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The impact of individual differences on network relations

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Chapter 2

How specialization can breed social exclusion: A model of strategic interaction between specialists and generalists in knowledge-intensive productive exchange

2.1 Introduction

Collaboration is a fundamental aspect of social and economic life. In collaboration with others people can often produce valuable goods more efficiently and achieve better results for all parties involved, than when acting on their own (Coleman, 1990). Examples of mutually beneficial collaboration range from scientific co-authorships (Jackson and Wolinsky, 1996) across R&D joint ventures between firms (Goyal and Moraga-González, 2001) to mutual help and advice in organizations (Agneessens and Wittek, 2012) or neighborhoods (Frieling et al., 2014).

One of the most prominent theoretical tools to study the emergence, process, and outcomes of cooperation is social exchange theory (Blau, 1964; Cook and Emerson, 1978; Homans, 1958). From the perspective of social exchange theory, collaborations can be analyzed as *productive exchanges* (Lawler et al., 2000; Molm, 1997). Productive exchanges are social interactions in which actors join their contributions into a common pool to achieve a goal together, aiming at outcomes greater than the aggregation of what each could have gotten separately (for a survey see Cook and Cheshire 2013). Thus, the reason for two actors to engage in a voluntary exchange is that both benefit from it (Blau, 1964).

Research on social exchange has highlighted how productive exchange benefits the actors

[0] This chapter is co-authored with Jacob Dijkstra, Rafael Wittek and Andreas Flache. It is currently in preparation to be submitted.

involved, but it also pointed to two "downsides" of social exchange. First, social exchange can increase inequality when the benefits of productive exchange are distributed highly unequally between those who participate. Second, since benefits of the exchange accrue exclusively to the participants, exclusion from exchanges can increase inequality between those who can participate in the exchange and those who are excluded from it ("principle of exclusion"; Komter 1996).

There are many examples for exclusion and inequality in exchange. In scientific co-authorships, some co-authors may attain a more favorable return on their investments in the joint product than others. Moreover, scientists who do not find co-authors to work with may lose in scientific standing because they are less productive than their peers who find co-authors to collaborate with (Jackson and Wolinsky, 1996). Similarly, in R&D collaborations bigger and more powerful firms often obtain a higher profit from a joint venture than their partner if they collaborate with a small start-up. But when big firms enter R&D collaborations with each other, this may harm the prospects of smaller firms to develop the same technology due to their lack of resources (Bojanowski et al., 2012; Goyal and Moraga-González, 2001). Finally, the exchange of help, advice or valuable information between employees in an organization is likely to benefit more those whose knowledge or skills are more in demand or who have a more powerful position in the informal network (Krackhardt and Hanson, 1993; Labun, 2012). In addition, these exchanges can marginalize those employees in the organization who are excluded because they fail to find others to exchange help, information or advice with (Balkundi and Kilduff, 2005).

Identifying mechanisms and conditions that lead to social exclusion and inequality is a major theme in theoretical and empirical research on social exchange (Cook and Cheshire, 2013). Broadly, three interrelated explanations for exclusion and inequality can be distinguished.

First, a range of theories focuses on the structural position of an actor in a network and how it can facilitate exclusion of others from exchanges and prevent exclusion of self (e.g. Skvoretz and Willer 1993; Willer 1999). The simplest example of a position that gives "strong power" (Skvoretz and Willer, 1993) is the center position in a network of three actors A-B-A . Here B can choose an exchange partner between the two A's, but the two A's have no alternative than to exchange with B.

Second, research on exchange behavior has highlighted the role of the strategic use of power resources. Molm (1990) and Molm et al. (1999) showed that the strategic use of "power-resources" through punishment and coercion can decisively affect whether resource advantages will actually be translated into favorable exchange outcomes.

Thirdly, a range of theoretical and empirical studies showed how resource inequality prior to the exchange generates unequal and exclusionary exchange structures. In exchange among unequal partners, those who are wealthy in resources are often also highly attractive as exchange partners (Flache, 2001; Flache and Hegselmann, 1999). As a result, the social inequality generated by the unequal distribution of resources is further increased through the social exclusion of relatively unattractive exchange partners, which occurs in exchanges among unequal actors.

Research on the role of network structures, the strategic use of power resources and the effects of resource inequality has importantly contributed to understanding how and under what conditions productive exchange entails social exclusion and inequality. Yet, these strands of thought do not capture a source of exclusion and inequality that is of particular relevance for productive exchange in *knowledge-intensive joint production*. Knowledge intensive joint production occurs when a high degree of in-depth specialized knowledge of a particular domain is required to be optimally productive in collaborative work.

A paradigmatic example of knowledge-intensive exchange is scientific co-production, but in certain realms like high-end technology, creative industries or the arts highly specialized knowledge may be similarly relevant. In fact, as the complexity of technology and knowledge required to be productive progresses in many domains, knowledge intensive joint production is becoming more and more important in those areas. We argue that in knowledge-intensive production, a fundamental difference in exchange outcomes can arise between actors with highly specialized expertise and those with less specialized expertise.

Our analysis builds on earlier work about the role of actors' expertise in the generation of scientific knowledge (Collins, 1990; Collins and Evans, 2002, 2007; Sellinger and Crease, 2006). This work suggests that there are two main types of actors' expertise in knowledge-intensive production: generalist and specialist expertise. Those experts who have a general knowledge of the domain but cannot solve highly complex problems within a specific subarea of the domain hold a generalist expertise. This means that generalist expertise can be very effective in simpler or more general problems, but the more specific knowledge is demanded, the less effective further investments of time and effort of a generalist will be. The *specialist* type of expertise, on the other hand, refers to actors who are proficient in a certain type of knowledge on a very specific subfield in their domain, so that their abilities allow them to perform a skilled practice of the task they are involved with, solving increasingly complex problems within that subfield.

A joint publication project between two scientists may serve as illustrative case. Consider collaboration between two experts, one is an expert in general sociological theory and the other a game theorist who is highly specialized in numerical methods for the identification of trembling hand perfect equilibria in network games. If the latter wants to make a new contribution to his specific subdomain of expertise, collaboration with the sociological theorist will be of little use for him. The theorist may invest a lot of time into the joint project, but his investments do little to foster the common project. Instead, the more time the two spend on working together, the less efficiently their common time will be invested. As they progress in addressing the highly specific problem of their paper, the specialist will need to spend more and more time on explaining details to the generalist. However, if the specialist collaborates instead with another specialist in the same subdomain, they can expect to be jointly more productive in making a new contribution than if they had worked separately. Moreover, as they progress in the project and invest more time, they may become even more productive because they can jointly focus on what they are really good at and further add to each other's knowledge.

This illustrative case suggests that experts with generalist knowledge may suffer from social exclusion if exchange is dominated by knowledge-intensive collaboration. At the same time, the example leaves open how this depends on the major preconditions of social exclusion and inequality that previous work has identified: social structure, strategic use of power resources and heterogeneity in resources. To answer these questions at a theoretical level, this chapter elaborates and analyzes a formal theoretical model of collaborative exchange in knowledge-intensive co-production. We draw upon and advance previous formal models of social exchange networks.

Our model moves beyond most existing formal modeling in the sociological literature on social exchange in its combination of four features. First, in order to address effects of social structure, actors in our model form a network of exchange relations by making decisions about who to collaborate with and who to leave out of their exchanges.

Second, we model the exchange network not as a given structure, but as resulting from strategic decisions that actors make about how to invest their expertise optimally into collaborative exchange relations. In this way, our model incorporates the strategic use of exchange resources that Molm (1990) and Molm et al. (1999) highlighted.

Third, our model conceptualizes actors' investment into collaborative exchange relations as a continuous variable. That is, actors make decisions about the allocation of their resources across a range of available potential partners, just like in decisions scientists make about the allocation of time across different co-authorships, or in decisions firms make about the allocation of financial resources across different R&D partnerships.

Fourth, our model allows for heterogeneity in the type of expertise between actors. In this way, we can address whether and under what conditions expertise heterogeneity implies similar structures of social exclusion than previous work found with regard to resource heterogeneity. At the same time, heterogeneity in expertise is different from resource heterogeneity, because experts differ not in how much of a specific resource (e.g. knowledge) they have, but they differ in how specialized that knowledge is on a particular domain.

In the remainder of this chapter, we first describe the theoretical framework of our modeling work and relate it to previous formal models of social exchange, in Section 2.2. In Section 2.3 the model is presented. Section 2.4 analyzes the network structures that emerge from the interactions of the different types of experts. In Section 2.5 we describe conditions under which different exchange network structures between actors arise and how these generate social exclusion and inequality in exchange outcomes. We conclude with a discussion of the implications and limitations of the study in Section 2.6.

2.2 Framework

2.2.1 Exchange and network emergence

Our theoretical framework builds on two lines of work examining how network structures emerge from actors' individual characteristics and their choices in partner selection: *social exchange theory* (from sociology) and *strategic link formation* (originating from economics but recently increasingly adopted by sociologists; see Dogan and van Assen 2009). Elaborations of the theories differ in the extent to which participants of an exchange are modeled as perfectly or imperfectly rational and strategic. However, both social exchange theory and the theory of strategic link formation start from the assumption that individual actors' strive to use their structural position and their resources in order to obtain optimal outcomes for themselves from the exchange. Our study integrates both lines of research for the particular case of collaboration networks in knowledge intensive production.

