who choose the disliked option (*frustrated*). When both actions coexist in the network, the Nash equilibrium portrays a *hybrid* case which can be satisfactory or frustrated as well.

Finally, we conclude our analysis with a relaxation of the assumption of complete information. Players are informed about the probability of being connected to different neighbors with different identities but do not know the preferences of these neighbors at the moment of choosing. As shown by Galeotti et al. (2010), this is a natural way to introduce incomplete information in network games, so that a player knows her preferences and has a good forecast of the number of her connections, but has incomplete information about the degrees of others and their individual preferences. We characterize the existence of pure

symmetric Bayesian equilibria for different distributions of identities. This means that, depending on how the population is distributed, the resulting configurations can either be specialized or frustrated: indeed, if there are enough players of both types, we show that they can play their action of choice, whereas when one large majority dominates over a small minority, we are able to establish monotonicity results which, for instance, lead to (almost) all players choosing the same action in the case of strategic complements. Interestingly, we are able to prove our main result weakening an assumption in Galeotti et al. (2010) while keeping their formation rule, thus obtaining a dramatic reduction of the complete information equilibria.

### 4.2.2 Network games

The literature studying strategic interactions in networks and its applications to the social sciences has grown increasingly for the past years. Our work contributes to this research program and in particular to the study of coordination network games. For detailed surveys of the literature see Goyal (2007); Jackson (2010); Vega-Redondo (2007); see also Roca et al. (2009) for a review of the literature from the evolutionary point of view.

The main objective of the chapter is to provide a tractable framework where heterogeneity in individual characteristics of the players as well as specifications of informational asymmetries can be analyzed in a single model. Other works that have considered identities of players in terms of individual productivity (Rogers, 2005) or linking costs (Galeotti et al., 2006) are also part of the research on heterogeneity in networks. Our model differs from these for they are focused on strategic link formation and our analysis considers players are located in a fixed network. Several authors have also modeled games with strategic complements or substitutes on networks, see for example Angeletos and Pavan (2007); Ballester and Calvó-Armengol (2010); Ballester et al. (2006); Bénabou (2013); Bergemann and Morris (2009); Bramouille and Kranton (2007); Calvó-Armengol et al. (2009); Glaeser and Scheinkman (2003); Goyal and Moraga-González (2001); Ilkilic (2011); Vives (1999). Our work belongs to this set of literature and complements it with the analysis of incomplete information and the characterization of the Bayesian-Nash equilibria. In particular, our work feeds and is closely related to Galeotti et al. (2010) for they consider games with strategic complements and substitutes and model equilibria for complete and incomplete information. The main difference in our analysis is the considerations of heterogeneity, by which players can differ in their preference for one action or another, leading to conflicting preferences between them.

Another work closely related to this study is Bojanowski and Buskens (2011), that investigates the strategic interaction of heterogeneous actors in coordination environments (i.e. networks). This work presents a very interesting characterization of stability in which no actor wants to change her relationships or her behavior (i.e. chosen convention) and no pair of actors who are not related have incentives to form a new relationship. Furthermore, apart from the analytical work, their paper provides computer simulations to study how heterogeneity affects the emerging relationships. A main difference between Bojanowski

and Buskens (2011) and the work presented in Chapter 4 is that we propose an analytical approach to a closely related model but characterize not only situations of complete information, where the preferences of the actors are visible, but also provide an analysis of situations where complete information about the preferences of those around us is not available.

The integration of heterogeneity in a simple manner into the analysis of social networks allows to disentangle the system of incentives of the players involved. Even when two action profiles in networks with the same configuration of links are identical, the distribution of identities in the population reveals key differences in terms of payoffs in their choice. The simplest example is a society where all individuals coordinate in the same choice. By the inclusion of an analysis on identities in terms of preferences over the available options, we observe that if one or more players are *frustrated* because their choice is based on the influence of their neighbors but not on their preference, they have stronger incentives to deviate. This interplay between individual preferences and social influence has been modeled in coordination games in networks by thresholds functions (Galeotti et al., 2010; Granovetter, 1978). For instance, when a person is deciding whether to acquire a specific technology or not, if more than a given number of her neighbors (i.e. the threshold) use that technology, this person would purchase the new technology, otherwise she would keep using the existing technology.

In this sense, our work also relates and contributes to the literature on threshold models<sup>1</sup>. The pioneering work by Granovetter (1978) referred to thresholds as the proportion of others (neighbors) who must adopt a certain behavior for a given player to do so. In this direction, works such as Morris (2000) and López-Pintado (2006) model coordination games in network structures where there is a contagion threshold that determines how an action spreads. Our work mainly differs from their analysis for we consider static games while they study best-response dynamics. However, we complement these works by assuming heterogeneity in the identities of players. The introduction of players with different preferences over the binary choice set allows for asymmetry in the payoffs between players, which in consequence causes the risk dominant action not to be the same for all players. As a consequence, although in a simplified static model, we study a richer setting where there is not one default action for all individuals, but each of them has preferences for one over the other. Note that this is different from homophily in the sense of Golub and Jackson (2012), where types control the probability of being linked, but do not make any difference in the preferences of individual players. In this sense, part of our contribution is that we obtain two different threshold functions, one for each identity of players for them to adopt the disliked action.

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<sup>1</sup>Equilibria in our model depend on a threshold of the chosen actions by a player's neighbors which also relates to the literature on conformism and social norms (e.g. Kandel and Lazear 1992), where there is a trade off between a player's choice for her preferred action and increasing the distance from the average behavior chosen by her neighbors. This is a relation between peer pressures and partnerships, where peer pressure arises when individuals deviate from a well-established group norm, i.e., individuals are penalized for working less than the group norm. Our threshold functions model when players are willing to *imitate* the behavior of others even if it is not their preferred one and conform to the average behavior or not.

