# Weak long distance contributions to the neutron and proton electric dipole moments

Bijnens, Johan; Pallante, Elisabetta

[Link to publication in University of Groningen/UMCG research database](#)

# Weak long distance contributions to the neutron and proton electric dipole moments

## Johan Bijnens, Elisabetta Pallante

*NORDITA, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark*

## Abstract

We evaluate the long distance weak contribution to the neutron and proton electric dipole moments using an effective Lagrangian framework. We estimate the coefficients needed by a factorization hypothesis and additional assumptions on $\gamma_5$ terms in the baryon lagrangian. We obtain $|d_e^n| \approx 5 \times 10^{-32}$ e·cm and $|d_e^p| \approx 4 \times 10^{-32}$ e·cm. The former estimate is similar to the quark model estimates done previously.

## 1. Introduction

The contributions to the electric dipole moment (EDM) of the neutron within the Standard model have been widely explored. In the Standard model the main contribution comes from the strong CP-violating $\theta$-term [1] and recently reconsidered in [2]. The experimental and theoretical situation can be found reviewed in [3]. The present experimental limits are

$$|d_e^n| \leq 11 \cdot 10^{-26} \, e \cdot cm \ [4] \quad \text{and}$$

$$d_e^p = (-3.7 \pm 6.3) \cdot 10^{-23} \, e \cdot cm \ [5] . \tag{1}$$

The neutron and proton EDM appear as an higher order effect of weak interactions. The one loop contribution with W exchange vanishes because of KM combinations and the two-W boson loops contribution is also vanishing as shown in [6] and references therein. First nonvanishing contributions are the so called transition quark electric dipole moments [6–8] and the insertion of penguin diagrams [9] within the baryon. Penguin diagrams can in fact produce the CP violating phases needed to generate the EDM term. The EDM is then generated by a two step process: the strong penguin diagram insertion which causes the transition $d \rightarrow s$ and weak radiative decay of the final strange baryon (e.g. $\Sigma^0, \Lambda \rightarrow n\gamma$). Already in [10] it was observed that penguin diagrams' contributions dominate the EDM. The evaluation of the long distance part of penguin insertions has been done up to now relying on quark models, like the one in [11]. See [3] for more references.

In this letter we propose an alternative derivation of the neutron and proton EDM based on a factorization hypothesis which leads to the derivation of the EDM within the framework of chiral perturbation theory for baryons.

We first describe our approach and perform the calculation. Here the assumptions made at various stages will also be explained. Then we present numerical results for both the proton and neutron electric dipole moment. A comparison with power counting in the heavy-baryon formalism for Chiral Perturbation Theory and a proof that our contribution is the leading one

are presented next. Finally, we recapitulate our main conclusions.

We do not attempt to ascribe an uncertainty to our results. However, contrary to the $p$-wave hyperon nonleptonic decays we have rather small cancellations between the different subamplitudes. We therefore expect higher orders to be of normal size. The main uncertainty is the assumption made in estimating the coefficients in the Lagrangian and the factorization ansatz.

## 2. The calculation

The gluonic penguin is the main source of CP violation in the weak $|\Delta S| = 1$ hamiltonian. The effective interaction which mediates the $d \rightarrow s$ transition in the EDM diagram involves the strong penguin four-quark operators of the effective weak $|\Delta S| = 1$ Lagrangian $\mathcal{L}_{\text{eff}} = -G_F/\sqrt{2} \sum_{i=3}^{6} C_i(\mu)Q_i(\mu)$ (we use the definitions of the $Q_i$ as in [12]). The operator $Q_6$ is defined as $Q_6 = (\bar{s}_\alpha d_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V+A}$, where $V \pm A$ stands for the combination $\gamma_\mu(1 \pm \gamma_5)$ and $\alpha, \beta$ denote colour indices. Using Fierz identities one can rewrite $Q_6$ as

$$Q_6 = -8 \sum_q \bar{s}_{\alpha_L} q_{\alpha_R} \ \bar{q}_{\beta_R} d_{\beta_L}$$
$$+ 4 \sum_q \bar{s}_{\alpha_R} \gamma_\mu q_{\alpha_R} \ \bar{q}_{\beta_L} \gamma_\mu d_{\beta_L} ,  \qquad (2)$$

where $q_{R,L} = (1 \pm \gamma_5)/2 \ q$. At this point we introduce a factorization hypothesis:

$$\langle B_i|Q_6|B_j\rangle = -8\langle \bar{d}_R d_L\rangle\langle B_i|\bar{s}_L d_R|B_j\rangle . \qquad (3)$$

Within the factorization hypothesis all the other operators $Q_i$, $i = 3, 4, 5$ do not contribute. The hypothesis is favoured by the substantial enhancement of the coefficient of the operator $Q_6$ by next-to-leading corrections and the enhancement of the $Q_6$ contribution to weak nonleptonic decays. For later use we introduce $\bar{q}\lambda_\pm q$ with $\lambda_\pm$ projection matrices defined in terms of the Gell-Mann matrices $\lambda_6, \lambda_7$ as $\lambda_\pm = (\lambda_6 \pm i\lambda_7)/2$.