Most research in social exchange theory has focused on effects that the structural position of actors in a network has on the distribution of exchange outcomes (Cook and Whitmeyer, 1992). Actors with more alternatives to obtain resources, due to their connections in the network, are less dependent and thus have more power (Cook and Emerson, 1978; Cook et al., 1983). When considering the network as a fixed structure of opportunities (Cook and Cheshire, 2013), there is a general and consistent finding in the theoretical and experimental research on social exchange: The relative position in a network of exchange relations produces differences in the relative use of power. This is typically manifested in the unequal distribution of rewards across positions in a social network of possible exchange relations (Bienenstock and Bonacich, 1992; Cook and Emerson, 1978; Cook et al., 1983; Friedkin, 1992; Markovsky et al., 1993, 1988; Molm and Cook, 1995; Skvoretz and Willer, 1993).

An actor's position in the network gives her a benefit in terms of the access she has to valuable resources and the power she can exert over others to achieve better results. Applied to collaborative exchange, this means that actors consider their own and others' abilities to influence outcomes within given structural constraints, when choosing *with whom* to collaborate (cf. Cook and Emerson 1978; Dijkstra and van Assen 2006, 2008; Emerson 1962; Molm and Cook 1995).

Social exchange research has predominantly assumed the network to be exogenously given (Cook and Cheshire, 2013; Cook and Whitmeyer, 1992), considering the pattern of connections in a network structure as a sort of a market with restrictions determining who can exchange with whom (van Assen, 2001). The limitation of this approach is, as pointed out by Jackson (2008), that one cannot investigate how networks are formed in the first place. Thus, by modeling the structure of social interactions as exogenously given, research on social exchange has not taken into account the strategic use of resources (Molm, 1990; Molm et al., 1999). Therefore, Molm's program integrating the strategic use of power resources with the effects of social structures can only be carried out incompletely if the structure of an exchange network is assumed to be static.

To study network formation with respect to productive social exchanges we integrate into our modeling the techniques from an increasingly growing body of research in economics (for surveys of this literature see Goyal 2007; Jackson 2010; Vega-Redondo 2007) on modeling strategic link formation in networks. More recently, sociologists have further built on and advanced this line of modeling (Braun and Gautschi, 2006; Dogan and van Assen, 2009; Dogan et al., 2009). In this line of work, actors are modeled as rational and strategic decision makers, albeit with limited ability to anticipate. Dogan and van Assen (2009) find that models of rational behavior can predict final allocations well, suggesting that actors' behavior in pure exchange situations is rational at the dyadic level. Their evidence also shows that exchange opportunities lead to large differences in actors' payoffs.

The common denominator of models of strategic link formation is that rational actors are not assumed to be governed by structural factors operating behind their backs (Schelling, 1978), i.e., a fixed network structure. Instead, theories of strategic link formation can be seen as an implementation of Coleman (1990)'s program to explain macro-level structures (i.e. patterns of network relations) through the individual actions and dyadic interactions that brought them about. This, in turn, allows us to address how actors' choices, with whom to exchange and how much of their resources to use in joint collaborations, produce social exclusion and inequality. Dogan and van Assen (2009) follows this approach by studying what happens to both the network structure and actor payoffs when actors change their links to maximize their payoffs. The authors observe that only few network structures are stable, so that no actor wants to create new relationships or terminate any existing connection.

Our model goes in line with these works and takes a similar approach to modeling the decisions of actors to change their links, in order to understand how individual behavior affects networks in productive exchange settings. Our work brings into the discussion of network stability a fundamental exchange relationship, where actors can simultaneously maintain different exchange relationships with others and use their resources and skills to jointly produce outcomes. Some main differences in our model, compared to the current work in sociology, relate to the exchange opportunities actors have. On one hand, we do not restrict interactions to a one-exchange rule, so commonly used in exchange literature. We consider situations in which actors can simultaneously maintain different exchange relationships with others, such as settings of co-authorship and R&D.

A study that is closely related to our work, in terms of the type of emergent network relationships that are modeled, is the co-author model by Jackson and Wolinsky (1996). In the co-author model, each actor is a researcher who spends time working on research projects. If two researchers are connected, then they are working on a project together. Each actor has a fixed amount of time to spend on research, and so the time that an actor spends on a given project is inversely related to the number of projects that she is involved in. This is what social exchange denotes as a *negatively connected* network (Dijkstra, 2009; Willer, 1999). The synergy between two researchers depends on how much time they spend together, and the more projects a researcher is involved in, the lower the synergy that is obtained per project. The main finding in this model is that there is a tension between individual incentives and social welfare, so that actors have individual incentives to increase the number of their projects, but when the population pursues such a behavior, society as a whole is worse off.

The co-author model allows integrating the strategic change of network relations in an exchange network with effects of the emergent structural positions of actors on their exchange outcomes. However, the co-author model lacks heterogeneity between actors in their type and resources, i.e., there are no (productive) differences between actors. The entire analysis focuses on the *number* of connections that are formed. All that matters is how many collaborative relationships an actor keeps up.

Heterogeneity between actors is an important ingredient for a useful model of productive social exchange. As various authors have pointed out, social exchange is often not among equals (e.g. Cook and Cheshire 2013). It is frequent for actors to have individual characteristics that create relevant distinctions between them. These distinctive characteristics are often also the reason for inequalities and relations of power and exclusion. An illustrative case is the system of social exchanges observed in Blau (1955)'s law enforcement agency, in which competent employees exchange their advice with less competent colleagues in return for social regard. Generally, if two actors A and B are involved in an exchange, but A is more dependent on B for the achievement of a valuable outcome than B is on A, this presents a relation in which B has power over A (Cook and Emerson, 1978; Emerson, 1962). Power, as a potential outcome of differentiation, is typically defined as involving some interdependence among actors (Brass, 1984). Differentiation in the way actors can use their resources attains broader social significance than just the mere power relationship at the dyadic level (e.g. in the relationship between A and B).

In many cases, patterns of relationships (i.e. network structures) emerge where social exclusion and inequality arise as a result of social exchange among unequal actors. For instance, exchange among unequal partners makes those who are wealthy in resources also highly attractive as exchange partners. Theoretical work (e.g. Flache, 2001; Flache and Hegselmann, 1999) has illustrated how actors' search for attractive exchange partners can then entail a mechanism in which wealthy actors can choose from a wide range of potential exchange partners and therefore end up in the "core" of exchange networks, exchanging with each other in mutually highly beneficial interactions. Actors with relatively little resources are left to exchange with each other, in exchanges that are less beneficial for them. The consequence is a further increase in social inequality. It starts from the unequal distribution of resources, where some are wealthier than others, and follows to the exclusion of those partners that appear relatively less attractive. Komter (1996)'s work on gift exchange can be seen as an empirical illustration of this mechanism of social exclusion. She showed how "those who give many gifts, receive many gifts in return, but those who do not give much - often because their social and material conditions do not allow them to do so - are also the poorest receivers" (p. 299). More recently, Offer (2012) highlighted a similar process in the dynamics of personal networks among low-income families.

For our study of collaborative exchange in knowledge-intensive production, we will extend the co-author model to include heterogeneous actors differing in productive ability. With this, the decisions about exchange relations actors make in our model are not restricted to whether or not a particular relationship exists. If actors affect outcomes differently, the *strength* of the relationships formed can vary. This points to the very fundamental problem that when actors choose with whom to collaborate (e.g. with few or multiple partners), the productive abilities, the type of expertise, of prospective partners will affect their choices of how many resources to invest in different joint collaborations. Thus, collaborative relationships of varying strength naturally emerge.

As stated above, by allowing heterogeneity in the types of expertise actors have, our model provides insight into the interaction between individual actions and aggregate network structure. Our model differs in particular from previous work in that we focus on the type of expertise as the main form of heterogeneity between actors. Next, we elaborate how the distinction of generalist and specialist expertise is integrated into the model and we formalize the model assumptions.

2.3 The model

In this section we formally model the way rational actors form productive exchange relationships between them. Actors are represented by the nodes in a network and a link between them represents a joint task (i.e. a collaboration) they perform. We are considering weighted networks in which the intensity of the links represents the amount of resources actors invest in the exchange relation under consideration. Moreover we integrate two choices actors make: whom they partner with (link *existence*) and *how much of their resources* they allocate to each of their collaborations (link *intensity*). Link existence and link intensity are decided simultaneously by the pair of allocation decisions made by two (potential) exchange partners. If at least one of them allocates no resource to the (potential) collaboration with the other, the collaboration does not come about. If both allocate resources to the collaboration, the size of these allocations determines the link intensity and the outcome of the productive exchange for each partner. The actors' *types of expertise* determine how their resources affect the value of the outcome from the productive exchange.

We first describe how generalist and specialist *types of expertise* are modeled. Then we proceed to the network game, which models how actors form their collaborative relationships.

2.3.1 Types of expertise

Differentiation in the types of expertise affects the structure of relationships that emerge, when actors can perform joint tasks and are able to choose with whom they want to do so. Collins and Evans (2007) distinguished two main types of actors' expertise: *generalists* and *specialist*. We consider actors' expertise to be a continuous range that in the lower levels' will be denoted as the generalist type and in the higher levels' will be denoted as the specialist type. Basically, what our modeling of expertise covers is that individuals go through different stages of expertise depending for instance on the phase they are in, in terms of their professional career. As time passes, and given the skills and talents of specific actors, the level of expertise might increase with regard to a specific domain of knowledge-intensive production. However, without loss of generality in our model we assume that actors have a given type of expertise for the task they are to perform.