We distinguish two threshold functions, which determine the tipping point where players switch from their liked to their disliked option. The threshold value is the minimum proportion of neighbors necessary to coordinate with to guarantee that choosing the preferred option gives greater payoffs. For example, in the case of choosing between two social events, say two parties, someone who likes party  $A$  over  $B$ , needs less of her friends to go to the first than to the second one for her to choose to go with them. Because of this, heterogeneity in preferences affects the structure and conditions of the game through the identity of the players interacting.

### 4.3 The model

Consider the undirected social network  $(N, g)$ . The set  $N = \{1, \dots, n\}$ , where  $|N| \geq 2$ , contains the players interacting in a game. This set is fixed throughout the analysis, so we represent the network by the set of links,  $g$ . Prior to the start of the game, players are informed about the size of the network and the identity of all players. The set of potential connections is the complete network,  $g^N$ , and any network configuration is part of the set  $G = \{g : g \subset g^N\}$ . In the network, if a pair of players  $i$  and  $j$  are connected by a link, it is denoted as  $ij \in g$ , and if there is no link between them, we say  $ij \notin g$ . The set of neighbors a player  $i$  has is  $k_i(g) = \{j : ij \in g\}$ . For simplicity we assume that  $ii \notin g$ , so that all neighbors in  $k_i(g)$  are different from  $i$ . The cardinality of  $k_i(g)$  is  $k_i$ , the degree of node  $i$  in the network, and is exogenously determined prior to the interactions.

Players from the set  $N$  interact in a network game denoted by  $\Gamma$ . Every player  $i \in N$  is *ex-ante* and exogenously endowed with an identity  $\theta_i \in \{0, 1\}$ . Players choose an action from the binary set  $X = \{0, 1\}$  which is the same for all players. A player  $i$  who has identity 1 (0) prefers action 1 over 0 (0 over 1). We denote  $x_{k_i}(g)$  as the vector of actions taken by  $i$ 's neighbors. The game is expressed through a linear payoff function,  $u_i(\Gamma)$ , that strategically depends on the choices made by connected players and their identities, as follows in Equation 4.1:

$$u_i(\theta_i, x_i, x_{k_i}(g)) = \lambda_{x_i}^{\theta_i} [1 + \delta \sum_{j \in k_i(g)} I_{\{x_j = x_i\}} + (1 - \delta) \sum_{j \in k_i(g)} I_{\{x_j \neq x_i\}}], \quad (4.1)$$

where  $I_{\{x_j = x_i\}}$  is the indicator function of those neighbors choosing the same action as player  $i$ , and  $I_{\{x_j \neq x_i\}}$  indicates neighbors choosing the opposite. The parameter  $\lambda$  is defined by  $\lambda_{x_i}^{\theta_i} = \alpha$  when a player chooses what she likes, and  $\lambda_{x_i}^{\theta_i} = \beta$  otherwise, where  $0 < \beta < \alpha$ . The class of game played is specified through the multiplier  $\delta$ , that takes value 1 if the game is of strategic complements (SC) and 0 if it is of strategic substitutes (SS).

The main feature of our utility specification is that it captures several strategic scenarios in a simple way, allowing for games of strategic complementarities or substitutes. As a result, we can observe the way players' payoffs are affected by the choices of others given their



individual preferences. This is motivated by our desire to develop an understanding of how the conflict of preferences interacts with the network structure. In addition, as discussed in the Introduction, by considering players' types we are extending the applicability of network game models to situations in which the preferences of different players may not be aligned.

We now consider a partial order  $\succeq_i$  on the action profiles of the neighbors for a given player  $i$ . Fix a player  $i$  with  $k_i$  neighbors and identity  $\theta_i$ , where  $x_{k_i}(g)$  and  $x'_{k_i}(g)$  are two action profiles of her neighbors. We say that  $x_{k_i}(g) \succeq_i x'_{k_i}(g)$  if  $\sum_{j=1}^{k_i} I_{\{x_j=1\}} \geq \sum_{j=1}^{k_i} I_{\{x'_j=1\}}$ . For player  $i$ , the actions of her partners can be ordered depending on the number of neighbors playing the action 1. When more individuals in  $i$ 's neighborhood play the action 1, the corresponding action profile is ordered in a higher position. The payoff function for a SC ( $\delta = 1$ ) game satisfies the following condition when the action of player  $i$  is  $1 = x_i > x'_i = 0$  and the action profiles of her neighbors are  $x_{k_i}(g) \succeq_i x'_{k_i}(g)$ :

$$u_i(\theta_i, x_i, x_{k_i}(g)) - u_i(\theta_i, x'_i, x_{k_i}(g)) \geq u_i(\theta_i, x_i, x'_{k_i}(g)) - u_i(\theta_i, x'_i, x'_{k_i}(g)). \quad (4.2)$$

Notice that when  $x_{k_i}(g) \succeq_i x'_{k_i}(g)$  then the number of zero actions in  $x'_{k_i}(g)$  is larger than in  $x_{k_i}(g)$ . By multiplying by  $(-1)$  we get  $u_i(\theta_i, 0, x'_{k_i}(g)) - u_i(\theta_i, 1, x'_{k_i}(g)) \geq u_i(\theta_i, 0, x_{k_i}(g)) - u_i(\theta_i, 1, x_{k_i}(g))$ . From both equations above, we conclude that in SC it is more profitable to play the action that your neighbors play more. Simply said, Equation 4.2 shows that our payoff function satisfies the conditions of games with strategic complementarities.

Analogously, when the game is SS ( $\delta = 0$ ), the payoff function  $u_i$  satisfies the following condition when  $1 = x_i > x'_i = 0$  and  $x_{k_i}(g) \succeq_i x'_{k_i}(g)$ :

$$u_i(\theta_i, x_i, x_{k_i}(g)) - u_i(\theta_i, x'_i, x_{k_i}(g)) \leq u_i(\theta_i, x_i, x'_{k_i}(g)) - u_i(\theta_i, x'_i, x'_{k_i}(g)). \quad (4.3)$$

Analogously than in SC case, we obtain that in SS, it is more profitable to play the action that your neighbors play less. Conversely, Equation 4.3 shows that our payoff function satisfies the conditions for games with strategic substitutes.