The baryon Lagrangian for strong interactions in presence of external scalar and pseudoscalar sources allows for the general form leading in the derivative expansion and in the light quark mass matrix:

$$\mathcal{L}_B = \text{Tr} \left(\bar{B}(iD_\mu\gamma^\mu - m_B)B\right) + b_1 \text{Tr} \left(\bar{B}\chi_+ B\right)$$
$$+ b_2 \text{Tr} \left(\bar{B}B\chi_+\right) + b_3 \text{Tr} \left(\bar{B}B\right) \text{Tr} (\chi_+)$$
$$+ b_1^5 \text{Tr} \left(\bar{B}\chi_- \gamma_5 B\right) + b_2^5 \text{Tr} \left(\bar{B}\gamma_5 B\chi_-\right)$$
$$+ b_3^5 \text{Tr} \left(\bar{B}\gamma_5 B\right) \text{Tr} (\chi_-) , \qquad (4)$$

where Tr stands for the trace over flavour indices and the covariant derivative in the case of interest contains the electromagnetic field $D_\mu B = \partial_\mu B + ie\mathcal{A}_\mu[Q, B]$. The field $\chi_\pm = \xi^\dagger \chi \xi^\dagger \pm \xi \chi^\dagger \xi$ contains the external scalar and pseudoscalar sources with $\chi = 2B_0(s(x) + ip(x))$ and, in the absence of meson field ($\xi = 1$), we have $\chi_+ = 4B_0 s(x)$ and $\chi_- = 4iB_0 p(x)$. $B_0$ is related to the scalar quark condensate through the identity $\langle 0|\bar{q}q|0\rangle_{q=u,d,s} = -B_0(f^2/2)(1 + O(\mathcal{M}))$, with $f \simeq f_\pi \simeq 132$ MeV and $\mathcal{M}$ the light quark mass matrix. The baryon fields are incorporated in the $3 \times 3$ matrix

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda \end{pmatrix},$$

and transform nonlinearly under $SU(3)_L \times SU(3)_R$. The electromagnetic coupling of the baryon field in the covariant derivative generates the ordinary magnetic moment for baryons.

Taking the functional derivative with respect to $s_{ij}, p_{ij}$ of the generating functional of the baryon Lagrangian including terms (4) it follows that

$$\langle B_k|\bar{q}_i q_j|B_l\rangle = \langle B_k| - 4B_0 b_1 \bar{B}_i B_j - 4B_0 b_2 (\bar{B}B)_{ji}$$
$$- 4B_0 b_3 \text{Tr} \bar{B}B\delta_{ij}|B_l\rangle,$$

$$\langle B_k|\bar{q}_i\gamma_5 q_j|B_l\rangle = \langle B_k|4B_0 b_1^5 \bar{B}_i\gamma_5 B_j$$
$$+ 4B_0 b_2^5 (\bar{B}\gamma_5 B)_{ji} + 4B_0 b_3^5 \text{Tr} \bar{B}\gamma_5 B\delta_{ij}|B_l\rangle . \qquad (5)$$

Both identities relate the two fermion matrix element (3) to the corresponding baryon Lagrangian. In the case of the flavour combination $\bar{B}_i B_j$ with $i, j = s, d$ also the ordinary baryon Lagrangian for $|\Delta S| = 1$ weak interactions has to be taken into account. Assuming octet enhancement it has two terms at leading order in the derivative expansion which transform as $(8_L, 1_R)$

$$\mathcal{L}_{\text{eff}}^{|\Delta S|=1} = a \text{Tr} \bar{B}\{\lambda_6, B\} + b \text{Tr} \bar{B}[\lambda_6, B], \qquad (6)$$

where for the case of interest the meson field is absent, i.e. $\xi = 1$. Then the full baryon Lagrangian which induces $s \to d$ transitions can be written as follows:

$$\mathcal{L}_B^{s \to d}$$

$$= 4B_0 \left( \frac{\alpha + \gamma}{2} b_1 \operatorname{Tr} \bar{B}\lambda_- B + \frac{\alpha + \delta}{2} b_2 \operatorname{Tr} \bar{B}B\lambda_- \right)$$

$$+ 4B_0 \left( \frac{\alpha^* + \gamma}{2} b_1 \operatorname{Tr} \bar{B}\lambda_+ B + \frac{\alpha^* + \delta}{2} b_2 \operatorname{Tr} \bar{B}B\lambda_+ \right)$$

$$- 4B_0 \frac{\alpha}{2} \left( b_1^5 \operatorname{Tr} \bar{B}\lambda_-\gamma_5 B + b_2^5 \operatorname{Tr} \bar{B}\gamma_5 B\lambda_- \right)$$

$$+ 4B_0 \frac{\alpha^*}{2} \left( b_1^5 \operatorname{Tr} \bar{B}\lambda_+\gamma_5 B + b_2^5 \operatorname{Tr} \bar{B}\gamma_5 B\lambda_+ \right). \quad (7)$$