The generalist type of expertise characterizes those actors who have a general knowledge to perform a task but their abilities associated to that knowledge, do not allow them to provide deep insight into solving complex problems within a specific knowledge-intensive domain. This is also denoted as *beermat knowledge*, as illustrated in Example 1, taken from Collins and Evans (2007):

Example 1 ***Generalist type of expertise:*** *Consider the following explanation of how a hologram works: A hologram is like a 3 dimensional photograph you can look right into. In an ordinary snapshot, the picture you see is of an object viewed from one position by a camera in normal light. The difference with a hologram is that the object has been photographed in laser light, split to go all around the object. The result is a truly 3 dimensional picture! This explanation, found on a beermat made for the Babycham Company in 1985, appears to give an answer to the question What Is a Hologram? It is capable, presumably, of making at least some people feel that they now know more about holograms. The words on the beermat are not simply nonsense nor could they be taken to be, say, a riddle or a joke. Presumably there are people now alive who have studied the beermat and, if asked: Do you know how a hologram works? would reply: Yes, whereas immediately before they had read the beermat they would have answered: No, to the same question (Collins and Evans, 2007, pg. 18).*

The hologram example is illustrative of the generalist type of knowledge. An actor who possesses it can have a *shallow* comprehension of a topic, which allows her to use it for basic applications. Nonetheless, this knowledge is not enough to apply it in a deeper way. Thus, the generalist type of expertise is such that an actor cannot go beyond a certain point of depth to perform more complex tasks without becoming less effective in her contribution to them. This means that generalist expertise can be very effective in simpler problems but the more there is demanded from it, the less effective it will be. In our model we therefore assume generalist expertise to yield *decreasing marginal returns* to effort, meaning that with such type of expertise an actor's extra unit of effort in a task will have a lower effect than the previous unit. In Figure 2.1(a) it is illustrated the returns of a generalist when she uses her resources in a specific knowledge-intensive domain.

The specialist type of expertise, on the other hand, refers to actors who are proficient in a certain type of knowledge. This type of expertise, according to Collins and Evans (2007) requires immersion such as that needed to master scientific or technical knowledge. Actors who are specialists have the ability to perform a skilled practice of the task they are involved with. This *skilled* performance refers to the ability of deepening knowledge in such ways that actors can tackle difficult tasks and solve complex problems. A limitation for the specialists is that they cannot connect to broader problems and therefore miss important input. However, we argue that for knowledge-intensive productive exchanges, the particular expertise in a domain is what makes the contribution of the specialist so effective.

In our model this type of expertise is represented by a production function (i.e. a relationship between effort and outcome) with *increasing marginal returns* to effort. Thus, an extra unit of effort exerted by an actor with specialist expertise will be *more* valuable than the previous unit, since it allows for more profound problem solving and task performance. As pointed before in our example of scientific co-authorship, an actor who is a specialist in a domain of knowledge can be inclined to invest even more time on a project in that domain, because she can expect to become more productive by focusing on what she is really good at . An illustration of this is shown in Figure 2.1(c).

Although the two main types of expertise addressed in the literature are the generalist and the specialist, we consider them to be the two extremes of a range of types of expertise. In other words: we conceive of expertise as a continuum, going from levels of generalist expertise all the way to levels of specialist expertise, passing through what we denote as *semi-specialist expertise*, see Figure 2.1(b).

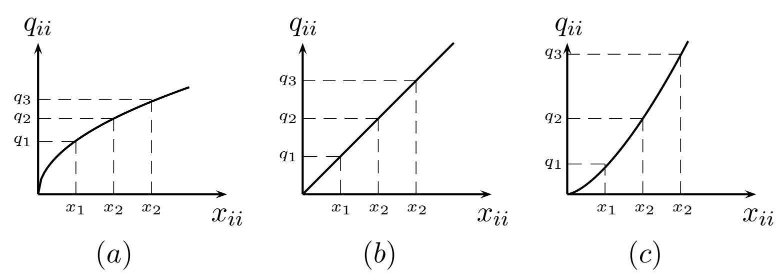

Figure 2.1: (a) generalist, (b) semi-specialist and (c) specialist type of expertise. The horizontal axis represents units of resources used by an actor in a task and the vertical axis levels of outputs achieved with these resources, given a fixed and strictly positive investment by an exchange partner.

2.3.2 Strategic link formation

We now proceed to present our model of productive exchange. Consider the undirected social network (N, g). The set $N = \{1, \ldots, n\}$, where $|N| \geq 2$, represents the actors interacting. In the remainder "actors" will be called "players", in accordance with game theoretical usage. Also, we will denote by productive projects (or just projects) the tasks or collaborations between players in our network game. The set of players is fixed throughout the analysis, so we represent the network by the set of links, g. This means that we assume, for simplicity, that no player will enter or leave the set of participants. Thus, the population is fixed.

Prior to the start of the game, players are informed about the size of the set of players and the type of expertise of all players. That is, the type of expertise is common knowledge. I know my type of expertise and I know the type of expertise any other player has. Also, all other players know their and my type of expertise. The set of potential partnerships is the complete network, g^N, and any network configuration is part of the set $G = \{g : g \subset g^N\}$. In the network, if a pair of players i and j are connected by a link, it means they perform a task together, which is denoted as $ij \in g$. If there is no link between them, we write $ij \notin g$. A player i can also have a link to herself denoted as $ii \in g$, representing her performing a task alone. The set of partners a player i has is $N_i(g) = \{j : ij \in g\}$, for all $j \in N$. The cardinality of $N_i(g)$ is n_i, (which is simply the degree of node i in the network) and is endogenously determined through the interaction (i.e. the mutual choices of all pairs of players). A player can have more than one connection simultaneously and at most n.

Players from the set N play a productive exchange network game denoted by Γ. Every player $i \in N$ is *ex-ante* and exogenously endowed with a fixed amount of resources $\Omega > 0$, equal for everyone, and with an individual type of expertise $\delta_i > 0$. A player i simultaneously chooses whom to perform a task with and the amount of resources to be invested in each of her productive collaborations, expressed by the vector of allocations $x_i = \{x_{i1}, \ldots, x_{i1}, \ldots, x_{in}\}$. The allocation of resources by i can be made to two types of productive projects (e.g. tasks): *individual*, x_{ii}, and *combined* with a partner j, x_{ij}. We denote $x_{N_i}(g)$ as the vector of allocations made to i by i's partners. That is, the amount of resources each other player j allocates to the joint project they perform with i. When a player j does not wish to collaborate with i she simply allocates no resources to i.

Payoffs in the game are determined by a Cobb-Douglas production function, $u_i(\Gamma)$, which depends on the choices made by all players and their types of expertise, as follows in Equation 2.1:

$$u_i(\delta_i, \delta_j, x_i, x_{N_i(g)}) = \rho x_{ii}^{\delta_i} + \sum_{j \neq i}^{n} x_{ij}^{\delta_i} x_{ji}^{\delta_j}. \tag{2.1}$$

where $\rho > 0$ is a prime on individual production that weights the relation between individual and combined outputs.[1] The type of expertise, δ_i, weights the allocations made by player i to her productive interactions, given the allocation of the exchange partner (or given ρ). Note how this utility function captures the essential feature of productive exchange, where players cannot benefit unless both partners to the exchange contribute. This condition on productive exchange is motivated by previous works (Lawler, 2001; Molm, 1990, 1994b), in which players do not bargain or negotiate the exchange of resources but

[1]For two players i and j, if $x_{ij} > 0$ and $x_{ji} = 0$, no link is formed between them and the resources invested by i in the failed exchange are lost. However, the resources invested by a player in individual production do not need reciprocity, but are multiplied by ρ. Coleman (1990) models his study of social exchange assuming a $\rho = 1$. In our case, by allowing for multiple values of the prime on individual production we cover a wider set of productive scenarios. For details of the analysis see Appendix A.

participate in reciprocal (and contingent) acts of giving and receiving resources of value. Particularly, the failure of reciprocity in such contexts results in the termination of the relationship. Thus, without reciprocity (zero allocations to joint projects) no productive exchange can occur.

Our utility specification captures several strategic scenarios in a simple way and allows us to observe how players' payoffs are affected by the choices of others given the distribution of individual types of expertise. Since payoffs motivate the choices players make (i.e. with whom to perform a task and how much to allocate in it), we can develop an understanding of how the resulting network structure is affected by expertise heterogeneity of interacting players.

In our setup, the productive interaction allows for endogenous differentiation of resource allocations per project. This means that one same player can be part of multiple joint projects without symmetrically distributing her resources between them (note that such symmetry is assumed in the co-author model of Jackson and Wolinsky 1996). Thus, we frame our network structures as weighted networks and define the output of a productive project between i and j by the link intensity q_{ij}, such that $q_{ij} = q_{ji} = x_{ij}^{\delta_i} x_{ji}^{\delta_j}$ (compare to Equation 2.1). The output of an individual productive project for player i is given by the link intensity q_{ii}, such that $q_{ii} = \rho x_{ii}^{\delta_i}$. In consequence, what an actor produces individually ranges between, $(0, \rho\Omega^{\delta_i})$ and what a pair of actors can produce together in a joint collaboration ranges between $(0, \Omega^{\delta_i+\delta_j})$. These ranges of production are illustrated by the intensity of the link actors have between them (joint projects) or to themselves (individual projects). Link intensity is particularly relevant as a measure of cohesion for the relationship between actors (Lawler, 2001; Lawler and Yoon, 1993, 1996, 1998). The higher the intensity of an output the greater the benefits the players involved will receive from it. This makes a relationship stronger, or in terms of social exchange theory, more cohesive.

