Topics in inhomogeneous Bernoulli percolation

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Summary

This thesis is an investigation of some mathematical aspects of inhomogeneous Bernoulli bond percolation in two different graphs $\mathbb{G} = (\mathbb{V}, \mathbb{E})$. In each of them, we consider a decomposition $\mathbb{E}' \cup \mathbb{E}''$ of the relevant edge set $\mathbb{E}$ and, given $p, q \in [0,1]$, we assign parameters p and q to the edges of $\mathbb{E}'$ and $\mathbb{E}''$, respectively. In such formulation, one of the sets, say $\mathbb{E}''$, is regarded as the set of inhomogeneities.

The first graph $\mathbb{G} = (\mathbb{V}, \mathbb{E})$ we consider is the one induced by the cartesian product of an infinite and connected graph $G = (V, E)$ and the set of integers $\mathbb{Z}$. We choose an infinite collection $\mathcal{C}$ of finite connected subgraphs of G and consider the Bernoulli bond percolation model on $\mathbb{G}$ which assigns probability q of being open to each edge whose projection onto G lies in some subgraph of $\mathcal{C}$ and probability p to every other edge. Here, we focus our attention on the critical percolation threshold, $p_c(q)$, defined as the supremum of the values of p for which percolation with parameters p, q does not occur. We show that the function $q \mapsto p_c(q)$ is continuous in $(0,1)$, provided that the graphs in $\mathcal{C}$ are "suffciciently spaced from each other" on G and their vertex sets have uniformly bounded cardinality.

The second graph is the ordinary d-dimensional hypercubic lattice, $\mathbb{L}^d = (\mathbb{Z}^d, \mathbb{E}^d)$, $d \geq 3$, where we define the inhomogeneous Bernoulli percolation model in which every edge inside the s-dimensional subspace $\mathbb{Z}^s \times \{0\}^{d-s}$, $2 \leq s < d$, is open with probability q and every other edge is open with probability p. Defining $q_c(p)$ as the supremum of the values of q for which percolation with parameters p, q does not occur and letting $p_c \in (0,1)$ be the threshold for homogeneous percolation on $\mathbb{L}^d$, we prove two results: first, the uniqueness of the infinite cluster in the supercritical phase of parameters (p, q), whenever $p \neq p_c$; second, we show that, for any $p < p_c$, the critical point $(p, q_c(p))$ can be approximated by the critical points of slabs, in the spirit of the classical theorem of Grimmett and Marstrand for homogeneous percolation.