The parameters $\gamma, \delta$ are defined as $2B_0\gamma b_1 = a+b$ and $2B_0\delta b_2 = a - b$, where $a, b$ are the couplings in (6). The Lagrangian (7) induces transitions $n \to \Sigma^0, \Lambda$ and $p \to \Sigma^+$ both parity conserving (p.c.) and parity violating (p.v.) i.e. with a $\gamma_5$ insertion. Here we have made the assumption that the parity violating part of the other operators besides $Q_6$ can be neglected. The EDM term of the neutron $\mathcal{L}_{\text{EDM}} = i\frac{d_e}{2}\bar{\psi}\sigma_{\mu\nu}F^{\mu\nu}\gamma_5\psi$ can be generated at tree level in the baryon theory by the vertices in (7) and through the insertion of the anomalous magnetic moment operator which we write as

$$\mathcal{L}_\mu = \frac{e}{4m_N} \left( \mu_D \operatorname{Tr} \bar{B}\sigma_{\mu\nu}F^{\mu\nu}\{Q, B\} \right.$$

$$\left. + \Delta\mu_F \operatorname{Tr} \bar{B}\sigma_{\mu\nu}F^{\mu\nu}[Q, B] \right). \quad (8)$$

The baryon magnetic moments receive the ordinary contribution from the leading electromagnetic coupling in (4) and the anomalous contribution from the next to leading $O(p^2)$ terms in (8).

At tree level in the full baryon theory the set of diagrams which contribute to the electric dipole moment of the neutron and proton are shown in Fig. 1. They are given by the insertion of a parity violating (p.v.) vertex and a parity conserving (p.c.) vertex from (7) and the insertion of the anomalous magnetic moment vertex in (8).

The Lagrangians (7) and (8) lead to the following tree level contribution for the neutron case:

$$d_e^n = -\frac{ie}{2m_N} \frac{\alpha - \alpha^*}{2} \left\{ \mu_\Lambda A_\Lambda A_\Lambda^5 \frac{1}{M_\Lambda^2 - M_n^2} \right.$$
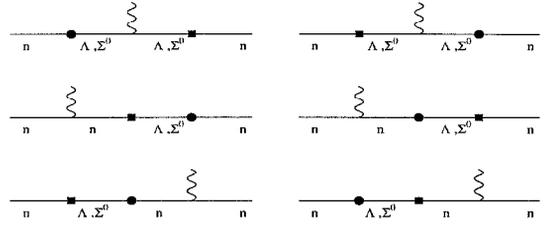


Fig. 1. Tree level diagrams which contribute to the neutron electric dipole moment. The circle vertex is the parity conserving vertex. The box is the parity violating vertex. The photon insertion is the anomalous magnetic moment vertex. In the proton case the same diagrams contribute where the $p \to \Sigma^+$ transition is allowed.

$$+ \mu_{\Sigma^0} A_\Sigma A_\Sigma^5 \frac{1}{M_{\Sigma^0}^2 - M_n^2} + \mu_n A_\Lambda A_\Lambda^5 \frac{M_\Lambda}{M_n(M_\Lambda^2 - M_n^2)}$$

$$+ \mu_n A_\Sigma A_\Sigma^5 \frac{M_{\Sigma^0}}{M_n(M_{\Sigma^0}^2 - M_n^2)}$$

$$+ \mu_{\Lambda\Sigma^0} A_\Lambda A_\Sigma^5 \frac{1}{(M_\Lambda - M_n)(M_{\Sigma^0} + M_n)}$$

$$+ \mu_{\Lambda\Sigma^0} A_\Sigma A_\Lambda^5 \frac{1}{(M_{\Sigma^0} - M_n)(M_\Lambda + M_n)} \bigg\}. \quad (9)$$

The last two terms are the mixed $\Sigma^0 - \Lambda$ exchange contributions. The $\mu_i$ are the magnetic moments of the neutral baryons in units of nuclear magnetons (1 nuclear magneton $= e/2m_N = 1.052 \cdot 10^{-14}$ e·cm) and we assumed as valid their tree level expressions in terms of $\mu_D$:

$$\mu_n = -\tfrac{2}{3}\mu_D, \quad \mu_\Lambda = -\tfrac{1}{3}\mu_D,$$

$$\mu_{\Sigma^0} = \tfrac{1}{3}\mu_D, \quad \mu_{\Lambda\Sigma^0} = \frac{1}{\sqrt{3}}\mu_D, \quad (10)$$

while the parameters $A_i$ are a short hand notation for

$$A_\Sigma = -\sqrt{2}B_0\delta b_2,$$

$$A_\Sigma^5 = 2\sqrt{2}B_0 b_2^5,$$

$$A_\Lambda = \sqrt{\tfrac{2}{3}}B_0(\delta b_2 - 2\gamma b_1),$$

$$A_\Lambda^5 = -2\sqrt{\tfrac{2}{3}}B_0(b_2^5 - 2b_1^5), \quad (11)$$

and are related to the weak nonleptonic hyperon decay amplitudes $\Sigma^- \to n\pi^-$, $\Lambda \to p\pi^-$ as it is explained in the next section. Note that the anomalous magnetic moment term with coupling $\mu_D$ is the only contribution to the magnetic moments of neutral baryons.