Players in our game Γ decide on the allocations of their resources across (potential) productive exchanges with all other players (including themselves). These allocations are denoted by vectors x_i. A unilateral deviation by player i changes her choice x_i to choice x_i', a reallocation of her resources between her projects, where $x_i \neq x_i'$. We call the collection of allocation vectors of all players (one for each player) an "allocation profile", and denote it by $(x_1, \ldots, x_n)$. When no player has incentives to unilaterally deviate from a given allocation profile $(x_1^*, \ldots, x_n^*)$, this profile is a Nash equilibrium. Formally:

$$u_i(\delta_i, \delta_j, x_1^*, \ldots, x_n^*) \geq u_i(\delta_i, \delta_j, x_1^*, \ldots, x_i{}', \ldots, x_n^*) \quad \forall \;\; x_i' \neq x_i{}^*, \quad i \in N.$$

An equilibrium, as illustrated by Schelling (1978) is a situation "in which several things that have been interacting, adjusting to each other and to each other's adjustment, are at last adjusted, in balance, at rest" (pg. 25). The equilibria in our model can be understood as the stable points in which the adjustments of the different players regarding their partner selection and resource allocation are at rest. Or, as pointed out by Coleman (1990), an equilibrium is the point in a social exchange in which the discrepancy between interest and control is reduced. No player wants to change her behavior given what the rest of the players are choosing at that point. The equilibrium requirement can be seen as a minimal condition for an exchange outcome to be consistent with the (myopic) rational self-interest of the players involved. If the outcome is not a Nash equilibrium, then at least some players would expect to gain from reallocating their resources and would do so.

Note that in our model a deviation (i.e. a change of behavior in the way a player uses her resources) can lead from one network structure to another without changing the set of links that are formed, but only the link intensities (i.e. the production outcomes). That is, even if the same productive exchanges are maintained, the resources allocated to each are not the same as before, resulting in differences of how cohesive relationships are between the players. Now we proceed to the description of equilibrium for the productive exchange interactions.

2.4 Equilibrium

In this section we describe the set of Nash equilibria for the network game Γ, NE(Γ). Our equilibrium analysis is carried out for the one-shot game with complete information. We first define the set of strategies players have. Then we address the 2-person game and extend the description to networks of size $n \geq 2$.

2.4.1 Strategies

A player in the network game Γ chooses an allocation vector x_i. Based on the types of projects a player invests her resources in we can group the allocation profiles. She either invests only in her *individual* project ($x_{ii} = \Omega; \sum_{j \neq i} x_{ij} = 0$), only in combined projects with others ($x_{ii} = 0; \sum_{j \neq i} x_{ij} = \Omega$), or both in individual and combined productive projects ($x_{ii} > 0; \sum_{j \neq i} x_{ij} > 0$), where $x_{ii} + \sum_{j \neq i} x_{ij} = \Omega$.

The strategies of all players together induce the resulting network structure. Thus, we have *four* possible network configurations as illustrated in Figure 2.2: (i) the *empty* network ($\emptyset$) where no player allocates resources to individual production and none of their intended combined projects is reciprocated (this configuration emerges due to complete coordination failure); (ii) *no exchange* networks (N_E), where at least one individual project is formed but no joint projects exist; (iii) the *full exchange* networks (F_E), where at least one joint project is formed but no individual projects exist; and (iv) the *hybrid exchange* networks (H_E) where at least one individual project and one joint project are formed.

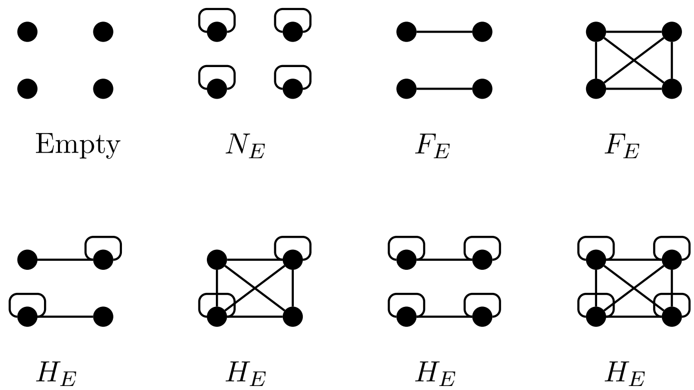

Figure 2.2: Resulting configurations. A connection between two nodes shows a combined project and a loop around a node shows an individual project. The defined strategies and resulting configurations illustrate the relation between allocation profiles and resulting networks. This categorization helps focus our attention on the relation between the productive projects that are created and equilibrium in the network game. Our main results are expressed in terms of the types of productive projects formed between the players and the level of cohesion each project portrays, expressed by the amount of resources exchanged between the parts involved in the collaboration.

2.4.2 Nash equilibrium

To describe the set of Nash equilibria, NE(Γ), we consider the optimization of the payoff function $u_i(\Gamma)$, constrained by the endowment of resources, $x_{ii} + \sum_{j \neq i}^{n} x_{ij} \leq \Omega$. That is, we assume rational actors who create productive collaborations with others and allocate resources to their projects in order to achieve the maximum individual benefit. Proposition 1 presents the best responses in terms of individual and combined productive projects for a player $i \in N$. After the descriptions of the best responses, Lemma 1 gives the conditions under which any dyadic interaction is formed. This is specified for any player i given her type of expertise $\delta_i \gtreqless 1$ (i.e, generalist, semi-specialist, or specialist). In Lemma 2 we give the conditions for multiple productive projects to form simultaneously; and the optimal number of projects a player is part of, given her type of expertise. Through these results we illustrate the Nash equilibria for our game.

Proposition 1 ***Best Responses in*** Γ***:*** *For a productive exchange network game, the proportion of resources a player i allocates to a project is equal to the proportional productivity of the given project compared to her total productive output in equilibrium. Therefore, the best response of player ii on her allocations to an individual project, x^*_{ii}, is:*

$$x^*_{ii} = \frac{\rho x_{ii}^{*\delta_i}}{\rho x_{ii}^{*\delta_i} + \sum_{j \neq i}^{n_i} x_{ij}^{*\delta_i} x_{ji}^{*\delta_j}} \ \Omega, \tag{2.2}$$

The best response of player i on her allocations to a combined project with j, x^*_{ij}, is:

$$x^*_{ij} = \frac{x_{ij}^{*\delta_i} x_{ji}^{*\delta_j}}{\rho x_{ii}^{*\delta_i} + \sum_{j \neq i}^{n_i} x_{ij}^{*\delta_i} x_{ji}^{*\delta_j}} \ \Omega. \tag{2.3}$$

Proof: The proof for Proposition 1 is presented in Appendix A.

In the productive exchange network game, a player wants to create individual and/or combined projects in order to maximize the aggregate output of her resources. The best response functions in Proposition 1 are implicit expressions of x^*_{ii} and x^*_{ij}. They shows that if a player i is best responding, the proportion of resources she allocates to a given project with j (Equation 2.3) is equal to the proportional productivity of that project, compared to her total productivity. The same holds for the allocation i makes to her individual project (Equation 2.2). Thus, the greater the output of a productive project, say $q_{ij} > q'_{ij}$, the more resources, $x_{ij} > x'_{ij}$, i allocates to such project.[2]

This result goes along the lines of the algorithm developed by Cook and Whitmeyer (1992), consistent with power-dependence theory, which they referred to as the "equi-dependence" formulation. It indicates equilibrium points in the network at which the dependence between exchange partners reaches a "balance". These authors argue that social exchanges in a network proceed toward an equilibrium point at which partners depend equally on each other for valued resources. For our productive exchange case this is measured by the link intensity of the exchange. That is, in our formulation power and dependence can be reached even between players who differ in their type of expertise (i.e. levels of power), through differences in the allocation of resources each puts into a combined project between them. The more productive the resources of a player, because she is more specialized to perform a certain task, the less she needs to allocate in a joint project to reach an equi-dependent equilibrium point with a partner.

The argument in Proposition 1 links directly to empirically observed findings in productive exchange experiments (Lawler, 2001). These studies have shown that the more cohesive a relationship is (in our case, the higher the productive output) the more inclined a player is to invest greater resources in that relationship. This in turn gives more incentives to her partner to do the same, giving cohesive relationships a higher likelihood of becoming stronger. In summary, Proposition 1 addresses two main aspects of our analysis. On one hand it points to the possibility that any player can collaborate with another player independently of their type of expertise, as long as the resources they use for their productive interaction are enough to reach a point of "equi-dependence". On the other hand, those players who establish stronger collaborations are more likely to receive higher resources from their productive partners, making these relationships more cohesive.

