Kac-Moody Symmetries and Gauged Supergravity

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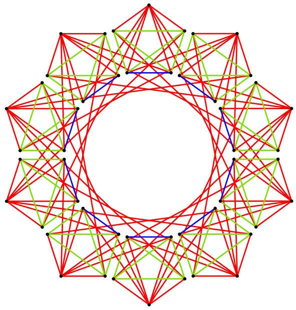

Conclusions

In this thesis we have shown that Kac-Moody symmetries play a unifying role in supergravity. In particular, we have given evidence that certain over- and very-extended Kac-Moody algebras are intimately related to gauged supergravity. Not only do the Kac-Moody algebras correctly predict all the physical degrees of freedom of the supergravities in all dimensions, but they also contain deformation- and top-form potentials that correspond to all the known gaugings of those supergravities. This is a promising unifying picture, although there are a few mismatches. First, the Kac-Moody algebra E_{11} predicts more top-forms than there are quadratic constraints in the corresponding gauged maximal supergravity. Secondly, matching the equations of motion succeeds only up to a certain level. Lastly, the infinite tower of exotic representations in Kac-Moody algebras have no obvious counterpart on the supergravity side.

The deformation potentials, or $(D-1)$-forms, that follow from over- or very-extended Kac-Moody algebras occur in precisely the same representations of the global symmetry group as the embedding tensors of gauged supergravity. Also, the way the $(D-1)$-forms are reached by commutators of the fundamental p-forms tells us whether they correspond to a 'traditional' gauged supergravity, or a massive deformation.

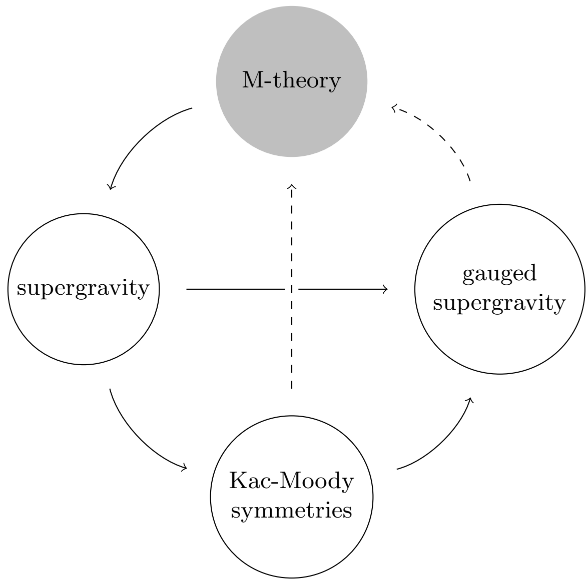

Figure 7.1: The interrelations between M-theory, (gauged) supergravity, and Kac-Moody symmetries.

Furthermore, all the known quadratic constraints of the embedding tensor have a top-form, or D-form, counterpart in very-extended Kac-Moody algebras. However, in certain cases, such as the $D = 3$ or IIB case for E_{11}, there are more top-forms than quadratic constraints. For the IIB case it is known that these extra top-forms can be interpreted to couple to D9-branes of Type IIB string theory [11]. A similar scenario could be possible for the $D = 3$ case, but such an interpretation is currently lacking.

As for the dynamic comparison, we have found that the equations of motion of $D = 3$ gauged maximal supergravity adapted to a one-dimensional language can in part be matched to the E_{10} equations, even though the latter have a priori a rather different form. For one thing, the absence of gauge-covariant derivatives on the E_{10} side agrees with the supergravity expressions, once a gauge-fixing condition which is inevitable for the comparison, has been imposed. Moreover, in spite of the fact that on the E_{10} side all fields appear with a 'kinetic' term, the (truncated) duality relation between vectors and scalars expected from supergravity naturally follows via integrating the one-dimensional equations of motion. Finally, the embedding tensor automatically appears as an integration constant in the right representation. In this sense, none of the essential ingredients of gauged supergravity have to be introduced by hand, but rather they naturally follow from the E_{10} sigma model.

However, a mismatch occurs at higher levels: the scalar potential of gauged supergravity is not fully reproduced by E_{10}. This is somewhat reminiscent to a discrepancy encountered in higher dimensions, once spatial gradients are introduced as the duals of higher-level fields [23].

In total we are led to conclude that further insights, like those of [27, 29], are required in order to understand the precise relation between supergravity theories and the E_{10} sigma model. It would be interesting to see whether modifications and/or extensions of the E_{10} model are possible to compensate for the present mismatches. We note that mismatches already occur before comparing to *gauged* supergravity and so an ultimate resolution of the present discrepancies must await a better understanding of the basic picture.

It is therefore too soon to claim that the Kac-Moody symmetries of E_{11} or E_{10} are truly the symmetries of M-theory, or are the fundamental symmetries of nature. One might conjecture that one of the missing pieces of the puzzle, the infinite tower of exotic representations, might encode some M-theory degrees of freedom. In the same direction it is thought that some of the gauge deformations of supergravity also encode M-theory degrees of freedom [96]. As E_{11} somehow 'knows' of the gauged deformations via deformation and top-form potentials, one might argue that it indirectly contains information on M-theory (see also Figure 7.1).

To conclude, the Kac-Moody symmetries provide an elegant unifying framework for the symmetries of supergravity, although the correspondence is not perfect. Future work might lift the discrepancies, and hopefully allow a sneak peak into the structure of M-theory.