In the proton case the only possible transition induced by (7) is $p \to \Sigma^+$. It involves only $b_2, b_2^5$ type of coefficients. In the charged case also the anomalous magnetic moment term proportional to $\Delta\mu_F$ does contribute. The expression for the electric dipole moment of the proton induced by Lagrangians (7) and (8) is

$$d_e^p = -\frac{ie}{2m_N}(\alpha - \alpha^*)\frac{A_\Sigma A_\Sigma^5}{M_{\Sigma^+}^2 - M_p^2}$$
$$\times \left(\Delta\mu_{\Sigma^+} + \Delta\mu_p \frac{M_{\Sigma^+}}{M_p}\right). \qquad (12)$$

Here $\Delta\mu_p = \Delta\mu_{\Sigma^+} = \mu_p - 1 = 1/3\mu_D + \Delta\mu_F$ are the anomalous magnetic moments of the proton and the $\Sigma^+$ in units of nuclear magnetons which are equal in the $SU(3)$ limit. They receive contributions from $\Delta\mu_F$ and $\mu_D$. The proton EDM involves only the anomalous magnetic moment terms. It is easy to verify that the tree level diagrams as in Fig. 1 where the anomalous magnetic moment vertex is replaced by the ordinary electromagnetic coupling in the proton case sum to zero.

The expressions (9) and (12) are the tree level contributions to the electric dipole moment of the neutron and the proton respectively. In Section 4 we explicitly show the power counting for the tree level and quantum corrections in the Heavy Baryon Chpt and that one loop corrections are naturally suppressed also in virtue of the absence of large cancellations at tree level.

## 3. Numerical results

The numerical estimate of the tree level contributions to the neutron electric dipole moment in (9) and the proton one in (12) requires the knowledge of the following set of weak and strong parameters: the combination $\alpha - \alpha^*$, the parameters $A_{\Lambda,\Sigma}$, $A_{\Lambda,\Sigma}^5$ and finally the magnetic moment coefficients $\mu_D, \Delta\mu_F$.

The latter can be extracted at tree level from the measured values of the magnetic moments of baryons. At tree level the $SU(3)$ symmetric Coleman-Glashow relations amongst magnetic moments are valid, while they are experimentally violated by about 0.25 nuclear magnetons in average [13]. For the leading quantum corrections to the magnetic moment of baryons in the Heavy Baryon expansion see e.g. [13].

If we use the experimental values of $\mu_p$ and $\mu_n$ to determine $\mu_D$ and $\Delta\mu_F$ and disregard quantum corrections we obtain $\mu_D = -\frac{3}{2}\mu_n = 2.87$ and $\Delta\mu_F = \mu_p + \frac{1}{2}\mu_n - 1 = 0.8365$ for the experimental values $\mu_p = 2.793$, $\mu_n = -1.913$ in units of nuclear magnetons.

The other magnetic moments in the tree level approximation are

$$\mu_\Lambda = \frac{1}{2}\mu_n \quad (-0.613 \pm 0.004 = -0.96),$$

$$\mu_{\Sigma^0} = -\frac{1}{2}\mu_n,$$

$$\mu_{\Lambda\Sigma^0} = -\frac{1}{2}\sqrt{3}\mu_n \quad (\pm 1.61 \pm 0.08 = 1.66),$$

$$\mu_{\Sigma^+} = \mu_p \quad (2.458 \pm 0.010 = 2.793). \qquad (13)$$

In brackets the latest experimental values are indicated [14] and compared with the $SU(3)$ symmetric value. The $\Sigma^0$ magnetic moment has not been measured. Even though the observed magnetic moments do not satisfy the $SU(3)$ relations very well, a more accurate treatment is unnecessary in view of the other uncertainties involved.

The complete determination of the $b_i$, $b_i^5$ parameters requires an additional assumption to relate the experimentally constrained $b_i$ to the unconstrained $b_i^5$. We impose $b_i^5 = b_i$ for $i = 1, 2$ ($b_3$, $b_3^5$ do not enter the EDM expression). This choice seems natural starting from the Lagrangian (4) and is our second main assumption.