Notice that so far, our model points to the idea that the more productive an exchange can be, the stronger the incentives players have to allocate resources to it. However, Proposition 1 does not imply that players necessarily use all their resources in a single relationship, neither that they will reciprocate all potential partners. It is possible that

[2] A unilateral deviation by a neighbor j, increasing her investment to a common project with i, gives incentives to i to make an increasing unilateral deviation as well, given $\frac{x_{ii}}{x_{ij}} = \frac{x_{ii}^{\delta_i}}{x_{ij}^{\delta_i} x_{ji}^{\delta_j}}$ and $\frac{x_{ij}}{x_{ik}} = \frac{\rho x_{ij}^{\delta_i} x_{ji}^{\delta_j}}{x_{ik}^{\delta_i} x_{ki}^{\delta_k}}$. Then, $\frac{\partial x_{ii}}{\partial x_{ji}} \leq 0$, $\frac{\partial x_{ik}}{\partial x_{ji}} \leq 0$, and $\frac{\partial x_{ij}}{\partial x_{ji}} \geq 0 \;\; \forall\, i \in N$ and $j, k \in N_i(g) : j \neq k$.

the same player forms multiple relationships with different levels of intensity or focalizes her devotion to a single exchange. For instance, organizational studies have observed that strong ties between business units facilitate the transfer of complex knowledge, whereas weak ties are sufficient for less complex knowledge (Brass et al., 2004; Hansen, 1999), suggesting scenarios where both types of relationships coexist. In the following section we present how the choices of having few more intense relationships *versus* having multiple less intense relationships take place for any dyadic interaction through the 2-person game.

2.4.3 Dyadic interactions: The 2-person game

In the 2-person productive exchange game, a player chooses to allocate her resources Ω, (i) in producing alone, *no exchange*, which gives her a payoff $u_i(N_E) = \rho\Omega^{\delta_i}$, (ii) in producing a combined project with j, *full exchange*, obtaining a payoff $u_i(F_E) = \Omega^{\delta_i} x_{ji}^{\delta_j}$, or (iii) in producing both types of productive projects, *hybrid exchange*, with a payoff $u_i(H_E) = \rho x_{ii}^{\delta_i} + (\Omega - x_{ii})^{\delta_i} x_{ji}^{\delta_j}$. Lemma 1 shows that a player i's best response to her interaction with a partner j, depends on i's and j's type of expertise. Note that the best response is expressed in terms of what player i invests in her own individual project.[3]

Lemma 1 ***Optimal allocations in a dyad:*** *There is a monotonic relation between the types of expertise of the players interacting in an exchange and the level of cohesion (productive output) of the exchange relationships. In equilibrium specialists can establish more cohesive productive relations than generalists. Therefore, the optimal allocations in a dyadic interaction for a player i who has a specialist type of expertise ($\delta_i > 1$), is:*

$$x_{ii}^* = \begin{cases} 0, & \textit{iff } x_{ji}^{*\delta_j} > \rho, \\ \Omega, & \textit{otherwise.} \end{cases} \tag{2.4}$$

The optimal allocation in a dyadic interaction for a player i who has a semi-specialist type of expertise ($\delta_i = 1$), is:

$$x_{ii}^* = \begin{cases} 0, & \textit{iff } x_{ji}^{*\delta_j} > \rho, \\ \in [0, \Omega], & \textit{iff } x_{ji}^{*\delta_j} = \rho, \\ \Omega, & \textit{iff } x_{ji}^{*\delta_j} < \rho. \end{cases} \tag{2.5}$$

The optimal allocations in a dyadic interaction for a player i with decreasing productive capacity ($\delta_i < 1$), is:

$$x_{ii}^* = \Omega[1 + (\frac{1}{\rho})^{\frac{1}{1-\delta_i}} x_{ji}^{*\frac{\delta_j}{1-\delta_i}}]^{-1} \quad \forall\ x_{ji}^{*\delta_j} > 0. \tag{2.6}$$

[3] As shown in the proof of Lemma 1 this analysis can be made, with no loss of generality, for $\hat{\Omega} \leq \Omega$ available resources.

Proof: The proof for Lemma 1 is presented in AppendixA.

Lemma 1 specifies how Proposition 1 can be understood for players with different levels of specialization. It shows that the productivity of a project differs considerably for different types of expertise, depending on the players' investments. The intuition behind Lemma 1 can be described in two parts: (*i*) a specialist expects higher allocations from a potential partner j than a generalist, before she reciprocates with a positive allocation to a combined project; and (*ii*) a specialist reciprocates to a combined project with a higher allocation than a generalist. For both cases the semi-specialist can behave as a specialist or a generalist depending on the allocation her partner makes. The first implication, (*i*), states that depending on her type of expertise a player i requires specific levels of allocations from a potential partner in order to reciprocate (see Equations 2.4 through 2.6). Observe that in order to give a positive allocation of resources to a combined project, players with specialist expertise (i.e. increasing returns to own effort: $\delta_i > 1$) require their potential partner's allocation to be greater than the prime on individual production. For players whose type of expertise is generalist (semi-specialist) the minimum contribution can be lower than (equal to) the prime. Thus, specialists expect more from a potential partner than generalists do, in order to team up with such a partner.

The second implication (*ii*) states that a player with specialist type of expertise, in equilibrium, has two corner solutions: allocate all resources either to her individual project (if the partner allocates insufficient resources to their joint task) or to the combined project (otherwise). Players with generalist type of expertise have a unique interior solution: allocate a positive amount of resources to both the individual *and* the combined projects. Players with semi-specialist type of expertise have either the two corner solutions or a continuum of interior solutions. If a semi-specialist player receives what is needed to sustain an exchange ($x_{ji}^{\delta_j} = \rho$), this player allocates also the necessary amount of resources ($x_{ij}^{\delta_i} = \rho$), and uses the rest for individual production (behaving similarly to generalist players). However, if this same player receives from her partner a greater amount of resources, she will best respond by allocating all her available resources into the combined project (behaving similarly to specialist players). Thus, once a player has chosen to team up with a partner and collaborate together, the specialists are more inclined to put higher levels of effort into the collaboration than the generalists, because this makes their project more productive.

From Lemma 1 we can summarize then a relation between levels of cohesion in the productive relations and types of expertise. Although it is necessary that a potential partner j allocates a greater amount of resources to a combined project when the focal player is a specialist (i.e. the "equi-dependence point" is harder to achieve given differences in power), once player i finds that she is better off reciprocating, her allocation is also greater if she is a specialist, compared to a generalist. Therefore, in equilibrium, there is a monotonic relation between the types of expertise of the players interacting in an exchange and how cohesive a productive exchange is.

For an illustration, take our example of a joint collaboration between two scientists (i.e, a general sociologists and one specialized in numerical methods) aiming to make a contribution to the specific domain of identifying the trembling hand perfect equilibria in network games. Implication (i) means that the numerical specialist would require from a co-author to have a certain level of knowledge and effort invested in the project that would not hinder the flow of work, so that her participation can improve the outcome of the research, instead of having to spend more and more time on explaining details to such co-author.

However, a generalist would not require the same minimum amount of knowledge and effort but a lower one. Implication (ii) means that once the numerical specialist has found a co-author who can guarantee the minimum level of contribution she needs to have in the project, her best choice is to put all of her time and attention to that project. As they progress and invest more time, they may become even more productive because they can jointly focus on what they are really good at and further add to each other's knowledge. The generalist however is better off not putting *all her eggs in the same basket*, because there is only so much she can contribute into a single project before she becomes less productive.

The intersections of the best responses presented in Lemma 1 result in the Nash equilibria of the 2-person game. For an illustration of the 2-person Nash equilibria see Figure 2.3. It is straightforward to see how, depending on the type of expertise of the players involved in the interactions, the emerging configurations vary; the types of productive projects that are created and the level of cohesion (i.e. output) of such projects are different. Although it is not depicted in Figure 2.3, the case where all players produce only individual projects, the *no exchange* network, is always a Nash equilibrium. If a player expects to receive nothing from her potential partners, she has no incentives to invest resources in a combined project with her. This allocation profile is Nash independently of the expertise of the players in the network.

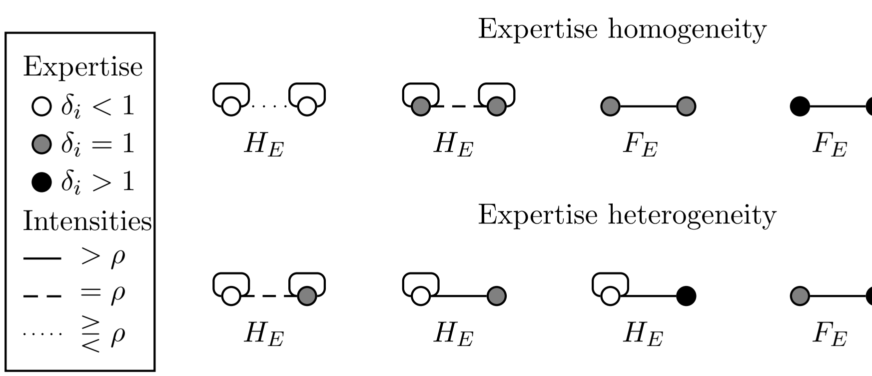

Figure 2.3: Characterization of the dyadic interactions in equilibrium. A connection between two nodes shows a combined project and a loop around a node shows an individual project. The color of the node represents its expertise. The type of line connecting two nodes represents the minimum allocation necessary by either one or both players to reciprocate to the combined exchange between them (i.e. level of cohesion).

2.4.4 Connectivity in the n-person network

From Lemma 1 we know that specialists have two corner solutions to exchange (all or nothing), generalists have an interior solution (allocate to both individual and combined exchanges), and players with semi-specialist type of expertise fall in a combination of interior and corner solutions depending on the allocations of their potential partners. Lemma 2 shows the number and classes of projects that can be created when an allocation profile is a Nash equilibrium given the expertise of the players. This Lemma gives conditions for networks in which there are $n \geq 2$ players interacting.