Players in our game, represented by  $\Gamma = \{N, \{g\}_{i,j \in N}, X, \{\theta_i\}_{i \in N}, \{u_i\}_{i \in N}\}$ , decide on an action from the binary choice set  $X$ . A unilateral deviation by player  $i$  changes her choice  $x_i$  to choice  $x'_i$ , where  $x_i \neq x'_i$ . When no player has incentives to deviate from an action profile  $(x_1^*, \dots, x_n^*)$ , it is a Nash equilibrium. Formally:

$$u_i(\theta_i, x_1^*, \dots, x_i^*, \dots, x_n^*) \geq u_i(\theta_i, x_1^*, \dots, x'_i, \dots, x_n^*) \quad \forall x_i^* \neq x'_i, \quad \forall i \in N.$$

Note that  $u_i(\theta_i, x_1^*, \dots, x_n^*) = u_i(\theta_i, x_i^*, x_{k_i}^*(g))$ , i.e., the actions of players that are not  $i$ 's neighbors do not change her payoff.

### 4.3.1 The 2-person game: Strategic complements and substitutes

We model games with strategic complements (SC) or strategic substitutes (SS) in a 2-person setting. Specifically:

**Definition 2 *Strategic Complements (Coordination)*:** Let  $SC$  be a 2-person game where every player has an identity  $\theta_i \in \{0, 1\}$  and the set of actions  $X$ . The payoff matrix<sup>2</sup>, where  $2\beta > \alpha > \beta > 0$ , depends on each player's choices and identity as follows:

		<b>0</b>			<b>1</b>		
		1	0		1	0	
<b>1</b>	1	$2\alpha, 2\beta$	$\alpha, \alpha$	<b>1</b>	1	$2\alpha, 2\alpha$	$\alpha, \beta$
	0	$\beta, \beta$	$2\beta, 2\alpha$		0	$\beta, \alpha$	$2\beta, 2\beta$
		$\theta_1 = 1; \theta_2 = 0$			$\theta_1 = 1; \theta_2 = 1$		
		<b>0</b>					
			1	0			
<b>0</b>	1	$2\beta, 2\beta$	$\beta, \alpha$	<b>0</b>	1	$\alpha, \beta$	$2\alpha, 2\alpha$
	0	$\alpha, \beta$	$2\alpha, 2\alpha$		0		
		$\theta_1 = 0; \theta_2 = 0$					

**Table 4.1:** Payoff matrices for SC games.

**Definition 3 *Strategic Substitutes (Anti-Coordination)*:** Let  $SS$  be a 2-person game where every player has an identity  $\theta_i \in \{0, 1\}$  and the set of actions  $x_i = \{0, 1\}$ . The payoff matrix, where  $2\beta > \alpha > \beta > 0$ , depends on each player's choices and identity as follows:

Each  $2 \times 2$  coordination (anti-coordination) game can be played between two players of equal or opposite identities. There are two Nash equilibria in pure strategies and one in mixed strategies. Let us first discuss the pure strategy equilibria. For the SC case the Nash equilibria in pure strategies  $psNE = \{(0, 0), (1, 1)\}$  present conflicting preferences when the two players have opposite identities given that each likes a different action and both want to coordinate. Thus, it is not possible to Pareto rank them. However, in games between players with equal identity there is no conflict in preferences because each one likes the same action, and the equilibrium when both choose the action corresponding to their identity is Pareto dominant in payoffs:  $(1, 1)$  Pareto dominates  $(0, 0)$  if two players

<sup>2</sup>We consider a payoff structure such that a player prefers to coordinate in the disliked option than staying alone. This payoff structure is observed in the game of the Battle of Sexes, for an example of the  $n$ -person game see Szidarovszky et al. (2008).

		<b>0</b>		<b>1</b>	
		1	0	1	0
<b>1</b>	1	$\alpha, \beta$	$2\alpha, 2\alpha$	1	$\alpha, \alpha$
	0	$2\beta, 2\beta$	$\beta, \alpha$	0	$2\alpha, 2\beta$
		$\theta_1 = 1; \theta_2 = 0$		$\theta_1 = 1; \theta_2 = 1$	
		<b>0</b>			
		1	0		
<b>0</b>	1	$\beta, \beta$	$2\beta, 2\alpha$		
	0	$2\alpha, 2\beta$	$\alpha, \alpha$		
		$\theta_1 = 0; \theta_2 = 0$			

**Table 4.2:** Payoff matrices for SS games.

with identity 1 are playing, and the opposite for two players with identity 0. For the case of SS the  $psNE = \{(0, 1), (1, 0)\}$  shows no conflicting preferences when the two players have opposite identities because both are better-off when choosing the action corresponding to each of their identities, which is the Pareto dominant Nash equilibrium of the game. For example:  $(1, 0)$  Pareto dominates  $(0, 1)$  if the first player is of identity 1 and the second is of identity 0. Conflicting preferences arise when two players with the same identity interact, because both of them like the same action and want to anti-coordinate.

Let us now consider the mixed strategy equilibrium. For the SC game, the probability to choose your favorite action when playing against a player of your same identity is obtained from the corresponding payoff matrix and is given by  $\underline{q} = (2\beta - \alpha)/(\alpha + \beta)$ . When playing against a player of different identity, the result is  $\bar{q} = (2\alpha - \beta)/(\alpha + \beta)$ . Following Morris (2000) and López-Pintado (2006), these probabilities can be understood as the adoption threshold function, i.e., the proportion of neighbors making a given choice required for a player to adopt that same action.