The linear combinations $B_0 b_1 m_s$ and $B_0 b_2 m_s$ enter the mass terms of the baryons as implied by (4). Defining $\bar{m} = (m_u + m_d)/2$ we use the combinations of baryon masses which are not affected by the isospin breaking effect at tree level. They are

$$m_N \equiv \frac{M_p + M_n}{2} = m - 4B_0 b_1 \bar{m} - 4B_0 b_2 m_s$$
$$(= 938.91897(28) \text{ MeV}),$$

$$M_{\Sigma^0} = \frac{M_{\Sigma^+} + M_{\Sigma^-}}{2} = m - 4B_0(b_1 + b_2)\bar{m}$$
$$(1192.55(8) = 1193.41(5) \text{ MeV}),$$

$$M_\Xi \equiv \frac{M_{\Xi^0} + M_{\Xi^-}}{2} = m - 4B_0 b_1 m_s - 4B_0 b_2 \bar{m}$$
$$(= 1318.07(11) \text{ MeV}),$$

$$M_\Lambda = m - \frac{4}{3}B_0(b_1 + b_2)(\bar{m} + 2m_s)$$
$$(= 1115.57(6) \text{ MeV}), \qquad (14)$$

where $m = m_B - 4B_0 b_3 (2\bar{m} + m_s)$ takes into account the contribution from the $b_3$ term in (4). The values in brackets are the latest experimental determinations [14]. Using the experimental values of the four masses $m_N, M_{\Sigma^0}, M_{\Xi}, M_\Lambda$ in (14) we can determine the combinations $B_0 b_1$ and $B_0 b_2$ with fixed $\bar{m}$ and $m_s$ (or alternatively the combinations $B_0 b_1 m_s$ and $B_0 b_2 m_s$ if we approximate $\bar{m} = 0$). Using the set $(M_{\Sigma^0}, m_N, M_{\Xi})$ we get

$$M_{\Sigma^0} - m_N = 4B_0 b_2 (m_s - \bar{m}) \simeq 253.63 \,,$$

$$M_{\Sigma^0} - M_{\Xi} = 4B_0 b_1 (m_s - \bar{m}) \simeq -125.52 \qquad (15)$$

and with $\bar{m} = 6$ MeV, $m_s = 175$ MeV we get $2B_0 b_1 = -0.3714$ and $2B_0 b_2 = 0.7504$. These values give $M_\Lambda \simeq 1107.15$ MeV, with $m$ extracted from $M_{\Sigma^0}$, which is a reasonable approximation of the real value. Alternatively if we use the set $(M_{\Sigma^0}, m_N, M_\Lambda)$ the numbers change to $2B_0 b_1 = -0.4088$ and $2B_0 b_2 = 0.7504$ and a slightly too small value for the $\Xi$ mass $M_{\Xi} = 1054.38$.

We still need an additional constraint to fix $b_1, b_2$ together with the coefficients of the weak Lagrangian $\gamma, \delta$, or equivalently $a$ and $b$. The latter enter the weak nonleptonic hyperon decay amplitudes. There are seven measurable amplitudes: $\Sigma^\pm \to n\pi^\pm, \Sigma^+ \to p\pi^0, \Lambda \to n\pi^0, \Lambda \to p\pi^-, \Xi^- \to \Lambda\pi^-, \Xi^0 \to \Lambda\pi^0$ and three isospin relations both for S-wave and P-wave amplitudes. For the chosen four independent S-wave amplitudes at tree level one has (for the standard definition of the S and P-wave amplitudes in weak hyperon decays see e.g. [15])

$$A^{(S)}(\Sigma^+ \to n\pi^+) = 0 \quad (0.06 \pm 0.01) \,,$$

$$A^{(S)}(\Sigma^- \to n\pi^-) = \frac{b-a}{f} \quad (1.88 \pm 0.01) \,,$$

$$A^{(S)}(\Lambda \to p\pi^-) = \frac{a+3b}{\sqrt{6}f} \quad (1.42 \pm 0.01) \,,$$

$$A^{(S)}(\Xi^- \to \Lambda\pi^-) = \frac{a-3b}{\sqrt{6}f} \quad (-1.98 \pm 0.01), \qquad (16)$$

where the last number in parenthesis on the r.h.s. is the corresponding experimental value in units of $G_F m_{\pi^+}^2$ (this is in agreement with Ref. [16] since experimental values for the decay parameters of hyperon nonleptonic decays are unchanged since the Particle

Data Book of 1990 [17]). We use the values $a = -0.58 \pm 0.21$ and $b = 1.40 \pm 0.12$ in units of $G_F m_{\pi^+}^2 f_\pi$ ($f_\pi \simeq 132$ MeV) [16] determined with a tree level least squares fit of the seven measured S-wave amplitudes. These determine the combinations

$$2B_0 \gamma b_1 \equiv a + b = 0.82 \, G_F m_{\pi^+}^2 f_\pi \,,$$

$$2B_0 \delta b_2 \equiv a - b = -1.98 \, G_F m_{\pi^+}^2 f_\pi, \qquad (17)$$

with $G_F m_{\pi^+}^2 f_\pi = 2.98 \cdot 10^{-8}$ GeV. Using instead only the measured values of the decay amplitudes of the two processes $\Sigma^- \to n\pi^-$ and $\Lambda \to p\pi^-$ for a tree level determination of $a$ and $b$ one gets: $a = -0.54$ and $b = 1.34$.

