Three-nucleon force effects in proton-deuteron break-up studied with BINA at 135 MeV

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2. Theoretical aspects of the break-up reaction

2.1 Introduction

It is well known that the electric force between two charged particles at rest or in very slow relative motion is proportional to their charges and to the inverse of the square of the distance between them, and its direction is along the line joining the two charges. According to this law, for charged particles very close to each other \( r \to 0 \) like protons inside nuclei, this force should go to infinity preventing the protons from binding. The nuclear force, which is the force between two or more nucleons, overcomes the Coulomb force and is responsible for binding protons and neutrons into atomic nuclei. To a large extent, this force can be understood in terms of the exchange of virtual light mesons, such as pions. The nuclear force is strong and is driven by quantum chromodynamics (QCD). It has been a long-standing paradigm that pair interactions between nucleons largely account for the low-energy properties of complex nuclei. Studies of strong and electromagnetic processes involving two nucleons provide the most direct information about the two-nucleon interaction. However, in addition to the firmly established one-pion-exchange (OPE) tail, nowadays, we know a lot about two-pion-exchange (TPE) and the shape of the two-nucleon potential as well [28, 29]. General field-theory considerations [30] suggest that the potential is non-local for nucleon separations less than about 0.5 fm. Furthermore, it is now established [30] that there are families of two-nucleon potentials having the same OPE tail but having different (generally non-local) intermediate and short-range behaviors, which give exactly the same fit to the deuteron binding energy and two-nucleon scattering phase shifts. These potentials usually underbind triton [31], so the hope is to use three-body scattering to determine missing elements. Beyond two-nucleon systems, three-nucleon systems are the next challenge for theoretical calculations. In this chapter, the basic ingredients to describe the scattering process in the three-body system will be discussed.

2.2 The nucleon-nucleon force

The Nucleon-Nucleon Potentials (NNPs) can be described in momentum space and in configuration space [5]. The potentials which are described in configuration space have been used the nonrelativistic Schrödinger equation. The potential must be invariant under rotation, reflection and time reversal. The model-independent terms
are written in the configuration space as a sum of six terms, \( V = \sum_{i=1}^{6} V_i P_i \), where

\[
\begin{align*}
P_1 &= 1, \\
P_2 &= \vec{\sigma}_1 \cdot \vec{\sigma}_2, \\
P_3 &= S_{12} = 3(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2, \\
P_4 &= \vec{L} \cdot \vec{S}, \\
P_5 &= Q_{12} = \frac{1}{2}[(\vec{\sigma}_1 \cdot \vec{L})(\vec{\sigma}_2 \cdot \vec{L}) + (\vec{\sigma}_2 \cdot \vec{L})(\vec{\sigma}_1 \cdot \vec{L})], \\
P_6 &= (\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \vec{L}.
\end{align*}
\]

(2.1)

Here, \( P_i \) is a common choice for the six operators which are referred to as the central, spin-spin, tensor, spin-orbit, quadratic spin-orbit, and antisymmetric spin-orbit operators, respectively. In general, \( V_i \) in configuration space is a function of the distance between two nucleons, \( r^2 \), and the operators of linear and angular momentum \( p^2 \) and \( L^2 \). In modern high-quality NNPs such as NijmI, NijmII and Reid93, up to \( E_{lab} = 350 \) MeV, \( V_i \) is also a function of \( S^2 \) and \( J^2 \), which results from the fact that each partial wave is parameterized separately [5].

After the discovery of heavy mesons in the 1960s, the NNPs were written as a sum of the potentials representing the exchange of bosons. The long range part (\( \gtrsim 2 \) fm) of the potential is governed by the \( \pi \)-meson and is, therefore, called the One-Pion-Exchange Potential (OPEP). As Eq. (1.1) shows, at shorter distances, the attractive medium-range, a heavier meson such as the \( \sigma \)-meson or two \( \pi \)-mesons could be exchanged. The short-range (\( \lesssim 1.4 \) fm) repulsive part of the NNP needs a heavy vector boson such as the \( \omega \)-meson or \( \rho \)-meson to be exchanged. We note that potentials derived from the Effective Field Theory (EFT) and AV18 potentials are as precise as any OBE potentials without any need to exchange heavy mesons.