Lemma 2 ***Optimal number of projects in equilibrium:*** *There is a monotonic relation between the type of expertise of a player i and the number of collaborations she forms, n_i. In equilibrium, specialists establish (i.e. reciprocate to) fewer projects than generalists. Therefore, in Γ a Nash equilibrium is an allocation profile such that networks portray:*

- *"No exchange" configurations if all players are specialists and resources are low ($\Omega \leq \rho$). Thus, $n_i = 1$ if $\delta_i > 1$.*
- *"Full exchange" configurations, formed by components of size 2, if all players are specialists and resources are high ($\Omega > \rho$). Thus, $n_i = 1$ if $\delta_i > 1$.*
- *"Hybrid exchange" configurations, if all players are generalists, where all players always create an individual project. Thus, $1 \leq n_i \leq n$ if $\delta_i < 1$.*
- *"Full exchange", "Hybrid exchange" or "No exchange" configurations if all players are semi-specialists, as a function of the size of the resources, $\Omega \gtreqless \rho$. Thus $1 \leq n_i \leq n$ if $\delta_i = 1$.*

Proof: The proof for Lemma 2 is presented Appendix A.

Lemma 2 characterizes network configurations in equilibrium, given the players' types of expertise and the allocations by their neighbors. The intuition behind this Lemma is that a specialist reciprocates to fewer projects than a generalist. For the case of generalists, both individual and combined projects are created in equilibrium, resulting in a hybrid exchange network. On the other extreme, with the specialists, a single combined project is formed for every pair, resulting in a full exchange network composed by dyads. Between these two *types* of players are the semi-specialists, who behave as a combination of specialists and generalists.

If there is expertise heterogeneity, players behave as in homogeneity but configurations vary in their levels of cohesion, depending on the types of expertise of the players. That is, the specialist focalize their resources and the generalist diversify them.

Thus, we can study the way Nash equilibria depend on the distribution of types of expertise in the network. Since a Nash equilibrium is any combination of best responses, it is clear there will be very many different equilibria in any given network. Examples of these equilibria are illustrated in Figure 2.4, for four-player networks.

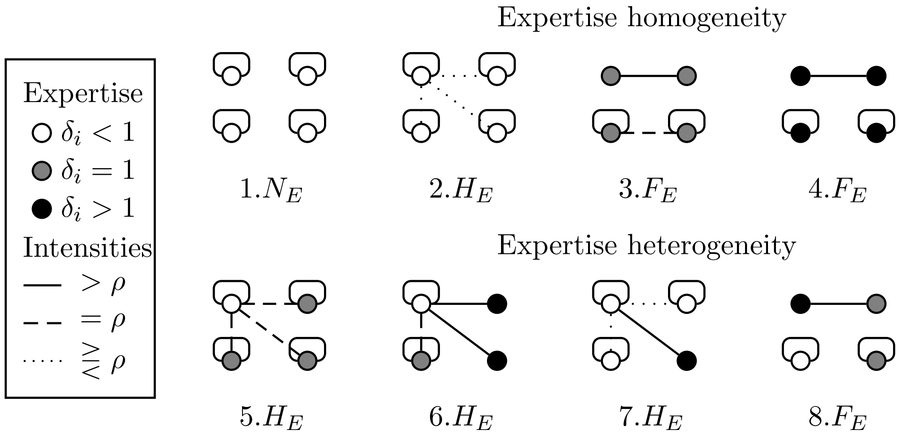

Figure 2.4: Examples of the set NE(Γ). A connection between two nodes shows a combined project and a loop around a node shows an individual project. The color of the node represents its type of expertise. The type of line connecting two nodes represents the minimum allocation necessary by either one or both players to reciprocate to the combined exchange between them.

As observed in the illustrated networks, the multiplicity of Nash equilibria varies for expertise homogeneity and expertise heterogeneity. Even when all players are identical (i.e. all are specialists or all are generalists), there is a wide range of network configurations and allocation vectors. The fist configuration in Figure 2.4 is the N_E network, which is always a Nash equilibrium for any distribution of expertise, as mentioned above. If all players are generalists, there is a wide range of equilibrium configurations, from the N_E where only individual tasks (i.e. self-links) are performed to any H_E where all players perform an individual task and they also perform up to $\frac{n(n-1)}{2}$ joint tasks.

Under homogeneity, generalists can reciprocate to productive exchanges even with the entire set of potential partners. For the case of semi-specialist players, the same is true if the allocation of resources each partner makes to a combined project is exactly ρ. Otherwise players will either form N_E networks where no combined projects are created or F_E networks where players only have one combined project and no individual production. Thus, the network is composed of dyads.[4] Finally, if all players are specialists, each participates in a single project so that the networks formed are either N_E or F_E. In the first type of network players perform a single task alone and in the second they perform a single task in a dyad with a partner.

As a matter of illustration, considering our example of general sociologists and numerical experts. If all actors are general sociologists, the depth of their knowledge is not such that it would be worth it for them to only work in one research project at a time. Instead, it is optimal for them to diversify into various collaborations, so that when their efficacy in one task runs low, they can better contribute to another. Thus, a network with only

[4]If n is odd, there is one player excluded from exchanging with others and will use her resources to produce alone.

general sociologists would portray multiple collaborations between them, such that each researcher would be part of different projects at a time. Instead, if all actors are numerical specialists, their knowledge is so specialized that they are better off either working alone if the available co-authors are not "good enough" for them, or putting all their efforts into a single research collaboration if they found a co-author who can guarantee a contribution that will help their work. Particularly, in a network with only specialists each researcher would be part of a single research project at a time, so that they can be most effective in their outcome.

When the distribution of types of expertise in the network is heterogeneous the incentives for individual players do not change compared to a network with homogeneous types of expertise. That is, the generalists can reciprocate to (multiple) projects for allocations of any size, as long as these allocations are positive. The specialists only reciprocate to a combined project if the allocation received by a partner is greater than the prime on individual production (ρ), and will create only a single productive project. The semi-specialists, depending on whether the allocations received from their partners are below, equal or above the prime on individual production create individual, multiple or a combined project, respectively. Moreover, notice that any pair of players can create combined projects as long as the conditions in Lemma 1 are satisfied, independently of their types of expertise. Specialists and generalists can collaborate together if their knowledge and resources are enough to satisfy the conditions presented above.

The main feature of expertise heterogeneity is not a difference in the individual incentives players face to form productive exchange relations compared to expertise homogeneity, but the allocation of resources (i.e. level of cohesion) across these projects that varies given the differences in the type of expertise of their productive partners. For example, observe in the illustrations of Figure 2.4 that the first three configurations under heterogeneity (networks 5, 6, and 7) have the same number of combined projects but the cohesion of the relationships (i.e. productive outputs) is not the same. It depends on the expertise of the players interacting.

In sum, specialized experts find it optimal to be part of fewer tasks than generalists. Thus, the numerical specialist, from our example, would focus on a small number of research projects at the same time. In doing this, she would be able to make ground-breaking contributions by investing her time and attention to deepening the problem at hand. Whether she would do it alone or with a partner would depend on the conditions of who the partner is and what can the partner contribute to the joint project. On the other hand, the general sociologist is better off having different projects simultaneously and giving each of them part of her attention. Given her knowledge is not so specialized the contribution of the generalist has a decreasing marginal effect. However, what our model also implies is that in equilibrium it is possible to observe different matchings between actors, as long as the above conditions are satisfied. Generalists collaborating between them or with specialists, and vice versa.

Therefore, we can say that in a setting of productive exchange, in equilibrium, players with specialist type of expertise focus on strengthening the cohesion of a single project and those with generalist type of expertise focus on the quantity of projects. Semi-specialist players, depending on the available resources exchanged, behave as either specialists or generalists. In the following section we conclude our study by analyzing how the possibility of forming small coalitions affects players behavior and the set of Nash equilibrium configurations.

2.5 Pairwise stable Nash equilibria

In the previous section we have described the network configurations that emerge in equilibrium given the types of expertise of the players in the game. To do so, we have used Nash equilibrium as the solution concept. However, in social and economic settings such as the productive exchanges studied here, players form and maintain relationships while having discretion in how many resources to devote to different relationships (weighted networks), *in a setting where a relation requires mutual agreement to be created*. Because these players are assumed to behave rationally (i.e. they are utility maximizers), it is fundamental that they form relationships that are (mutually) beneficial and to drop relationships that are not. For instance, one would expect players to talk to each other and form a productive exchange if it is in their mutual interest.[5] That is, players communicate about mutual improvements and coordinate their behavior to achieve them (i.e. coalition behavior). The idea of coalition formation for the improvement of well being has been widely used in social exchange because it is illustrative of the way actors reach points in which they are not interested in changing neither their partners nor the way they are using their resources. For instance, Emerson (1972) argued that coalition formation was a key mechanism in order to stabilize relationships.