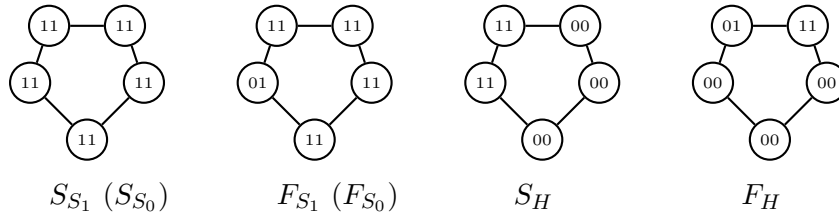
Heterogeneity in preferences gives a new insight to this by showing that the  $q$  needed varies depending on the identity of the player choosing, but not on the identity of the player(s) she is interacting with. That is, there exist  $\underline{q} < \bar{q}$ , where  $\underline{q}$  is the probability of choosing the liked action and  $\bar{q}$  the disliked action. The intuition of this result relates directly to the Nash equilibrium configurations of the network games, and it is associated to many social scenarios where the utility of affiliation is based on choices of others and not on a player's preferences, but the utility of the individual is based both on her choice and her preference. A similar result holds for the SS case exchanging the probabilities.

## 4.4 Equilibrium: Complete information

In this section of complete information we characterize the set of Nash equilibria for our network game  $\Gamma$ ,  $\text{NE}(\Gamma)$ . We develop in detail the analysis for games with SC, which symmetrically hold for SS unless the opposite is specified.

### 4.4.1 Strategies

A player in the network game  $\Gamma$  chooses an action in the set  $X = \{0, 1\}$ , the same for all her connections. The action profiles in the network are such that either all players coordinate on one action (*specialized*) or both actions are chosen by different players (*hybrid*). Having in mind the identity of the players, there are two possible categories, depending on whether all players coordinate in choosing the action for which  $x_i = \theta_i$  (*satisfactory*) or at least one player chooses  $x_i \neq \theta_i$  (*frustrated*). Thus, we have four possible configurations: (i) satisfactory specialized ( $S_S$ ) where all players coordinate on the same action, which is their preferred choice,  $x_i = \theta_i$ ; (ii) frustrated specialized ( $F_S$ ), where all players coordinate on the same action, but at least one of them is choosing her disliked option,  $x_i \neq \theta_i$ ; (iii) satisfactory hybrid ( $S_H$ ), where all players choose the action they prefer but there is at least one player with an identity different from the rest, so that both actions are present; and (iv) frustrated hybrid ( $F_H$ ) which portray both actions and at least one player chooses her disliked option. Figure 4.1 illustrates these categories for games with SC.



**Figure 4.1:** Types of configurations. The first digit refers to the identity and the second to the action.

### 4.4.2 Nash equilibrium

We characterize the set of Nash equilibria in our network game,  $\text{NE}(\Gamma)$ , in relation to players' unilateral deviations. To that end, we will call the number of player  $i$ 's neighbors choosing action 1:  $\chi_i$ ; correspondingly the number of her neighbors choosing action 0 is  $k_i - \chi_i$ . Because the threshold functions represent a number of neighbors, we will denote by  $\lceil \dots \rceil$  and  $\lfloor \dots \rfloor$  respectively the minimum higher integer and the maximum lower integer of the real number considered. Proposition 3 presents the best responses for games with SC, characterizing the thresholds to change actions. Note that for SS the relation of  $\chi_i$  with the thresholds is inverted.

**Proposition 3 Thresholds and best responses:** For an SC game, let

$$\underline{\tau}(k_i) = \lceil \frac{\beta}{\alpha + \beta} k_i - \frac{\alpha - \beta}{\alpha + \beta} \rceil, \quad (4.4)$$

$$\bar{\tau}(k_i) = \lfloor \frac{\alpha}{\alpha + \beta} k_i + \frac{\alpha - \beta}{\alpha + \beta} \rfloor, \quad (4.5)$$

defined for any degree  $k_i \in \{1, \dots, N - 1\}$ . The best response of player  $i$  with identity  $\theta_i = 1$  and degree  $k_i$ ,  $x_i^*$ , is

$$x_i^* = \begin{cases} 1, & \text{iff } \chi_i \geq \underline{\tau}(k_i), \\ 0, & \text{otherwise.} \end{cases} \quad (4.6)$$

The best response of player  $i$  with identity  $\theta_i = 0$  and degree  $k_i$ ,  $x_i^*$ , is

$$x_i^* = \begin{cases} 0, & \text{iff } \chi_i \leq \bar{\tau}(k_i), \\ 1, & \text{otherwise.} \end{cases} \quad (4.7)$$

*Proof:* For simplicity we develop the proof in terms of action 1 for the case of SC, however it extends naturally for action 0 and also for the case of SS.

Suppose that  $\chi_i \geq \underline{\tau}(k_i)$  for a player  $i \in N$  with identity  $\theta_i = 1$ . She gets a payoff

$u_i(1, 1, ((x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n)))$ . Following the payoff functions in Equation 4.1, we have:  $u_i(1, 1, ((x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n))) = u_i(1, 1, x_{k_i(g)})$ . Therefore,

$$\begin{aligned} u_i(1, 1, x_{k_i(g)}) &= \alpha(1 + \chi_i) \geq \alpha(1 + \underline{\tau}(k_i)) = \alpha \left( 1 + \lceil \frac{\beta}{\alpha + \beta} k_i - \frac{\alpha - \beta}{\alpha + \beta} \rceil \right) \geq \\ &\geq \alpha \left( 1 + \frac{\beta}{\alpha + \beta} k_i - \frac{\alpha - \beta}{\alpha + \beta} \right) = \alpha \left( 1 + k_i - k_i + \frac{\beta}{\alpha + \beta} k_i - \frac{\alpha - \beta}{\alpha + \beta} \right) = \\ &= \alpha \left( 1 + k_i - \left[ k_i \left( \frac{-\beta}{\alpha + \beta} + 1 \right) + \frac{\alpha - \beta}{\alpha + \beta} \right] \right) = \\ &= \alpha \left( 1 + k_i - \left[ k_i \frac{\alpha}{\alpha + \beta} + \frac{\alpha - \beta}{\alpha + \beta} \right] \right) \geq \beta(1 + k_i - \bar{\tau}(k_i)) \\ &\geq \beta(1 + k_i - \chi_i) = u_i(1, 0, x_{k_i(g)}) \end{aligned}$$

The remaining cases can be proven straightforwardly in the same manner.