The combinations (17) determine the values of the parameters $A_i$ defined in (11). We obtain

$$A_\Sigma = -\sqrt{2} B_0 \delta b_2 \simeq 1.40 \, G_F m_{\pi^+}^2 f_\pi \,,$$

$$A_\Lambda = \sqrt{\tfrac{2}{3}} B_0 (\delta b_2 - 2\gamma b_1) \simeq -1.48 \, G_F m_{\pi^+}^2 f_\pi, \qquad (18)$$

while the numerical values obtained by using directly the experimental values for $\Sigma^- \to n\pi^-$ and $\Lambda \to p\pi^-$ decays are $1.33 \, G_F m_{\pi^+}^2 f_\pi$ and $-1.42 \, G_F m_{\pi^+}^2 f_\pi$ respectively. Using then $b_i = b_i^5$ and the values in (15) we also get

$$A_\Sigma^5 = 2\sqrt{2} B_0 b_2^5 = \frac{M_{\Sigma^0} - m_N}{\sqrt{2}(m_s - \bar{m})} \simeq 1.06 \,,$$

$$A_\Lambda^5 = -2\sqrt{\tfrac{2}{3}} B_0 (b_2^5 - 2b_1^5) = -\frac{2M_{\Xi} - M_{\Sigma^0} - m_N}{\sqrt{6}(m_s - \bar{m})}$$

$$\simeq -1.22, \qquad (19)$$

while for $A_\Lambda^5$ one gets $-1.28$ if using the experimental values of $m_N, m_\Sigma$ and $m_\Lambda$.

The last parameter to be estimated is $\alpha - \alpha^*$. In terms of the Wilson coefficient function of the effective four-quark operator $Q_6$ we have

$$\frac{\alpha - \alpha^*}{2} = -8i \langle \bar{d}_R d_L \rangle \frac{G_F}{\sqrt{2}} \, \mathrm{Im} \, C_6, \qquad (20)$$

where

$$C_6(\mu) = V_{ud} V_{us}^* z_6(\mu) - V_{td} V_{ts}^* y_6(\mu) \,, \qquad (21)$$

$$\mathrm{Im} \, C_6(\mu) = -\mathrm{Im} \, V_{td} V_{ts}^* \, y_6(\mu)$$

$$= c_{23} s_{23} s_{13} \sin \delta_{13} \, y_6(\mu). \qquad (22)$$

The estimate of the size of the coefficient $y_6(\mu)$ is affected by large uncertainty. We use the renormalization scheme independent definition in [12] where the next-to-leading corrections at a given $\mu$ to the effective hamiltonian are shifted into the Wilson coefficient functions. As noticed there the coefficient $y_6(\mu)$ is a very sensitive function of $\Lambda_{\overline{MS}}$ and $\mu$ being next-to-leading corrections sizable. We use the approximate value $y_6 \simeq -0.13$ at $\mu \simeq 1$ GeV and $\Lambda_{\overline{MS}} \simeq 300$ MeV. The CKM matrix elements are [14] $s_{23} = |V_{cb}| = 0.040 \pm 0.005$, $s_{13} = |V_{ub}| \simeq 0.0032$ extracted from $|V_{ub}/V_{cb}| = 0.08 \pm 0.02$. We approximate cosines to unity and put $\sin\delta_{13} \simeq 1$ [18]. Using for the scalar quark condensate $\langle \bar{d}_R d_L \rangle = -1/2 \cdot (0.235)^3$ GeV$^3$, this gives $(\alpha - \alpha^*)/2 = -i\,5.48 \times 10^{-12}$ GeV.

For the final prediction we use $A_\Sigma = 1.40\,G_F m_{\pi^+}^2 f_\pi$, $A_\Lambda = -1.48\,G_F m_{\pi^+}^2 f_\pi$, $G_F m_{\pi^+}^2 f_\pi = 3.0 \times 10^{-8}$ GeV and the experimental values for the magnetic moments and baryon masses in the EDM formulas (9) and (12). We use $\mu_{\Lambda\Sigma^0} = +1.61$. We predict the following value for the neutron EDM:

$$d_e^n = -\frac{ie}{2m_N}\frac{\alpha - \alpha^*}{2}\Big\{-9.18 \times 10^{-8}\ \text{GeV}^{-1}$$
$$+\,7.89 \times 10^{-8}\ \text{GeV}^{-1} - 34.02 \times 10^{-8}\ \text{GeV}^{-1}$$
$$-\,20.04 \times 10^{-8}\ \text{GeV}^{-1} - 20.19 \times 10^{-8}\ \text{GeV}^{-1}$$
$$-\,15.87 \times 10^{-8}\ \text{GeV}^{-1}\Big\}, \tag{23}$$