Some standard game theoretic equilibrium notions are not well-suited for the study of network formation, as they do not properly account for the communication and coordination that is important in the formation of social relationships in networks. Thus, they do not take into account how coordination and coalition formation can occur. Jackson (2010) states that "Nash equilibrium-based solution concepts, do not capture the fact that forming a relationship or link between two players usually involves mutual consent, while severing a relationship only involves the consent of one player. Therefore, these solution concepts fail to capture the possibility that if two players each want to engage in a relationship then we should expect them to" (pg. 203). Specifically, a solution concept that accounts only for unilateral deviations (i.e. individual and uncoordinated reallocation of one's resources), includes some equilibria that are easy to be unreasonable, such as the *no exchange* network, which is always a Nash equilibrium regardless of payoffs. For an

[5] Communication between players, prior to deciding whether to exchange or not, is a natural way to address their uncertainty. This underlying communication process can be understood as what Crawford (1990) calls *explicit bargaining* or Farrell and Rabin (1996) call *cheap-talk*. That is, those cases where players communicate by sending non-binding messages with no direct payoff implications on their decisions. Tacit bargaining, as opposed to explicit bargaining, implies that players can only communicate by making offers and counteroffers that directly affect their payoffs, see Nash (1953).

illustration see Example 2:

Example 2 ***Equilibria in No Exchange***. *Consider the 2-person game where each player is endowed with a set of resources $\Omega > \rho$. Independently of the resources each player allocates to individual production, there is an equilibrium where both players invest resources to create a combined project between them, and another where both players choose not to allocate to the combined project. The last vectors of allocations form a Nash equilibrium since neither player has an incentive to make any reallocation, given the (correct) anticipation that the other player will not allocate to the combined project (see Figure 2.5). This second equilibrium does not make much sense in a social setting, where we would expect the players to talk to each other and create the combined project if it is in their mutual interest.*

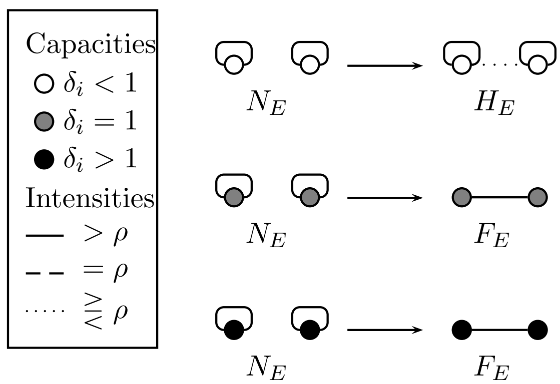

Figure 2.5: Equilibria in dyads. A connection between two nodes shows a combined project and a loop around a node shows an individual project. The color of the node represents its type of expertise. The type of line connecting two nodes represents the minimum allocation necessary by either one or both players to reciprocate to the combined exchange between them.

Jackson and Wolinsky (1996) proposed *pairwise stability* as an alternative to the Nash concept that captures mutual consent (see also Jackson and Watts 2001, 2002). A network is pairwise stable if no player wants to eliminate a relationship she has and no two players both want to form a relationship between them if they do not have one. This concept has led to the notion of pairwise stable Nash equilibrium (PNE), so that a network is PNE if it is Nash *and* pairwise stable. Particularly, PNE can be related to a game theoretic solution concept used for a wide variety of exchange situations: the *core*[6] (see Bienenstock and Bonacich 1992, 1993; Dijkstra 2009). The core is based on the assumption that no individual or coalition of players in an exchange with others should accept an outcome if by forming a coalition, they can all do better. If there is a situation where all players are assured that they cannot guarantee themselves more by leaving their current situation, that situation is said to be in the core. Applied to exchange networks, this

[6]The core was first introduced in Edgeworth (1881) and later by Von Neumann and Morgenstern (1944).

assumption involves only dyads (Bonacich and Bienenstock, 1995), so that pairs of players form coalitions. Thus, if in a network this criterion cannot be satisfied by all dyads simultaneously, such network has an *empty core*. The same dyadic strategic behavior is what defines PNE as a solution concept.

We can argue then that PNE is based on two conceptions of rationality: individual and dyadic rationality (Rapoport, 1970). Individual rationality is needed to ensure Nash equilibrium outcomes where no player will choose an allocation profile that gives her a lower outcome than what she could achieve, given the allocation of the other partners. Dyadic rationality is the same assumption with respect to pairs of players. It ensures pairwise stable Nash equilibrium outcomes.

This notion has been widely used in strategic network formation models, where links are either present or not. However, in settings such as productive exchanges, players decide how much of their resources to devote to various collaborations. Thus, the question is not only whether a connection exists or not, but what is the intensity of such connections. Particularly, in our setting of weighted networks, we need to adapt the notion of pairwise stability from a binary choice set to a continuous choice set. Thus, we use an *adapted* definition of pairwise stability in productive exchange, presented in Definition 1:

Definition 1 ***Pairwise stability in productive exchange:*** *A network is pairwise stable if no player i would benefit by any reallocation of her resources in vector x_i, and no pair of players i and j would benefit by a reallocation in x_i and x_j.*

With this in mind, Proposition 2 characterizes the equilibrium configurations that are pairwise stable, PNE(Γ):

Proposition 2 ***Pairwise stable Nash equilibria:*** *The set of PNE(Γ) is a subset of NE(Γ) where specialists form exchanges in such a way that the most expert players connect between them, then those who follow in their level of expertise, and so on, such that:*

- *Under homogeneity only N_E or F_E configurations emerge if players have specialist (semi-specialist) type of expertise, when $\Omega \leq \rho$ ($\Omega < \rho$) and $\Omega > \rho$ ($\Omega \geq \rho$), respectively. Only H_E configurations, maximally connected, emerge if players have generalist type of expertise.*

- *In heterogeneity a player best responds as if in homogeneity given her type of expertise, and the only configurations that emerge are those in which segregation by types of expertise arises.*

Proof: The proof of Proposition 2 is presented in Appendix A.

The definition of pairwise stable Nash equilibria (PNE) implies that if a network is PNE it is also a Nash equilibrium. Thus, it is straightforward that the set of PNE(Γ) is a subset of NE(Γ). Less obvious is, however, the specific pattern of productive relations and resource allocations that emerges when assuming that players will pursue, bilaterally, relationships that if formed will benefit both parts (PNE). For settings with expertise heterogeneity, PNE networks are configurations where the specialists exclude the generalists. As a consequence, these networks portray specific connection patterns in which segregation arises.

To illustrate this result let's consider some cases with homogeneous expertise. As pointed in Example 2, if players are specialists and resources are high enough to guarantee joint exchanges, a Nash equilibrium can be either a N_E network where all players have a single individual project or a F_E where all have a single combined project. But, the only Nash equilibrium that is pairwise stable is the second one. In the same way, the set of PNE networks is smaller for players with semi-specialist type of expertise. Specifically, there are no hybrid exchange networks in equilibrium. For these players, as shown in Equation 2.5, if the available resources are greater than the prime but the allocation received by a partner to a combined project is equal to the prime, a semi-specialist player best responds by reciprocating also with an allocation equal to the prime and using the remaining resources to produce alone.

However, if both players can agree on a bilateral deviation, they would be better off exchanging all their resources and investing no resources at all in individual production. Conversely, if the resources are not enough to guarantee a profitable exchange (lower than ρ), only an equilibrium where all players produce alone is PNE. Finally, if players are generalists, they always produce individually in a Nash equilibrium. In addition, they create combined projects with all other players. Thus, the only PNE is a hybrid exchange configuration completely connected. All these configurations emerge because players have the possibility to follow deviations by pairs, so that all the projects that can benefit two unconnected players are formed, resulting in the complete network.

Under heterogeneity players follow similar strategies as in homogeneity, meaning that those with specialist and semi-specialist types only create a single project with another player, and generalists create multiple projects with many other players. The main difference, between NE and PNE is that in Nash any pair of players can interact, independently of their types of expertise, as long as the amount of resources necessary for an exchange to be reciprocated is met. For example, consider a case where a player i is a specialist and two potential neighbors, j and k, both allocate more than the prime on individual production to a combined project with her. There is an equilibrium in which i reciprocates to j and another in which i reciprocates to k. However, in PNE there is only one point of equi-dependence in which i reciprocates to j if her total allocation is greater than that of k ($x_{ji}^{\delta_j} > x_{ki}^{\delta_k}$). If the allocations are equal i is indifferent.

To illustrate better what PNE implies, consider our example of how co-authors can decide on the research projects they want to be part of and with whom. As shown before, it is possible that a generalist and a specialist could collaborate together in a complex project, as long as the generalist had enough resources (e.g. knowledge, time and effort) to make the project better, otherwise the generalist could require constant explanation which would delay the process instead of boosting it up. If such an interaction becomes available and the two researches decide to collaborate, the best option for the specialist is to put all of her effort in this task.

What PNE brings into consideration is that, if there were not one but two potential partners for the specialist, the only likely outcome is such in which the specialist pairs up with the most productive partner. Thus, in a setting in which generalists and specialists interact; which is a very plausible setting in a research environment, a problem of exclusion would arise. Generalists have the intention and availability to collaborate with other generalists as well as with specialists, but the specialists would only be interested in joining efforts with other specialists, excluding generalists from their collaboration networks. As a consequence the network of relationships would result in segregation by types of expertise.

In the same vein it is natural to observe that the most specialist player connects with the player closest to her in degree of specialized expertise. This process is repeated until all specialists are paired. If the number of players with this expertise is odd, there is one who connects with a semi-specialist player, until also all semi-specialist are paired. If a semi-specialist is not paired with another player with her type of expertise, she connects with a generalist player, while all other players who are generalists connect between each other. Thus, while in Nash equilibrium a heterogeneous population can lead to multiple equilibria in which players with different expertise form combined projects, in PNE the only equilibria is such in which segregation between types of expertise arises.