In SC a player  $i$  wants to coordinate with the highest number of neighbors making the same choice, and prefers coordination on the action corresponding to her identity. Players with identity  $\theta_i = 1$  have incentives to choose the action they like when  $\chi_i \geq \underline{\tau}(k_i)$ . Thus, players with identity  $\theta_i = 0$  choose  $x_i = 0$  if  $\chi_i \leq \bar{\tau}(k_i)$ . We illustrate this in Figure 4.2.

In games with SS a player  $i$  wants to anti-coordinate with the highest number of neighbors making the opposite choice. This means that no specialized configuration is Nash in pure

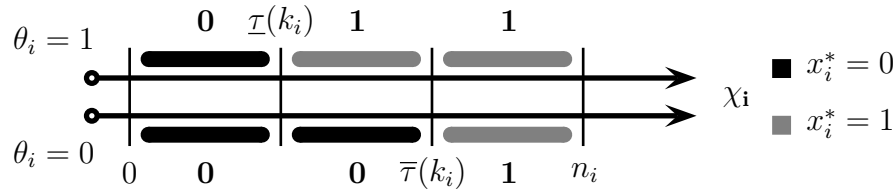


Figure 4.2: SC Thresholds

strategies. The threshold functions in SS are inverted compared to SC. A player has incentives to choose the action she likes when  $\chi_i \leq \bar{\tau}$  for  $\theta_i = 1$ , and  $\chi_i \geq \tau$  for  $\theta_i = 0$ .

With Proposition 3, we have characterized the best response of every player in terms of her identity and the actions of her neighbors. It is important to stress that this best response does not depend on the identities of her neighbors, but only on their actions. This result allows us to analyze the Nash equilibria and how do they depend on the distribution of identities, actions and network characteristics. From the best responses, it is clear that there will be very many different equilibria. Examples of these equilibria are illustrated in Figure 4.3.

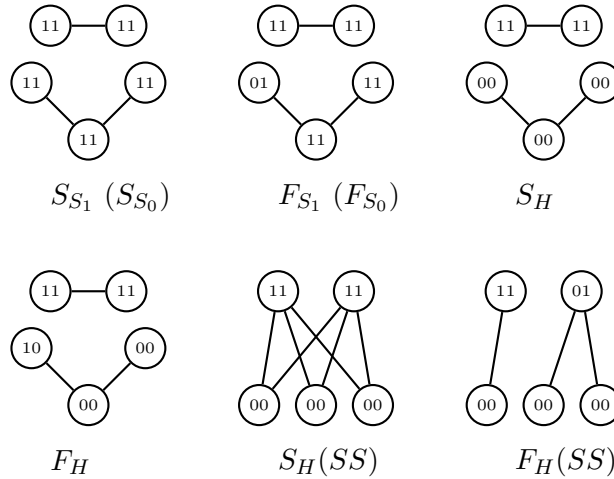


Figure 4.3: Examples of Nash equilibria. The first digit refers to the identity and the second to the action. The two bottom-right configurations are examples for strategic substitutes.

As a specific example, consider the following case: Satisfactory specialized equilibria are very restrictive. Indeed, let us assume that all players have identity 1. Then, in a  $S_S$  equilibrium, all players choose action 1. However, if for any reason a player or group of players play action 0 and for them the condition  $\chi_i \geq \tau(k_i)$  is not satisfied, we would again have a Nash equilibrium, but it would not be satisfactory, i.e., it would be frustrated hybrid. In general, if all players have the same identity, if an equilibrium is satisfactory it has to be specialized with players choosing the action they like.

There is another manner in which specialized equilibria emerge, namely when the distribution of identities is not homogeneous and condition either  $\chi_i \leq \underline{\tau}(k_i)$  or  $\chi_i \geq \bar{\tau}(k_i)$  holds for all players. As a consequence, for the same distributions of links and identities, two players with opposite identities can best respond with the same action and vice versa.

## 4.5 Equilibrium: Incomplete information

The above section is devoted to characterize the Nash equilibrium set when players know the realized neighbors' actions, i.e., the complete information framework. We study now how the structure of the network and the configuration on identities affect the set of equilibria. In this setup we introduce an incomplete information framework where players know their own identity and degree, but have no information about the whole network. Players beliefs about the rest of the network are given by a probability distribution over the connections (the formation rule) and the distribution of identities.

The consideration of a framework of incomplete information in our setup is motivated by our aim to obtain a more realistic approach of the interaction of conflicting preferences in coordination problems. Based on the assumption of a network structure *a la* Erdős-Renyi, where the size of the network is very big, incomplete information characterizes the reduction of information into a local level. Moreover, it has been shown by results found in Galeotti et al. (2010) that an incomplete information framework in network games reduces the multiplicity of Nash equilibria obtained when information is complete. As we will show below, in our case and in spite of the fact that we use weaker assumptions, the set of equilibria is much more drastically reduced, allowing for configurations in which every player chooses her preferred action when there is enough heterogeneity in the network.

We assume that both processes, formation rule and allocation of identities in the network, are independent. The probability that player  $i$  has  $k$  neighbors depends on the formation rule, which in our setup accounts for an Erdős-Renyi network structure. In addition, a player  $i$  knows that with probability  $0 \leq \rho \leq 1$ , each player in her neighborhood is of identity 1 and with probability  $1 - \rho$  is of identity 0. Then, the incomplete information set up could be understood as players having local knowledge of the network.