In the Nijmegen phase shift analysis, the short-range repulsive part is short enough to be screened by a centrifugal barrier for the higher partial waves [4]. In other words, the partial waves with higher angular momenta do not need to be parameterized and only the partial-waves of low angular momenta, normally up to \( J = 5 \), are parameterized. For the partial waves of lower angular momenta, the short-range part is represented by an energy dependent square-well with a range of \( b = 1.4 \) fm. The depth of square-well is independent of \( r \) but energy dependent which is, therefore, different for each of the partial-waves. In general, the NNP is written as a sum of an electromagnetic part, an OPEP part and an intermediate- and short-range phenomenological part:

\[
v(NN) = v_{EM}(NN) + v_\pi(NN) + v_I(NN) + v_S(NN).
\]

(2.2)

The OPEP for \( pp \) scattering is given by

\[
v_\pi(pp) = f_\pi^2 V(m_{\pi^0}),
\]

(2.3)
and for $np$ scattering it is

$$v_\pi(np) = -f_\pi^2 V(m_{\pi^0}) + (-)^{I+1} 2f_\pi^2 V(m_{\pi^\pm}), \quad (2.4)$$

where $I$ is the isospin and

$$V(m) = \left( \frac{m}{m_{\pi^\pm}} \right)^2 \frac{1}{3} mc^2 [V_S(r) \vec{\sigma}_i \cdot \vec{\sigma}_j + V_T(r) S_{ij}]. \quad (2.5)$$

$V_S(r)$ and $V_T(r)$ are the Yukawa and tensor functions:

$$V_S(r) = \frac{e^{-\mu r}}{\mu r},$$

$$V_T(r) = \left( 1 + \frac{3}{\mu r} + \frac{3}{(\mu r)^2} \right) \frac{e^{-\mu r}}{\mu r}, \quad (2.6)$$

where $\mu = mc/h$ [32] and $f_\pi^2 = 0.075$ is the pion-nucleon coupling constant [33]. The intermediate- and short-range phenomenological part of the potential are expressed, as in the Argonne $v_{14}$ model, as a sum of central, $L^2$, tensor, spin-orbit, and quadratic spin-orbit terms. For more details, see Ref. [32]. In spite of the good performance of the high-quality NNPs, the potentials still have problems which need further attention. When the potentials are evaluated in momentum space and then Fourier transformed to configuration space, a form-factor is needed to handle the singularity at the origin [5]. It is good to mention that this problem is not only limited to coordinate space but when we solve the LSE with the phenomenological potentials in momentum space, we still have to introduce ad-hoc form factors. This form-factor is not fixed by theory or experiment. Every model takes a form-factor that fits better to the data-set. One of the scalar mesons, the $\sigma$-meson, which is necessary in OBE potentials to explain especially the medium-range attraction has not yet been unambiguously definitely observed experimentally. There are several high-quality nucleon-nucleon potentials (NNPs) based on a OBE. They have been fitted to the world Nucleon-Nucleon (NN) scattering data-set up to 350 MeV and they are all non-relativistic. Here, some aspects of these models, which were used in predicting the observables of the present experiment, are explained briefly.

The Nijmegen group has introduced NijmI, NijmII, Nijm93 and Reid93 potential models and A partial-Wave Analysis,PWA93 [4]. The PWA93 has 39 parameters and has been fitted to almost 4301 data points of all $pp$ and $np$ scattering data with $\chi^2/\text{datum} \approx 0.99$. This can be compared to, for instance, Nijm93 [4] which fits the NN data with $\chi^2/\text{datum}=1.87$ and has only 15 parameters.

The newest Charge-Dependent (CD) Bonn potential, which is the successor of the original Bonn potential, is called CD-Bonn [28]. This high-quality potential fits 2932 $pp$ scattering data points with $\chi^2/\text{datum}=1.01$ and 3058 $np$ scattering data points with $\chi^2/\text{datum}=1.02$. The Bonn group also accounted for Charge-Independence Breaking (CIB) and Charge-Symmetry Breaking (CSB) effects, since the $pp$ and $nn$
interactions are different even after removing the electromagnetic interaction (CSB).

Another high-quality NNP is Argonne $v_{18}$ (AV$_{18}$) [32]. This potential has 18 operators, 14 of which are charge-independent and correspond to an updated version of $v_{14}$ [34]. Three additional charge-dependent and one charge-asymmetric operators have been added along with a complete electromagnetic interaction. The AV$_{18}$ potential has 40 parameters which have been adjusted via fitting the model to 4301 $pp$ and $np$ scattering data points from the Nijmegen database with a $\chi^2/\text{datum}=1.09$ in the range 0-350 MeV.

A different approach is provided by the Hannover-Lisbon theory group, where the $\Delta$-isobar is treated on the same footing as the nucleon, resulting in a coupled-channel potential CD-Bonn+$\Delta$ [35, 36] with pairwise nucleon-nucleon and nucleon-isobar interactions mediated through the exchange of $\pi$, $\rho$, $\omega$, and $\sigma$ mesons.

The first NN potential has been presented at next-to-next-to-next-to-leading order (N$^3$LO) of chiral perturbation theory [37]. Entem and Machleidt showed that since NN potentials of order three and less are known to be deficient in quantitative terms, the fourth order is necessary and adequate for a NN potential reliable up to 290 MeV. The accuracy of this method is comparable to the AV$_{18}$ potential. Thus, the NN potential at N$^3$LO is the first to meet the requirements for a reliable input potential for exact few-body and microscopic nuclear structure calculations, including chiral 3NF consistent with the chiral 2NF [37].