where the numerical value in each line corresponds to the relative expression in (9). This shows the absence of large cancellations. This gives

$$d_e^n \approx 5.3 \times 10^{-32}\ \text{e} \cdot \text{cm}. \tag{24}$$

For the proton we have

$$d_e^p \approx -3.6 \times 10^{-32}\ \text{e} \cdot \text{cm}, \tag{25}$$

where we used the experimental values for $\Delta\mu_{\Sigma^+} = 1.458$ in units of nuclear magnetons, $M_{\Sigma^+} = 1189.37(6)$ MeV. Both the neutron and proton electric dipole moments acquire the opposite sign if we use instead $b_i^5 = -b_i$.

## 4. Power counting and loops

The purpose of this section is to derive the power counting rules for quantum corrections to the tree level

electric dipole moment contributions. They can be consistently derived within the Heavy Baryon chiral perturbation expansion (HBChPt).

One comment is in order concerning the heavy baryon mass limit of our tree level contribution to the electric dipole moment term. In the ordinary Heavy Baryon ChPt the EDM term is one of the possible counterterms which appears in the tree level Lagrangian at order $p^2$ in the derivative expansion. As an example the first tree level diagram shown in Fig. 1 of the full baryon theory leads to the following contributions in the heavy baryon mass limit:

$$-2i\frac{\bar{B}_f^v(v^\mu S_v^\nu - v^\nu S_v^\mu)F_{\mu\nu}B_i^v}{m_i^2 - m_{\text{pole}}^2}$$
$$+\,2i\frac{\bar{B}_f^v(k^\mu S_v^\nu - k^\nu S_v^\mu)F_{\mu\nu}B_i^v}{(m_i^2 - m_{\text{pole}}^2)(m_i - m_{\text{pole}})}. \tag{26}$$

These terms are enhanced by one inverse power of the baryon mass splitting respect to the ordinary EDM counterterm appearing at order $p^2$ in the derivative expansion.

The heavy baryon mass limit in (26) of the tree level contribution to the electric dipole moment derived in the full theory shows that the leading term appears at order $p$ in the derivative expansion, while the usual first counterterm to the electric dipole moment appears at order $p^2$ both in the full theory and in the HBChPt. This power counting for (26) is consistent with the fact that baryon propagators count as $1/p$ in the derivative expansion and that the leading parity violating vertex in the HBChPt appears at order $p$ i.e. $\bar{B}_v \gamma_5 \gamma_\mu D^\mu B_v \rightarrow -2\bar{B}_v S_v^\mu D_\mu B_v$. In this case the octet mass splitting, proportional to the off-shellness of the baryon propagator, counts as $O(p)$, while in the strange quark mass expansion it is $O(m_s)$.

The full baryon Lagrangian contributing to the one loop corrections to the electric dipole moment term includes: the Lagrangian which mediates $|\Delta S| = 1$ weak interactions, given by (6) with the inclusion of the meson field through the substitution $\lambda_6 \rightarrow \xi^\dagger \lambda_6 \xi$, the usual strong interaction Lagrangian with the inclusion of meson interactions starting at order $p$, the magnetic moment term at order $p^2$, the electric dipole moment counterterm at order $p^2$ and the Lagrangian (7) with the inclusion of the meson field through the substitution $\lambda_- \rightarrow \xi\lambda_-\xi$, $\lambda_+ \rightarrow \xi^\dagger\lambda_+\xi^\dagger$. Both octet and de-
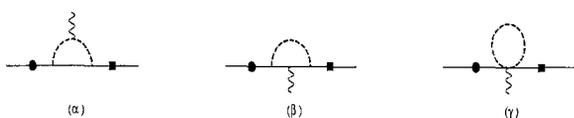
Fig. 2. One loop diagrams of class a) which contribute to the neutron and proton electric dipole moments. The circle vertex is the parity conserving (p.c.) vertex. The box is the parity violating (p.v.) vertex. The photon insertion in ($\beta$) and ($\gamma$) is the magnetic moment vertex. Internal baryon propagators can be also decuplet.



Fig. 3. One loop diagrams of class c) which contribute to the neutron and proton electric dipole moments. The photon insertion is always the EDM counterterm that appears at order $p^2$ in the chiral perturbation expansion.

cuplet states can contribute inside the loop.

One loop diagrams contributing to the EDM term can be divided into four classes: a) corrections to the one loop contribution to the magnetic moment through the insertion of the p.v. and p.c. vertices of (7), b) one loop correction to the p.c. vertex and to the p.v. vertex in (7), c) one loop corrections to the EDM vertex appearing at order $p^2$, d) one-loop diagrams with the meson loop bridging several of the p.v., p.c. and magnetic moment vertices, including the case where the p.v. and/or the p.c. vertices emit the meson line. Diagrams with the insertion of a photon-meson-baryon-baryon vertex do not contribute. The first class is shown in Fig. 2 and the one loop corrections to the magnetic moment of baryons in the HBChPt have been derived in [13].