Given a formation rule for a network  $\Gamma$ , we denote by  $P(k_i)$  the probability that player  $i$  has  $k_i$  neighbors. Given that probability, a player  $i$  can compute the conditional probability of the degree of her neighbors given that she has  $k_i$  neighbors. The distribution of these conditional events with dimension  $k_i$  is represented by  $P(\mathbf{k}_{N(i)}|k_i)$ . For instance, if player  $i$  only has one neighbor,  $k_i = 1$ , player  $i$  should consider that her neighbor may have  $1, 2, \dots, N - 1$  neighbors. Therefore, player  $i$  will compute  $P(1|1), P(2|1), \dots, P(N - 1|1)$ . If player  $i$  has two neighbors,  $k_i = 2$ , it is necessary to consider all possible degree combination for both neighbors of player  $i$ . For example,  $P(1, 1|2), P(2, 1|2), P(2, 2|2), P(3, 1|2)$  and so on. Notice that each player with degree  $k$  has the same information of any other player with the same degree. It implies that we can assume anonymity among

the players. Having expectations over the degree of one's neighbors, as argued by Galeotti et al. (2010) is key for threshold models with incomplete information. Player  $i$  expects to choose her preferred action given her degree and the distribution of types, knowing that also her neighbors with equal or higher degree will choose their favorite action. The expectation on  $i$ 's neighbors' degree follows from the formation rule.

Henceforth, the information structure given a rule of formation and  $\rho$  is denoted by the family of anonymous conditional probabilities  $\mathbf{P} = \{P(\mathbf{k}, \boldsymbol{\theta}|k, \theta)_{\mathbf{k} \in \mathbb{N}^k, \boldsymbol{\theta} \in \{0,1\}^k}\}$ .

Given the above ingredients, the network game with incomplete information is represented by  $(\Gamma, \mathbf{P}, \rho)$ . In our benchmark individuals have two identities 0 or 1 and this identity establishes a unique pure-best response. Under incomplete information the type of each player is defined by her degree and her identity  $\theta_i$ , since it is her private information. Notice that the available information for each player includes her own identity  $\theta_i$  but not the realization of her neighbors' identities. Therefore, the bayesian game has the following features:

- The set of players  $N = \{1, \dots, n\}$
- The binary set of actions  $X = \{0, 1\}$  for  $i \in N$ . Denote by  $\mathcal{X} = \Delta(X)$  the set of mixed strategies on the support of  $X$ .
- The type set is  $T = \{0, 1, \dots, N - 1\} \times \Theta$  and denote by  $t_i$  the type of player  $i$ .
- The beliefs  $\mathbf{P} = \{P(\mathbf{k}, \boldsymbol{\theta}|k, \theta)_{\mathbf{k} \in \mathbb{N}^k, \boldsymbol{\theta} \in \{0,1\}^k}\}$ .
- Payoffs come from the expected utility criterium.

A strategy of player  $i$  is a function from her private information, i.e., her type to her action set:  $\sigma_i : \{0, 1, \dots, N - 1\} \times \Theta \rightarrow \mathcal{X}$ . A bayesian Nash equilibrium is a strategy profile  $\sigma$  such that each player plays her best response given the strategy profile of the other players. Notice that it is crucial on the valuation on the best response of player  $i$ , the computation of her expected payoff which depends on her beliefs. The beliefs rely on the network structure and the distribution of  $\theta$ , or in other words, on the formation rule which generates the network and the distribution of identities that depends on the probability of being of type 1 ( $\rho$ ) or 0 ( $1-\rho$ ).

Given that player  $i$ 's type is  $t_i = (k_i, \theta_i)$ , her beliefs are  $\mathbf{P}(\cdot|k_i, \theta_i)$ . The strategy profile of the other players is  $\sigma_{-i}$  generating an action profile of length  $k_i$ . Each of these sequences is included in the expected payoff of player  $i$  with the corresponding probability denoted by  $P_{-i}(t_{-i}|t_i)$  induced by the beliefs  $\mathbf{P}(\cdot|k_i, \theta_i)$  and  $\sigma$ . Hence, the expected payoff from choosing the action  $x_i \in \mathcal{X}$  is:

$$U_i(\theta_i, x_i, k_i, \sigma_{-i}) = \sum_{t_{-i} \in T_{-i}} u_i(\theta_i, x_i, \sigma_{-i}(t_{-i})) P_{-i}(t_{-i}|t_i).$$

The rule of formation taken into account in this chapter consists of creating a link independently with equal probability between two nodes. This is the well-known Erdős Renyi



graph formation. This rule of formation has a significant property. If the formation rule is *i.i.d* then the conditional probability when a player has  $k_i$  neighbors represented by  $P(\mathbf{k}_{N(i)}|k_i)$  exhibits increasing beliefs with respect to the partial order on  $T_{-i}$ . In other words, if one player has more neighbors, she may think that the network is more connected. Moreover as the distribution  $\rho$  is independent of the formation rule, the two conditions together may be interpreted as if player  $i$  has  $k_i$  neighbors, then any node has the same degree and  $\rho k_i$  of them have identity 1 and  $(1 - \rho)k_i$  have identity 0.

The next proposition characterizes pure symmetric bayesian equilibria fixing the distribution of identities 0 and 1. It does not give information on the existence of other non symmetric equilibria. We justify this class of equilibria since the belief formation is anonymous, therefore it is natural to assume that any player with the same private information will react with the same behavior. Namely, for a range of  $\rho$  and the above formation rule states that the different class of equilibria: non-decreasing, non increasing and both. We here focus only on the SC case for the sake of brevity, but it is apparent from what follows that a similar result with reversed inequalities holds for SS for the cases of hybrid action profiles<sup>3</sup>.

**Proposition 4 *Equilibria in incomplete information:*** *Consider the game with incomplete information  $(\Gamma, \mathbf{P}, \rho)$  with SC characteristics and the formation rule a la Erdős-Renyi. Then,*

- *There exists a pure symmetric equilibrium  $(\sigma_i^*)_{i \in N}$ .*
- *If  $\frac{\alpha}{\alpha+\beta} > \rho > \frac{\beta}{\alpha+\beta}$  then every symmetric equilibrium  $(\sigma_i^*)_{i \in N}$  is  $\sigma_i^*(k, \theta_i = 1) = 1$  and  $\sigma_i^*(k, \theta_i = 0) = 0$  for all  $k$ .*
- *If  $\rho > \frac{\alpha}{\alpha+\beta}$  then  $\sigma_i^*(k, \theta)$  is non-decreasing in  $k$  for all  $\theta$ . Moreover  $\sigma_i^*(k, \theta_i = 1) = 1$  for all  $k$ .*
- *If  $\rho < \frac{\beta}{\alpha+\beta}$  then  $\sigma_i^*(k, \theta)$  is non-increasing in  $k$  for all  $\theta$ . Moreover  $\sigma_i^*(k, \theta_i = 0) = 0$  for all  $k$ .*

*Proof:* The game  $\Gamma$  is a game with a finite number of players and a binary set of actions for each player. Moreover, the belief space depends on the Erdős-Renyi formation rule, which implies that the neighbors' degrees are stochastically independent, therefore the player's beliefs about the degree of her neighbors are weakly increasing in her own degree. As stated by Galeotti et al. (2010), this “simply embodies the idea that higher degrees for a given player are weakly correlated with higher degrees of all her neighbors” (pg. 229).