The high-quality models, such as CD-Bonn, NijmI, NijmII, Reid93, AV$_{18}$, Nijm93, CDB+$\Delta$ and EFT are all charge dependent and have a $\chi^2$ per degree of freedom very close to 1 (with the exception of Nijm93, which is less perfect). Possible deviations from experimental 3N data can no longer be attributed to defects in the description of the NN data and it can therefore, be caused by 3NF effects, which includes changes of NN forces due to the presence of the third nucleon [8]. These are some of the issues that leave room for more developments in the construction of NNPs.

### 2.3 Three-nucleon force

Even before the introduction of two-nucleon potentials that could make reliable predictions for physical observables, H. Primakov and T. Holstein showed that three-nucleon forces (3NF) should exist [38]. The first 3NF model was developed by Fujita and Miyazawa (FM) [39]. They described for the first time the Two-Pion Exchange (TPE) by incorporating an intermediate $\Delta$-isobar excitation, as shown in the right diagram in Fig. 2.1. The main ingredient of TPE-models is the $\pi N$ scattering amplitude illustrated as $T_3$ in the left diagram of Fig. 2.1. Later, more refined ingredients have been added such as the Tucson-Melbourne (TM) [40] and corrected Tucson-Melbourne (TM') 3NFs [41, 42, 43] allowing for additional processes contributing to the re-scattering of the mesons.

Theoretical predictions for the observables of interest were calculated by three
2.3. Three-nucleon force

Figure 2.1: The Feynman diagram for the Two-Pion Exchange (TPE) potential. In the left diagram, $T_3$ represents $\pi N$ scattering amplitude in which the nucleon is nucleon 3. In the right diagram, a TPE with a $\Delta$-isobar excitation is shown as one of the contributions for $T_3$.

groups. The Bochum-Krakow group used the modified-TM-model which is called TM’ (or TM99) as 3NP. They added this 3NP to the NN models, CD-Bonn, NijmI, NijmII, and AV$_{18}$. They have also combined the Urbana IX 3NP with the AV$_{18}$ NNP. The Hannover-Lisbon group has also calculated various observables for the $\vec{p} + d$ break-up reaction with the $\Delta$-isobar treated dynamically [35, 36]. The third approach for describing three-body systems is based on Chiral Perturbation Theory ($\chi$PT) [44], which includes TPE in the form of TM’ and the most general short-range contributions.

2.3.1 Tucson-Melbourne potential

The Tucson-Melbourne potential [40] was one of the first serious attempts, to build a complete three-nucleon force with the inclusion of short- and long-range parts of the interaction based on two-pion exchange. The TM potential includes the $\pi N$ scattering amplitude in which the pions are off-mass-shell$^1$, i.e. $E^2_\pi \neq p^2_\pi + m^2_\pi$, where $m_\pi$ is the pion mass. The most recent form of the TM potential is TM99 [43] which contains one parameter, $\Lambda_{TM}$, used as a cut-off to regularize its high-momentum behavior. The value of $\Lambda_{TM}$ is adjusted for each particular combination of the NN force and the TM99 3NF to match the value of the $^3H$ binding energy [45]. For the four NN potentials ($AV_{18}$, CD-Bonn, NijmI and NijmII) the corresponding values of $\Lambda_{TM}$, in units of the pion mass $m_\pi$, are 4.764, 4.469, 4.690 and 4.704, respectively. It should be noted that the adjustment of $\Lambda_{TM}$ is just a rough method to take into account other processes, which are ignored in the construction of the 3NF model. In the meson-exchange potential, these additional processes are different meson exchanges ($\pi - \rho, \rho - \rho$, etc.) and different intermediate excited states. The TM 3N potential has the following form:

$$V^{TM}_{ijk} = A^{SW}_{2\pi} O^{2\pi,SW}_{ijk} + A^{PW}_{2\pi} O^{2\pi,PW}_{ijk}, \quad (2.7)$$

$^1$ This is needed for the exchange of virtual space-like pions in a nuclear-force diagram.
where the terms represent TPE contributions from s-wave and p-wave $\pi N$ scattering corresponding to (a) and (b) three-body force Feynman diagrams of Fig. 2.2. The parameters $A_{2\pi}^{SW}$ and $A_{2\pi}^{PW}$ represent the strengths of the terms in this formula. There is another approach given by Brazilian group namely the Brazil 3NP. The Brazilian group derived a Two-Pion-Exchange (TPE) 3NP using an effective Lagrangian [46]. They considered the s-wave and p-wave potentials and their relative importance. The 3NP of the Brazil potential model in momentum space is quite similar to that of the TM’ potential.

