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## Kac-Moody Symmetries and Gauged Supergravity

Nutma, Teake Aant

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*Document Version*

Publisher's PDF, also known as Version of record

*Publication date:*

2010

[Link to publication in University of Groningen/UMCG research database](#)

*Citation for published version (APA):*

Nutma, T. A. (2010). *Kac-Moody Symmetries and Gauged Supergravity*. s.n.

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# Kac-Moody symmetries and gauged supergravity

Proefschrift

ter verkrijging van het doctoraat in de  
Wiskunde en Natuurwetenschappen  
aan de Rijksuniversiteit Groningen  
op gezag van de  
Rector Magnificus, dr. F. Zwarts,  
in het openbaar te verdedigen op  
maandag 13 september 2010  
om 11.00 uur

door

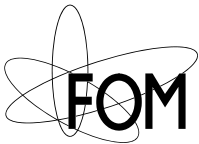
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geboren op 1 juni 1981  
te Heerenveen

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The work described in this thesis was performed at the Centre for Theoretical Physics of the University of Groningen. This work is part of the research programme of the Foundation for Fundamental Research on Matter (FOM), which is part of the Netherlands Organisation for Scientific Research (NWO).

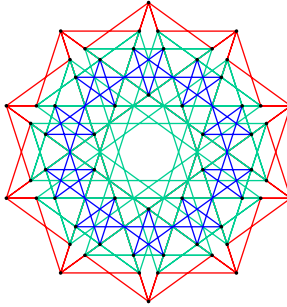
On the cover: the root system of the hyperbolic Kac-Moody algebra  $E_{10}$  up to height 124 projected onto the Coxeter plane of a regular  $E_8$  subalgebra.

Printed by GVO drukkers & vormgevers B.V. | Ponsen & Looijen

ISBN 978-90-367-4473-7 (printed version)

ISBN 978-90-367-4474-4 (electronic version)

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