Given the above conditions, applying Milgrom and Shannon (1994) and Van Zandt and Vives (2007) the existence of at least one pure symmetric equilibrium holds. Let  $(\sigma_i^*)_{i \in N}$  be a symmetric pure equilibrium.

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<sup>3</sup>Recall that the case of specialized action profiles where all players choose one same action is not an equilibrium in pure strategies for games with SS. Therefore, only conditions 1 and 2 in Proposition 4 are satisfied for SS.

Let us now characterize a class of symmetric pure equilibria.

Denote by  $(\rho k(1), (1 - \rho)k(0))$  an action profile with  $\rho k$  ones and  $(1 - \rho)k$  zeros. In view of Equations 4.6 and 4.7, given  $\frac{\alpha}{\alpha + \beta} > \rho > \frac{\beta}{\alpha + \beta}$ , the best response under complete information, we can check that  $\rho k > \underline{\tau}(k)$  and  $\rho k \leq \bar{\tau}(k)$  (note that  $\rho k$  plays the same role as  $\chi_i$  in complete information). Therefore, for player  $i$  with identity  $\theta_i = 1$  ( $\theta_i = 0$ ), her best response is  $x_i^* = 1$  ( $x_i^* = 0$ ):

$$u_i(1, 1, (\rho k(1), (1 - \rho)k(0))) - u_i(1, 0, (\rho k(1), (1 - \rho)k(0))) \geq 0.$$

As a consequence, under incomplete information we get the same order since the beliefs do not change the increasing difference obtained under complete information:

$$U_i(1, k_i, 1; \sigma_{-i}^*) \geq U_i(0, k_i, 1; \sigma_{-i}^*).$$

In other words, when the distribution on identities is quite heterogeneous:  $\frac{\alpha}{\alpha + \beta} > \rho > \frac{\beta}{\alpha + \beta}$  then, the best response of any player  $i$  with  $k$  neighbors will be her own realization of identity. Consequently, the equilibrium strategy  $(\sigma_i^*)_{i \in N}$  coincides with her identity for all  $k$ .

To prove the third point, let us assume that  $(\sigma_i^*)_{i \in N}$  is not trivial, *i.e.*, there exists a  $k'$  and an action such that  $x' \in \mathbf{supp}(\sigma_{k'}^*)$

**Case  $\theta = 1$ .** We study now the case of players whose identity realization is 1. For that we will compute the difference between the expected payoff for player  $i$  when she has  $k + 1$  nodes with respect to the case in which she has  $k$  nodes. Recall that  $\Gamma$  is a game with SC and  $\rho > \frac{\alpha}{\alpha + \beta}$ . We then have

$$\begin{aligned} & u_i(1, 1, (\rho(k + 1)(1), (1 - \rho)(k + 1)(0))) - u_i(1, 0, (\rho(k + 1)(1), (1 - \rho)(k + 1)(0))) \\ & \geq u_i(1, 1, (\rho k(1), (1 - \rho)k(0))) - u_i(1, 0, (\rho k(1), (1 - \rho)k(0))). \end{aligned}$$

Actually, the above condition holds even for a smaller  $\rho$ ,  $\rho > \frac{\beta}{\alpha + \beta}$ . Moreover,  $u_i(1, 1, (\rho k(1), (1 - \rho)k(0))) - u_i(1, 0, (\rho k(1), (1 - \rho)k(0))) \geq 0$  then 1 is best response for  $k$  since  $\theta = 1$ . Then 1 is best response for  $k + 1$ .

Given the condition for complete information, we can now express the expected payoff with the same order since the beliefs do not change the direction of the inequality in expected terms because the player's beliefs about the degree of her neighbors are weakly increasing in her own degree:

$$U_i(1, k + 1, 1, \sigma_{-i}^*) - U_i(1, k + 1, 0, \sigma_{-i}^*) \geq U_i(1, k, 1, \sigma_{-i}^*) - U_i(1, k, 0, \sigma_{-i}^*) \geq 0. \quad (4.8)$$

Therefore,  $U_i(1, k + 1, 1, \sigma_{-i}^*) - U_i(1, k + 1, 0, \sigma_{-i}^*) \geq 0$  and consequently 1 is as well the best response when the type of player  $i$  is  $(k + 1, 1)$  and players play the symmetric equilibrium  $(\sigma_{-i}^*)$ .

**Case  $\theta = 0$ .**

Similarly to the previous case, given that  $\rho > \frac{\alpha}{\alpha+\beta}$  for all  $k$ , 1 is the best response under complete information. Then we get for incomplete information the fixed strategy  $\sigma_i^*(k, \theta) = 1$ .

In actual realizations of networks, the degree  $k$  is a non-negative integer and  $\bar{\tau}(k) \geq 1$ . Therefore, isolated players with  $\theta = 0$  will choose action 0, and for this reason we cannot conclude that the action is fixed for all players.

The last part of the proposition holds in a manner completely similar to the third point.