2.3.2 Urbana-Illinois potential

After the study of 3N system dynamics with the AV$_{18}$ NN potential, the Urbana-Argonne collaboration in Illinois introduced new phenomenological 3NPs, namely Urbana IX (UIX) [47] and Illinois1-5 [6]. This collaboration has shown that the AV$_{18}$ NN potential alone predicts some key features of the nuclear structure correctly, such as the proper ordering of excited states and the rapid saturation of the binding energy for nuclei with mass number, $A$, larger than 4. But, as was shown in Fig. 1.3, with increasing $A$, it underestimates the nuclear binding energies. The three-nucleon potentials which have been used in the Illinois model have the following schematic form:

$$V_{ijk}^{UIX} = V_{ijk}^{TM} + A_{3\pi}^{\Delta R} O_{ijk}^{3\pi,\Delta R} + A_{R} O_{ijk}^{R},$$

(2.8)

where $V_{ijk}^{TM}$ is defined as Eq. 2.7. In Eq. 2.8, the second and the third terms are three-pion-exchange contributions from ring diagrams with one $\Delta$ in the intermediate states and a repulsive term as illustrated in the Feynman diagrams of (c) and (d) in Fig. 2.2. The parameters $A_{3\pi}^{\Delta R}$ and $A_{R}$ are the strengths of the terms. The Urbana IX potential does not have the TPE s-wave and the three-pion-exchange terms [6].

The spin-isospin and spatial dependence of the terms were obtained from diagrams shown in Fig. 2.2 but the strength of the terms was fixed by fitting to the binding energies of light nuclei and nuclear matter.
2.3.3 Hannover-Lisbon potential

The Hannover-Lisbon group takes the $\Delta$-isobar excitation into account to obtain an effective 3NP from any NNP. In this model, an explicit $\Delta$-isobar is added to the nucleonic Hilbert space of a three-nucleon system \cite{48}. The $\Delta$-isobar is considered as a stable particle rather than a dynamic $\pi N$ system \cite{49}. The $\Delta$-degree of freedom is treated in a coupled-channel approach. This approach emphasizes the need for consistency between the two-nucleon interaction and the correction mechanisms of microscopic nuclear structure due to $\Delta$-isobar excitation. A two-baryon coupled-channel potential was developed which couples two-nucleon states to nucleon-$\Delta$ states. The transition potential from NN to N$\Delta$ states is derived from $\pi$- and $\rho$-exchange. The Coulomb interaction between protons has been included in the potential in 2005 for the first time \cite{25}. The CD-Bonn+$\Delta$ potential was refitted to the Nijmegen database. This potential was then employed to describe elastic and inelastic 3N scattering processes \cite{35}.

2.3.4 Chiral perturbation theory ($\chi$PT)

A new approach for the description of the three-nucleon system is based on chiral perturbation theory \cite{50,51}. This theory provides the most general Lagrangian involving pions and low-energy nucleons and $\Delta$s consistent with a spontaneously broken symmetry. Nucleons with momenta greater than a scale, $\Lambda$, are integrated out, which makes this Lagrangian $\Lambda$ dependent. For the moment, it will be enough to specify that $\Lambda$ is a low-energy scale such as the momenta of the external particles or light masses and is substantially less than the mass of nuclei, $m_N$ \cite{52,53}. In a version for energies below the $\Delta$-nucleon mass difference, the $\Delta$ can be integrated out. In this case the three-nucleon terms appear, for the first time, at Next-to-Next-to Leading Order (NNLO). Figure 2.3 shows the ordering and the hierarchy of all diagrams in the $\Delta$-less chiral perturbation theory. The chiral perturbation Lagrangian is organized in powers of $\nu$ (chiral order), $\mathcal{L}(\Lambda^{-\nu})$, and is written as a sum of contributions from different orders \cite{52},

$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{\nu=0} + \mathcal{L}^{\nu=1} + \mathcal{L}^{\nu=2} + \cdots.$$  \hspace{1cm}(2.9)

The three-nucleon vertices that appear at NNLO are shown in Fig. 2.3. In comparison with the ad-hoc approach of adding two-nucleon potentials and some three-nucleon force, chiral perturbation theory clearly has the advantage of being a self-consistent theory.

2.4 Faddeev equations

The 3NFs can be investigated in a process such as nucleon-deuteron scattering. Here we present a formalism \cite{8} to describe this process. In the nucleon-deuteron hadronic
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**Figure 2.3**: A schematic representation of the hierarchy of the nuclear forces in the $\chi$PT approach. Solid (dashed) lines represent nucleons (pions). Solid dots, filled/open squares and filled circles depict vertices from $\mathcal{L}_{\text{eff}}$. Scattering $|1\rangle + |23\rangle$, four possible outgoing channels exist:

\[
\begin{align*}
|1\rangle + |23\rangle & \rightarrow |1\rangle + |23\rangle & (1) \\
|1\rangle + |23\rangle & \rightarrow |2\rangle + |13\rangle & (2) \\
|1\rangle + |23\rangle & \rightarrow |3\rangle + |12\rangle & (3) \\
|1\rangle + |23\rangle & \rightarrow |1\rangle + |2\rangle + |3\rangle & (4)
\end{align*}
\]