The magnetic moment one loop contribution is of order $p^3$ in diagram ($\alpha$) and of order $p^4$ in diagrams ($\beta$) and ($\gamma$). So the full contribution to the nucleon EDM is of order $p^2$ in diagram ($\alpha$) and of order $p^3$ in diagrams ($\beta$) and ($\gamma$). In the strange quark mass expansion the counting is somewhat anomalous because the leading tree level contribution starts at order $1/m_s$. One loop diagrams give nonanalytic corrections in the strange quark mass. The one loop in diagram ($\alpha$) gives a correction $\sqrt{m_s}$ to the tree level diagram, while the one loop in diagrams ($\beta$) and ($\gamma$) gives a $m_s \ln m_s$ correction.

Diagrams of class b) have the same counting as the corresponding diagrams of class a). They include also the electric charge vertex insertion in the proton case.

Diagrams of class c) start at order $p^4$ and are shown in Fig. 3. Diagrams of class d) have the same counting as those of a) and only appear at order $p^2$.

This shows that within the chiral perturbation expansion the tree level contributions to the electric dipole moment are in fact the leading contributions. One loop corrections are suppressed both in the derivative and strange quark mass expansion.

## 5. Conclusions

In this letter we have provided a new way of deriving the long distance weak contribution to the proton and neutron electric dipole moments. Our final results are

$$d_e^n \approx \pm 5.3 \times 10^{-32} \ e \cdot cm,$$

$$d_e^p \approx \mp 3.6 \times 10^{-32} \ e \cdot cm. \tag{27}$$

The opposite sign is a consequence of the opposite sign in the relevant anomalous magnetic moments. These numbers are quite comparable to those derived earlier in the quark model and again show that the weak contribution to the electric dipole moments is small and of order $10^{-32}$ e · cm. There are relatively few cancellations involved in this calculation. We therefore do not expect very large higher order corrections. The main uncertainties in the result come from the underlying assumptions: parity violating terms in the weak $|\Delta S| = 1$ Lagrangian are negligible, parity violating terms in the strong light quark mass sector are of the same size as the parity conserving ones. The other source of uncertainty is the estimate of the parameter $y_6(\mu)$ in the Wilson coefficient function of the effective four-quark operator $Q_6$, which we determined according to the renormalization scheme independent definition in [12]. This result should not be added to those obtained in the quark model. The value of $\langle \bar{q}q \rangle$ is related to the production of the constituent quark mass and the contribution as estimated here is thus related to the one obtained in the quark model.

## 6. Acknowledgements

# References

[1] R.J. Crewther et al., Phys. Lett. B 88 (1979) 123; B 91 (1980) 487 (E).

[2] A. Pich and E. de Rafael, Nucl. Phys. B 367 (1991) 313.

[3] N.F. Ramsey, Ann. Rev. Nucl. Part Science 40 (1990) 1; S.M. Barr and W.J. Marciano, in: CP Violation, ed., C. Jarlskog, Advanced Series on Directions in High Energy Physics, Vol. 3 (World Scientific, 1989).

[4] I.S. Altarev et al., Phys. Lett. B 276 (1992) 242.

[5] D. Cho, K. Singster and E.A. Hinds, Phys. Rev. Lett. 63 (1989) 2559.

[6] E.P. Shabalin, Sov. J. Nucl. Phys. 32 (1980) 228.

[7] B.F. Morel, Nucl. Phys. B 157 (1979) 23.

[8] D.V. Nanopoulos, A. Yildiz and P. Cox, Phys. Lett. B 87 (1979) 53.

[9] A.I. Vainstein et al., JETP Lett. 22 (1975) 55; M.A. Shifman et al., Nucl. Phys. B 120 (1977) 316; M.B. Wise and E. Witten, Phys. Rev. D 20 (1979) 1216.

[10] M.B. Gavela et al., Phys. Lett. B 109 (1982) 215.

[11] M.B. Gavela et al., Phys. Lett. B 101 (1981) 417.

[12] A.J. Buras et al., Nucl. Phys. B 370 (1992) 69.

[13] E. Jenkins, M. Luke, A.V. Manohar and M.J. Savage, Phys. Lett. B 302 (1993) 482.

[14] Particle Data Book, Phys. Rev. D 50 (1994) 1173.

[15] J. Bijnens, H. Sonoda and M.B. Wise, Nucl. Phys. B 261 (1985) 185.

[16] E. Jenkins, Nucl. Phys. B 375 (1992) 561.

[17] Particle Data Book, Phys. Lett. B 239 (1990).

[18] A. Pich and J. Prades, Phys. Lett. B 346 (1995) 342.