It is natural to compare this result to the homogeneous case studied by Galeotti et al. (2010). To begin with, our proposition relies on an assumption that is weaker than the one used in that paper. Indeed, in Galeotti et al. (2010), adding neighbors who choose one of the actions, e.g., 0 for the SC case, does not change the payoff for any player. In contrast, in our setup, this condition does not apply since depending of the player's identity and always using SC as a specific example, action 0 may be the preferred action for player  $i$  and then her payoff improves. On the other hand, if the player's identity is 1, the payoff may still change if the player has few connected nodes and  $\rho$  is at most  $\frac{\beta}{\alpha+\beta}$ . In spite of the fact that we do not have an analogue of what Galeotti *et al.* (2010) call property A, our result holds because when links are added, a fraction of them will be of identity 1, with probability  $\rho$ , and the remaining will be of identity 0, with probability  $1 - \rho$ , which suffices to guarantee the proper order on differences of payoffs. Note also that our proposition does use the same independent formation rule as Galeotti et al. (2010), so we can rightfully compare both results.

Importantly, the above proposition has crucial implications on the nature of the equilibria allowed under incomplete information. In particular we note the following: First, when the distribution on identities is very heterogeneous,  $\frac{\alpha}{\alpha+\beta} > \rho > \frac{\beta}{\alpha+\beta}$  then satisfactory hybrid configurations appear as a consequence of symmetric equilibrium. Moreover, an immediate implication of the second statement of the proposition is that the specialized equilibrium, where all players choose the same strategy, is never a symmetric bayesian equilibrium. Second, when the distribution on identities is extreme, for instance, when there is a large majority of players of identity 1 (a small minority of players of identity 1)  $\rho > \frac{\alpha}{\alpha+\beta}$  ( $\rho < \frac{\beta}{\alpha+\beta}$ ) and  $k$  is large enough, then frustrated specialized configurations are the result of symmetric equilibria.

Our result is quite strong since it holds for  $k \geq 1$ , i.e., unless there are isolated nodes, the frustrated specialized configuration are equilibria, or in other words, the strategy in which all players choose the same action arises as the symmetric bayesian equilibrium. We thus see that the introduction of identities we are proposing in our framework leads to a drastic reduction of the set of equilibria possible under complete information. In contrast with the case of Galeotti et al. (2010), our set up, in particular the existence of large enough heterogeneity, adds the satisfactory hybrid configurations as equilibria where everyone plays her favorite action.

## 4.6 Discussion

Networks of economic, technological or social interaction are nowadays recognized as a key structure to understand how actors behave and contribute to the general economic activity. A great part of the work carried out so far on this subject has focused on modeling and understanding the effects of having a (possibly complex) network of interactions among anonymous actors, where the only source of difference is the pattern of connections a given actor has.

The main novelty of this chapter is the introduction of heterogeneity in the preferences actors have over their available choices to understand how individual differences can lead to inequality and social exclusion between them. This follows from the idea that as actors differ in their preferences, they do not attain the same payoffs depending on the action chosen by them and by those around them in the social network. Thus, the interplay between the gains an actor obtains by coordinating with others and her individual preferences, can lead to inequalities in the benefits actors can achieve. In this direction, our model provides a very simple framework to understand how inequalities between actors depend on how heterogeneous in preferences the network is. While, admittedly, this is still a very simplified model, our results show that allowing for heterogeneity in the interactions on the network leads to a wealth of interesting results even when sufficiently detailed local information is available.

We here introduce a theoretical model in which actors are considered to have different individual preferences, and their preferences are based on the gains they get for coordinating in a specific choice. In many social and economic situations, actors pursue coordination (or anti-coordination) with their neighbors on a network, but also have intrinsic preferences among the available options. While actors have individual preferences for specific behavior, the presence of the benefits exerted from coordinating with others makes it profitable in some cases for actors to choose the option they prefer less and be compensated by what they gain for coordinating with others. This conflict in preferences can naturally lead to inequality in the benefits actors gain from their choices, given the interplay of the complementarities gained by choosing alike with others and one's individual preferences.

The results of this study show that the choices actors make, given their preferences and the choices of those around them, are determined by two thresholds that represent the number of neighbors they need to coordinate with in order to choose what they like or what they dislike. Intuitively, these thresholds illustrate that an actor requires less support, from those around her, to choose according to what she prefers but more pressure to choose the disliked option. As a consequence, the pressure exerted on an actor to influence her behavior by her peers is not the same when introducing individual heterogeneity in the characteristics of the actors, compared to settings where actors are homogeneous. In order to better capture the influence of heterogeneity on the micro-processes that lead to patterns of behaviors that can separate neighbors depending on their choices or integrate them, we defined the degree of heterogeneity as the proportion of actors, in the network,

having preferences for one action or the other.

In cases where actors do not know the preferences of those around them in the network (incomplete information), the model provides conditions for inequality in the benefit actors can achieve given their own individual preferences. For high degrees of heterogeneity in the population, the predicted outcome is such that every actor can choose her preferred action. Thus, the more heterogeneous the population, the more likely actors find enough support to choose the action they like. However, if the degree of heterogeneity is not high so that there is clearly a minority, actors in the minority perceive greater pressure from those in the majority. In turn, all actors are more likely to choose the behavior prescribed by the majority, although for some this is not their preferred behavior.

Thus, our study has shown theoretically that heterogeneity in preferences can lead to exclusion when the different sets of actors are of a similar size, because neither can exert enough influence on the other to make them choose different to what they prefer. This exclusion, recall, is expressed as the impossibility to interact and gain profit with those choosing differently but only with those actors choosing the same social convention. However, if the sizes differ so that a set of actors with a specific preference is a majority, the majority exerts a stronger pressure on those actors in the minority. Thus, the pattern of behaviors is such that actors are choosing the same convention although a portion of the population is not choosing what they like but what is better for them given the influence of those around them.

These results complement the findings on coordination games in homogeneous populations by showing how differences in individual preferences, in cases when there is conflicting preferences, can nonetheless result in outcomes where all actors are choosing the same behavior (i.e. no exclusion). As a consequence, although the existence of heterogeneity implies conflict and seems at first hand to trigger exclusion given that actors require more pressure to coordinate with those who are different from them and less influence to do so with those who are alike, it is the level of heterogeneity (i.e. how big the set of actors preferring one choice is compared to the other) that can actually lead to separation between them or total integration into one common choice.