Channel 1 corresponds to elastic scattering, while channels 2 and 3 represent the rearrangement channels. Channel 4 is the break-up channel. In the calculations of three-nucleon systems one has to include all four channels. In the following, we show the calculation of three-nucleon systems using the approach developed by Faddeev, who provided one of the basic equations in the few-body problem. In Ref. [8] one
can find a formal derivation of the Faddeev equations with the inclusion of the 3N interaction. The principal object of interest is the amplitude $U_0$, describing the transition from the initial state of a deuteron and a nucleon to the final state consisting of three nucleons. The nucleons are treated as identical particles, differing only in the third component of the isospin. The initial state is described by the wave function $\Phi_m$, with $m=1,2,3$ enumerating the incident (unbound) nucleon. The three possible situations are shown in Fig. 2.4, accounting for the fact that the initial state has to be properly antisymmetrized if one considers the action upon it by the operator $U_0$. The three pieces of the full amplitude we call $U_1$, $U_2$, $U_3$, with the subscript corresponding to the free nucleon in the entrance channel. Thus, the full amplitude is given by

$$U_0\Phi_1 = U_1\Phi_1 + U_2\Phi_2 + U_3\Phi_3. \quad (2.10)$$

We first consider the case of a two-body potential. The Faddeev method is to split again these three amplitudes into three pieces each, ordered by the last interaction occurring in the system. This can be any one of the NN interactions within pairs 2-3, 3-1 or 1-2. As an example, we describe the pairwise interaction between nucleons $j$ and $k$ by $V_i$, with $i,j,k=1,2,3$ and $i \neq j \neq k$. This decomposition is shown in Fig. 2.5, where each of the three rows corresponds to one of the diagrams on the right-hand side of Fig. 2.4. All the amplitudes in Fig. 2.5 are denoted by $U_m^n$, with the subscript $m=1,2,3$ enumerating the initial state and the superscript $n=1,2,3$ pointing to the last pairwise interaction. In the operator notation this decomposition can be expressed by three equations:

$$U_m\Phi_m = U_m^1\Phi_m + U_m^2\Phi_m + U_m^3\Phi_m. \quad (2.11)$$

A new set of three transition operators $T_n$ is defined, such that

$$T_n\Phi_1 = U_1^n\Phi_1 + U_2^n\Phi_2 + U_3^n\Phi_3, \quad (2.12)$$
which corresponds to sums in columns of the graphs on the right-hand sides of the equalities in Fig. 2.5.

\[ U_0 \Phi_1 = \sum_{m=1}^{3} \sum_{n=1}^{3} U_m^n \Phi_m = \sum_{n=1}^{3} T_n \Phi_1. \]  

(2.13)

Then

\[ U_0 \Phi_1 = T_1 \Phi_1 + T_2 \Phi_1 + T_3 \Phi_1. \]  

(2.14)

Introducing a sum of cyclic and anti-cyclic permutations operator

\[ P = P_{12}P_{23} + P_{13}P_{23}, \]  

(2.15)

and considering \( T_2 = P_{12}P_{23}T_1 \) and \( T_3 = P_{13}P_{23}T_1 \), the full transition amplitude for
the break-up reaction is obtained as

\[ U_0 = (1 + P)T_1. \]  

(2.16)

For simplicity, however, we only demonstrate the principle by working from now on with the \( T \) operators and dropping the wave function. This is in general equivalent to assuming implicitly the antisymmetrized initial state. Therefore in the diagrammatic representation also the deuteron state is omitted [54]. Since now the situation is fully symmetric, it is enough to consider, for example, the details of \( T_1 \). All possible sequences of interactions are depicted in Fig. 2.6. In the first row of the second equality on the right-hand side all occurrences of interactions in only pair 2-3 (i.e. of \( V_1 \)) are grouped together. In the second and third rows the multiple scattering series due to the same \( V_1 \) potential is preceded by the sequence “any interaction” ending with \( V_2 \) or \( any \ interaction \) ending with \( V_3 \). The action of \( any \ interaction \) followed by \( V_n \) is, per definition, described by the transition operator \( T_n \) and thus in algebraic notation we have

\[
T_1 = V_1 + V_1 G_0 V_1 + V_1 G_0 V_1 G_0 V_1 + \cdots \\
+ V_1 G_0 T_2 + V_1 G_0 V_1 G_0 T_2 + \cdots \\
+ V_1 G_0 T_3 + V_1 G_0 V_1 G_0 T_3 + \cdots ,
\]