In summary, the main results of our model are that in a collaborative exchange in knowledge-intensive co-production, in equilibrium: (*i*) optimal investment in a collaborative project is proportional to the overall productivity of the project (Proposition 1); (*ii*) specialists expect a higher allocation from a potential partner than generalists, before it is optimal for them to reciprocate and give a positive allocation to a combined project between them (Lemma 1); and (*iii*) specialists allocate more resources to a collaborative project than generalists, once they have found a partner to team up with (Lemma 1); (*iv*) specialists form fewer projects than generalists in equilibrium (Lemma 2); and (*v*) when actors are able to choose their partners and form coalitions to improve their well being, social exclusion will be manifested as segregation between types of expertise in the pattern of collaborations (Proposition 2).

2.6 Discussion

We have argued that differences between actors, in their types of expertise, are important in knowledge-intensive joint production, because they can decisively affect outcomes. To elaborate this argument, we proposed a model in which actors are differentiated by the level of specialization they have on the domain of the specific task they are to perform. Following sociological research on actors' expertise and the generation of scientific knowledge (Collins and Evans, 2007), we assumed that in knowledge-intensive production actors' expertise ranges across a continuum from *generalists* to *specialists*. The generalists are actors who have a general knowledge in the domain of the task they are to perform but cannot solve highly complex problems within a specific subarea. The specialists are actors who can perform a skilled practice of the task they are involved with, solving increasingly complex problems within that subarea of domain.

We applied our model to analyze populations with different expertise composition, such as those in which all actors are generalists or specialists, or those in which actors with different types of expertise interact together. This allowed us to tackle the problem of whether and under what conditions specialization in knowledge, i.e., type of expertise, implies structures of social exclusion. We first comment on our theoretical results (*i*), (*ii*) and (*iii*), which are results at the individual level. Then we address results (*iv*) and (*v*) which inform about the emerging network structure.

Result (*i*) indicated that the resources an actor invests in a collaboration with a potential partner depend on how productive the collaboration can be. Thus, it is possible that any actor can collaborate with another actor independently of their type of expertise, as long as the resources they use for their productive interaction are enough to reach a point in which they find the collaboration mutually beneficial (equi-dependence point; see Cook and Whitmeyer 1992). Moreover, within beneficial collaborations, the actors who establish stronger collaborations, say because they allocate more resources, are more likely to receive more resources from their productive partners, making these relationships more cohesive.

Results (*ii*) and (*iii*) specify differences in the conditions of forming a collaboration in terms of the type of expertise of the actors involved. Depending on how specialized they are, actors require specific levels of allocations from a potential partner in order to form a collaboration with them. Also, they reciprocate with different levels of resources once they find it profitable to join efforts in a project together. A potential partner needs to allocate a greater amount of resources to form a combined project with a specialist than with a semi-specialist or a generalist, before the specialist is willing to reciprocate. In addition, once that actor is better off reciprocating, the amount of resources she reciprocates with is also greater when she is a specialist than a generalist.

Thus, combining (*i*), (*ii*) and (*iii*) we can say that stronger collaborations are formed between actors that can influence outcomes in ways that make them more beneficial, and those actors are the specialists. For instance, in complex settings, a generalist can attempt to collaborate with a partner who possesses a highly advance knowledge about a difficult task they aim to develop. But, as our model has pointed out, unless the less specialized actor can provide a certain level of contributions (i.e. because she has a minimum level of knowledge necessary to perform the task) her involvement would actually slow down the effectiveness of the specialist. So, the expert would rather work alone or with someone else.

Following the results at the micro-level, result (*iv*) gives insight into how networks are configured in equilibrium, given the distribution of actors' types of expertise. We found that if actors pursue optimality in performing a task, behavior can differ dramatically between those who are specialists or generalists. In cases in which populations are homogeneous, say all actors are either specialists or generalists, networks will be less connected for the first and more densely connected for the second. Specialists are more productive by participating in fewer collaborations. For this reason, the optimal structure is that in which each actor forms a single productive exchange with a partner (or alone) and invest all her time and resources in achieving a strong outcome out of it. Generalists, on the other hand, reciprocate to multiple projects, so that instead of focalizing in a single collaboration they diversify and achieve better outcomes by aggregating the results of multiple exchanges.

Notice that our results, although intuitive, are surprising in some ways. At first sight, one could think that a specialist could, with relatively limited effort, add his specialty to a project. Therefore, she should also be able to be involved in relatively many projects, because her input is small and focused. Conversely, a generalist should be involved in few projects because his specialty is so restricted that it requires much effort to contribute in a significant way to a project. These considerations are precisely the arguments our model addresses. Our results point to a pattern of behavior such that, in equilibrium, specialists focus on strengthening the cohesion of a single project and generalists focus on the quantity of projects, changing the number of projects and the intensity of such projects in a network, depending on the composition of the population in terms of their types of expertise.

Our model of productive exchange also implies that when individual decisions are not coordinated, there is not necessarily exclusion in exchange network. For homogeneous populations, if all actors are specialists, they are interested in exchanging between them but focus on very cohesive relationships so that only one exchange takes place at a time for every actor involved (i.e. the network is shaped by dyads). If all actors are less specialized (i.e. generalists), they are also interested in collaborating together and aggregating multiple productive exchanges between them, instead of focalizing in a single strong collaboration.

For heterogeneous distributions of types of expertise in the network, so that there are specialist and generalists at the same time, the incentives for individual players do not change compared to a network with homogeneous types of expertise (in equilibrium). The patterns of connections show that the less specialized actors are, the less effective their contributions become for given tasks and therefore, the better for them to diversify their investments into multiple collaborations. Thus, the more generalists there are in a network, the higher the number of projects. For the case of the specialists, actors whose expertise to perform a task is greater are more inclined to pursue greater outcomes out of their collaboration interactions. This is due to their ability, which affects positively a more intense dedication to performing a task, because the knowledge and expertise of such an actor can lead to ground-breaking results that are not achievable in more cautious approaches. Thus, the more specialists in a network the lower the number of projects there are, but the stronger the collaborations will turn out (i.e. more intense).

So far, results (*i*) to (*iv*) have addressed the conditions actors need to exchange and participate jointly in knowledge-intensive collaborations. Our final section extended the analysis of our model to coalition formation and considered what happens if actors communicate about mutual improvements and coordinate their behavior to achieve them. That is, if two actors each want to engage in a relationship then we should expect them to, even if that meant rearranging the way they are using their resources. Thus, actors are free to rearrange their connections if a more profitable option is available.

With this in mind, result (v) states that when actors pursue, bilaterally, relationships that if formed will benefit both parts, networks configurations will be such that the specialists exclude the generalists, so that segregation would arise. Our model suggested that the more able an actor is to influence outcomes (i.e. specialists over semi-specialists over generalists) the more attractive that actor is as an outside option for those she is not collaborating with. In this same direction, those who are already exchanging with such an actor are less attracted to outside options.

As a consequence, the way this ranking of preferences connects individual behavior at the macro-level results in all actors preferring to exchange with others who are more specialized experts or are at least as specialized as they are. This is what generates a pattern of segregation by degree of specialization in the network. Actors are not interested in collaborating with those who are not as specialized experts as them, because they are not attractive partners, which results in actors mostly exchanging with those similarly or even more specialized. Specialists connect in this case with specialists and generalists connect with the generalists, even though the generalists would be interested in collaborating with the specialists. Particularly, once the networks are segregated, they are connected in the way pointed in result (iv). That is, networks are more connected when the population is composed by more generalists than by less.

According to result (v) the pattern of relationships, in heterogeneity, that leads to social exclusion is the following: under heterogeneity the most specialist actor collaborates with the actor closest to her in degree of specialization. This pattern in the same for all specialists until they are paired together. The specialists connect among each other, which leaves the generalists connecting only within their type. As a consequence, the equilibria that result are such in which segregation between types of expertise takes place. Therefore, those who need it the most are less likely to be reciprocated by others, especially by those with higher levels of expertise. This result mirrors the segregation by attractiveness of helping partners that previous theoretical studies of exchange networks found (e.g. Flache and Hegselmann 1999), but it extends it to the new realm of knowledge-intensive collaboration in areas with heterogeneous types of expertise.

Some potential limitations of our work warrant further discussion. Compared to other works on productive exchange (see Lawler et al., 2000), we model collaborations as dyadic interactions while other works have assumed them to be group interactions (i.e. more than three actors). Our aim was to understand link formation and the emergence of network structures using the tools developed in the game theoretic studies of networks. Accordingly, we decided to maintain the structural assumptions central to this approach. Dyadic interactions are common collaboration settings and our model can naturally be extended to larger groups in further research.

Our work also points to interesting approaches that can be pursued in further research. On one hand, testing experimentally the theoretical predictions of our model can give light on the process of how networks come about. Laboratory experiments present a useful a powerful technique for analyzing social and economic questions. Experimental studies on network formation have become more common in the past decades given they provide a main advantage in controlling variables (such as heterogeneity) that could possibly influence individual and aggregate behavior; for a survey see Kosfeld (2004).

Particularly, by studying how experimental subjects interact, we can discover in more depth how certain network structures are more likely to emerge than others, while controlling how the types of expertise in the population are distributed. On the other hand, theoretically there are many aspects that can be extended and dealt with in more detail. For instance relaxing the assumption that expertise is always observable can provide a fruitful framework in studying adverse selection. Modeling situations of uncertainty about the level of expertise actors possess can help us understand conditions under which actors have incentives to signal different expertise levels than their own to achieve higher positions of power.