(2.17)

where \( G_0 \) represents the free propagator of the 3N state. We observe that the multiple scattering series within the subsystem of nucleons 2 and 3 can be summed up according to the Lippmann-Schwinger equation (LSE), introducing the two-nucleon transition operator, \( t_1 \), acting in the 3N environment:

\[
t_1 = V_1 + V_1 G_0 V_1 + V_1 G_0 V_1 G_0 V_1 + \cdots = V_1 + V_1 G_0 t_1.
\]

(2.18)

Substituting Eq. (2.18) to Eq. (2.17) leads to

\[
T_1 = t_1 + t_1 G_0 T_2 + t_1 G_0 T_3.
\]

(2.19)

The same result can be obtained starting from first row of Fig. 2.6. Since any interaction in the system finishes with the action of the pairwise potentials, \( V_n \), that decomposition can be written as

\[
T_1 = V_1 + V_1 G_0 T_1 + V_1 G_0 T_2 + V_1 G_0 T_3.
\]

(2.20)

Ordering the terms with \( T_1 \) on the left-hand side, one obtains

\[
(1 - V_1 G_0)T_1 = V_1 + V_1 G_0 T_2 + V_1 G_0 T_3.
\]

(2.21)

Multiplying both sides of Eq. (2.21) with \((1 - V_1 G_0)^{-1}\) and using the LSE (2.18), one obtains again Eq. (2.19). Using Eq. (2.15) and considering the same equations for
operators $T_2$ and $T_3$ including the incident channel wave function notation, which
was omitted in the above presentation, one finally arrives at a compact formulation
of the Faddeev-like equation, representing the set of three coupled equation:

$$T\Phi = tP\Phi + tPG_0T\Phi. \quad (2.22)$$

Having the NN potential and 3N forces, the scattering problem in the 3N system is
stated in the form of a Faddeev-like integral equation for the amplitude $T$, being a
generalization of Eq. (2.22) to include also a 3NF:

$$T = tP\Phi + (1 + tG_0)V^{(1)}_{3\text{NF}}(1 + P)\Phi +$$

$$+ tPG_0T + (1 + tG_0)V^{(1)}_{3\text{NF}}(1 + P)G_0T, \quad (2.23)$$
where the initial channel state $\Phi$ is composed of a deuteron and a momentum eigenstate of the projectile nucleon. The 3N potential $V_{3\text{NF}}$ can always be decomposed into a sum of three parts:

$$V_{3\text{NF}} = V_{3\text{NF}}^{(1)} + V_{3\text{NF}}^{(2)} + V_{3\text{NF}}^{(3)},$$

where each part $V_{3\text{NF}}^{(i)}$ singles out nucleon $i$. The parts are symmetric under the exchange of the other two nucleons $j$ and $k$, with $j \neq i \neq k$. One can see that in Eq. (2.23) only one part, $V_{3\text{NF}}^{(1)}$ appears explicitly. The other parts enter via the permutations contained in $P$. As already stated, the physical break-up amplitude $U_0$ is obtained from $T$ according to Eq. (2.16). Iterating the Faddeev-like equation (2.23) and inserting the resulting $T$ into Eq. (2.16) yields the multiple scattering series, in which each term contains some number of interactions among nucleons via NN and 3N forces with free propagation in between. The relation mechanism is thus transparently mirrored.

## 2.5 Observables in the break-up reactions

In this section, we discuss the kinematics of the break-up reaction which is the subject of this thesis and introduce the break-up analyzing power and cross section. The most common observable in a scattering experiment is the cross section. To measure the cross section of a reaction, we need a beam of particles impinging on a target and a detector to observe the outgoing particles. In a simple scattering experiment with an unpolarized beam and target, the cross section depends on the polar angle, $\theta$, and the beam energy. If the beam of projectiles is polarized, then the cross section depends not only on the polar angle but also on the azimuthal angle, $\phi$. This dependence can easily be observed by putting two detectors at the same polar angle but on the opposite sides of the beam. The $\phi$-dependent part of the scattering cross section is characterized by the observable called vector analyzing power.

The concepts of polarization and experiments involving polarized beams and details about the derivations can be found in Ref. [55]. The polarized source at KVI is an ion source, and is described in more detail in Sect. 3.1. The polarized beam produced by an ion source has an axial symmetry because of the presence of a solenoidal magnetic-field in the source at the location where the polarized atoms are ionized. It is convenient to define the $Z$-axis of quantization along this magnetic field direction. The $X$ and $Y$ axes are arbitrary because of the axial symmetry. This $XYZ$ frame is called the polarization frame [55]. Polarized protons produced by the ion source have the states, $|j, m\rangle$, of spin-up, $|\frac{1}{2}, \frac{1}{2}\rangle$, and spin-down, $|\frac{1}{2}, -\frac{1}{2}\rangle$. If the populations of spin-up and spin-down in an ensemble are denoted by $N_+$ and $N_-$,
respectively, the vector polarization of the ensemble is defined by:

\[ p = \frac{N_+ - N_-}{N_+ + N_-}. \]  

(2.25)

There is a variety of spin-observables in the scattering process which can be studied. The form of the observables for the reactions studied in this thesis will be shown here. Suppose that the initial spin state of a scattering reaction is described by

\[ \chi_i = \sum_j a_j \phi_j, \]  

(2.26)

where \( \phi_j \) designates the basis for the spin states before the reaction, \( j \) is the dimension of the space and \( i \) denotes the initial state. The final state can be represented by

\[ \chi_f = \sum_j b_j \phi'_j, \]  

(2.27)

where \( \phi'_j \) designates the basis for the spin states after the reaction, and \( f \) denotes final state. The scattering amplitude, represented by \( M \), relates the initial and final states:

\[ \chi_f = M \chi_i \quad \text{and} \quad b_j = \sum_k M_{jk} a_k, \]  

(2.28)

where the \( M \) is given by

\[ M_{ij} = -\frac{2}{3} m(2\pi)^2 \langle \phi'_j | U_0 | \phi_i \rangle, \]  

(2.29)

where \( m \) is the nucleon mass and \( \phi', \phi \) are the final and initial states, respectively. The initial and final density matrices are defined for an ensemble of \( N \) particles as

\[ \rho_i \equiv \sum_{n=1}^{N} \chi_i^{(n)} [\chi_i^{(n)}]^\dagger \]  

(2.30)

and

\[ \rho_f \equiv \sum_{n=1}^{N} \chi_f^{(n)} [\chi_f^{(n)}]^\dagger. \]  

(2.31)

By substituting Eq. (2.28) into Eq. (2.31) we obtain

\[ \rho_f = M \rho_i M^\dagger. \]  

(2.32)

If \( \rho_i \) is normalized to unity, the differential cross section for a polarized beam is given by

\[ I(\theta, \phi) = Tr \rho_f = Tr M \rho_i M^\dagger. \]  

(2.33)
2.5. Observables in the break-up reactions

Expanding the density matrix in terms of the set of Pauli matrices, $\sigma_1, \sigma_2, \sigma_3$, and the unit matrix $I$ gives:

$$\rho_i = \frac{1}{2} \left( I + \sum_{j=1}^{3} p_j \sigma_j \right).$$

For any operator $\Omega$, we have the expectation value of that operator, $\langle \Omega \rangle = Tr \rho_i \Omega$. Therefore:

$$p_j \equiv \langle \sigma_j \rangle = Tr \rho_i \sigma_j$$

is the $j^{th}$ component of the beam polarization. By substituting Eq. (2.34) into (2.33) we obtain

$$I(\theta, \phi) = I_0(\theta) \left( 1 + \sum_{j=1}^{3} p_j A_j(\theta) \right),$$

where

$$A_j(\theta) \equiv \frac{Tr M \sigma_j M^\dagger}{Tr MM^\dagger},$$

is the $j^{th}$ component of the nucleon analyzing power, and

$$I_0(\theta) = \frac{1}{2} Tr MM^\dagger$$

is the cross section for an unpolarized beam.

For an unpolarized beam, the cross section usually depends on the polar scattering angle and it is independent on the azimuthal direction. In other words, it has an axial symmetry around the direction of the incoming beam. But, for a polarized beam, the spins of the projectile particles (beam) are aligned in a given direction which imposes a preferred azimuthal direction in the system. Usually, the scattering observables are studied in a reference frame that is called the projectile helicity frame. In this coordinate system, the $\hat{z}$ axis is taken along the direction of the projectile motion, $\hat{k}_{in}$, the $\hat{y}$ axis is taken along $\hat{k}_{in} \times \hat{k}_{out}$, where $\hat{k}_{out}$ represents the direction of the scattered particle, and the $\hat{x}$ axis is chosen to form a right-handed coordinate system (see Fig. 2.7).

If the incident beam is made up of spin-$\frac{1}{2}$ particles with a transverse polarization $\vec{p}$, the cross-section relation is written as:

$$I(\theta, \phi) = I_0(\theta)[1 + A(\theta) \hat{p} \cdot \hat{n}],$$

where $I_0(\theta)$ is the cross section for scattering of an unpolarized beam at scattering angle $\theta$, $A(\theta)$ is the analyzing power of the reaction, and $\hat{n}$ is a unit vector along $\hat{y} = \hat{k}_{in} \times \hat{k}_{out}$ in the helicity frame. We use the symbol $\phi$ to denote the angle between $\vec{p}$ and $\hat{n}$; that is $\vec{p} \cdot \hat{n} = |p| \cos \phi$. If $\phi = 0^\circ$, $\hat{p}$ is along $\hat{n}$ and for $\phi = 180^\circ$, $\hat{p}$ is opposite to $\hat{n}$. With this, the cross section can be written as:

$$I(\theta, \phi) = I_0(\theta) \left[ 1 + A(\theta) p \cos \phi \right],$$
The projectile helicity frame is used to study the scattering observables. In this coordinate system, the $\hat{z}$ axis is taken along the direction of the projectile motion, $\hat{k}_{\text{in}}$, the $\hat{y}$ axis is taken along $\hat{k}_{\text{in}} \times \hat{k}_{\text{out}}$, where $\hat{k}_{\text{out}}$ represents the direction of the scattered particle, and $\hat{x}$ axis is chosen to form a right-handed coordinate system. The scattering angle, $\theta$, is the angle between the direction of the incoming beam, $\hat{k}_{\text{in}}$, and the outgoing particle, $\cos \theta = \hat{k}_{\text{in}} \cdot \hat{k}_{\text{out}}$.

where $p$ represents the magnitude of the polarization. We denote the cross section for the scattering to the left by $\phi = 0$ and to the right by $\phi = \pi$. It is common to normalize the cross section of the reaction for the polarized beam to that of the unpolarized beam to eliminate geometrical and detector asymmetries and then calculate the analyzing power according to,

\begin{align*}
(\phi = 0) & \rightarrow I_L = I_0(\theta)(1 + pA), \\
(\phi = \pi) & \rightarrow I_R = I_0(\theta)(1 - pA),
\end{align*}

so that

\begin{equation}
A = \frac{1}{p} \left( \frac{I_L - I_R}{I_L + I_R} \right).
\end{equation}

The two forward-scattered particles of the break-up reaction are shown. In this right-handed coordinate system, $Z$-axis is along the beam direction. The energy of the particles, $E_1, E_2$, the scattering angles, $\theta_1, \theta_2$, and the relative azimuthal angle between the two particles, $\Delta \phi = \phi_1 - \phi_2 = \phi_{12}$, are shown.
For a system of three particles (two protons \((p_1, p_2)\) and a neutron \((n)\)) in the exit channel, there are in total nine parameters: \((E_1, \theta_1, \phi_1)_{p_1}, (E_2, \theta_2, \phi_2)_{p_2},\) and \((E_3, \theta_3, \phi_3)_{n}\). Figure 2.8 shows six of these parameters out of nine. Here, \(E\) is the energy of the particle, \(\theta\) is the polar scattering angle, and \(\phi\) is the azimuthal angle in the helicity frame [55], respectively. From the energy and momentum conservation laws, there are 4 inhomogeneous equations. Therefore, by measuring at least 5 of these parameters in an experiment, we can obtain the rest of the parameters uniquely. In our break-up experiments, we have identified two forward-scattered protons. We measured the energies and scattering angles, \((E, \theta, \phi)\), of both of these particles unambiguously. Therefore, we have measured six parameters, whereas, to solve the equations, we need to measure only five parameters. Measuring the sixth parameter helps to suppress the background. Here, we explain the correlation between the observables for the simpler case with non-relativistic kinematics. Using the non-relativistic expressions for the energy and momentum conservation, we obtain the relation [56, 57]:

\[
(m_1 + m_3)E_1 + (m_2 + m_3)E_2 - 2\sqrt{m_p m_1 E_p E_1} \cos \theta_1 \\
- 2\sqrt{m_p m_2 E_p E_2} \cos \theta_2 \\
+ 2\sqrt{m_1 m_2 E_1 E_2} \cos \theta_{12} \\
- m_3 Q + E_p (m_p - m_3) = 0,
\]

where \(m_1, m_2\) are the masses of the first and second detected particles, \(m_3\) is mass of the undetected particle, \(m_p\) is the projectile mass (here, proton), \(Q\) is the \(Q\)-value of the reaction, \(E_1, E_2, E_p\) are the energies of the first, second, and the projectile particles, respectively, and \(\theta_{12}\) is the opening angle between the first and second protons, so that \(\cos \theta_{12} = \cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2 \cos (\phi_1 - \phi_2)\). From Eq. (2.43) one can derive a relation between the energies of the protons \((E_1, E_2)\). The relativistic version of this relation is graphically presented in Fig 2.9 and is referred to as the S-curve. The S-curve contains all the kinematically allowed combinations of \(E_1\) and \(E_2\). The corresponding kinematical variable, \(S\), represents the arc-length along the S-curve with the starting point, \(S = 0\), defined by the proton with the minimum value for \(E_1\) and it increases in the counterclockwise direction. The parameter \(S\), therefore, represents the energy correlation between the two protons and it is expressed in units of energy. In the next chapters we will present the experimental results for the cross sections and the analyzing powers for various kinematical configurations. Furthermore, we will compare the experimental data with the most recent theoretical predictions which we briefly described in this chapter.
Figure 2.9: The energy of the second proton as a function of the energy of the first proton is represented as the S-curve. A few S-curves are shown for various kinematics, \((\theta_1, \theta_2, \phi_{12} = |\phi_1 - \phi_2|)\). Angles are given in degrees.