# Compositional analysis and control of dynamical systems

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# 9

# Conclusions

## 9.1. Summary

Compositional techniques are widely used in formal verification to analyze complex computer programs. The difficulty lies in the structure of these programs which often consist of large numbers of interacting concurrent processes. As a result, the state dimension is huge since it grows exponentially with the number of components. Models of physical systems are often equally complex as they are characterized by networks of interacting subsystems as well. In this thesis we show that compositional analysis methods developed for the verification of concurrent programs can be used to simplify analysis and synthesis problems in systems theory and control.
In Chapter 3 we adopt reasoning schemes based on simulation theory to analyze properties of interconnected linear systems. As one of the main results of this thesis we prove that circular assume-guarantee reasoning is sound and complete for linear systems. We extend this result to other types of interconnections and arbitrary numbers of components involved. Moreover, compositional and assume-guarantee reasoning schemes are also valid when using bisimulation instead of simulation relations. Altogether, this yields a comprehensive theory of compositional analysis methods for linear systems. We illustrate these findings with an example from circuit theory using the fact that simulation relations give rise to abstractions of linear systems. This makes it possible to replace a large interconnected system by a lower-dimensional one consisting of abstractions of the original components. Since abstractions computed by simulation can replace the original systems the verification task thus becomes less complex. Most of the results for linear systems can be generalized to nonlinear systems. However, as shown in Chapter 4, additional constant rank assumptions are needed to prove soundness of compositional and assume-guarantee reasoning for nonlinear simulation. In Chapter 5 we discuss passivity properties of both linear and nonlinear systems using compositional analysis techniques. The pivotal observation is that passivity can be captured as a nonlinear simulation relation between the system to be checked and the corresponding one-dimensional system given by the differential dissipation inequality. Compositionality of nonlinear simulation then allows to

infer passivity properties of interconnections from passivity properties of the components involved. In the case of open feedback, also the converse holds true. Since passivity is related to other concepts such as Lyapunov stability, the results of Chapter 5 can serve as a paradigm how to verify properties of complex interconnected systems using compositional analysis techniques.
Complementary to compositional analysis is the notion of decentralized control. The basic principle is to delegate a global control task to local controllers interacting with subsystems of the overall plant. In Chapter 6 we use compositional and assume-guarantee reasoning to derive decentralized control schemes. Combined with conditions for achievable simulation we obtain two bottom-up and one top-down scheme. The latter is particularly interesting since it proves that the existence of a global diagonally decoupled controller is equivalent to the existence of local feedback controllers with respect to the same global control target provided the global specification can be decomposed.
The last two chapters are dedicated to the study of switching linear systems as a special class of hybrid systems. Combining elements of discrete and continuous dynamics, the study of hybrid systems is an intrinsically interdisciplinary field. Like in previous chapters, our approach unites a representation stemming from systems theory – we describe the continuous part of the hybrid dynamics by means of differential equations rather than trajectories – with analysis methods borrowed from formal verification. This allows us to use structural hybrid (bi)simulation relations to prove soundness of compositional reasoning schemes for switching linear systems. Before these results can be generalized to switching linear systems with location invariants a bisimulation theory for these systems has to be developed. In particular, the influence of the continuous on the discrete dynamics due to polyhedral constraints on the state spaces requires extra conditions for bisimulation equivalence. As a first step, we characterize bisimulation relations for constrained linear systems. We then derive a structural notion of bisimulation relations for switching linear systems with location invariants which allows to directly apply the previous results to hybrid case.

## 9.2. Recommendations for future work

To extend the results obtained in this thesis for compositional analysis and control of dynamical systems, we recommend to investigate the following problems in the future:

- In Chapter 5 we mainly used compositional reasoning to infer passivity properties of both linear and nonlinear control systems. A natural extension would be to consider deduction schemes based on assume-guarantee reasoning. E.g., consider two nonlinear systems $\Sigma_i, i = 1, 2$, and assume that $\Sigma_i$ can be stabilized by arbitrary passive systems $\Xi_i$,

$$\begin{array}{lccc} S_1 : & \Sigma_1\|_{\mathrm{cl}}\Xi_2 & \preccurlyeq & \Xi_1\|_{\mathrm{cl}}\Xi_2 \\ S_2 : & \Xi_1\|_{\mathrm{cl}}\Sigma_2 & \preccurlyeq & \Xi_1\|_{\mathrm{cl}}\Xi_2 \end{array} . \tag{9.1}$$

Does (9.1) then imply that the interconnection $\Sigma_1\|\Sigma_2$ is also stable, in other words does there exist a full simulation relation $S$ of $\Sigma_1\|\Sigma_2$ by $\Xi_1\|_{\mathrm{cl}}\Xi_2$?

- For most parts of this thesis we assumed that a global specification is given as the interconnection of subspecifications related to subsystems of the global plant. For some applications, however, it would be useful to have available methods to decompose global specifications, i.e., given a specification $\Sigma_Q$ and a plant $\Sigma_P = \Sigma_{P_1}\|\Sigma_{P_2}$, under which conditions and how can $\Sigma_Q$ be decomposed into two subspecifications $\Sigma_{Q_i}, i = 1, 2$, such that $\Sigma_Q \preccurlyeq \Sigma_{Q_1}\|\Sigma_{Q_2}$?

- In computer science, implementations and specifications can usually be expressed in the same language, e.g. as labeled transition systems. The proof rules for compositional and assume-guarantee reasoning are based on this idea. In this thesis, we assumed throughout that the specification (the property to be checked) can be expressed as an input-state-output system. Chapter 5 showed how to this is done for passivity and, at least partially, for stability as well. It would be interesting to further investigate whether system theoretic properties such as observability, control performance etc., can be formalized as control systems themselves.

- The area of hybrid systems remains a promising application for compositional techniques. In particular, deriving checkable conditions and algorithms to compute structural hybrid (bi)simulations for general systems with location invariants and guard conditions is an open problem.

# A

# Soundness of circular assume-guarantee reasoning for linear systems

## A.1. Proof of Lemma 3.11

We give the proof with respect to $S_I + \bar{S}_I$, the result for $S_{II} + \bar{S}_{II}$ follows from symmetry.
Take any $(x_{P_1}, \bar{x}_{Q_2}, x_{Q_1}, -x_{Q_2}) \in S_1$. Since all components fulfill

$$C_{P_1} x_{P_1} = C_{Q_1} x_{Q_1} = 0, C_{Q_2} x_{Q_2} = -C_{Q_2} \bar{x}_{Q_2} = 0 \tag{A.1}$$

and

$$H_{P_1} x_{P_1} = H_{Q_1} x_{Q_1} = 0, H_{Q_2} x_{Q_2} = -H_{Q_2} \bar{x}_{Q_2} = 0\,, \tag{A.2}$$

condition (iii) in Theorem 3.4 is fulfilled. By Definition 3.2 there exists a $(x_{P_1}, x_{Q_2}, x_{Q_1}, \bar{x}_{Q_2}) \in S_I$ and since $S_I$ is a simulation relation, condition (ii) in Theorem 3.4 ensures that there exists a $(w_{P_1}, w_{Q_2}, w_{Q_1}, \bar{w}_{Q_2}) \in S_I$ such that

$$\begin{bmatrix} A_{P_1} x_{P_1} \\ A_{Q_2} x_{Q_2} \\ A_{Q_1} x_{Q_1} \\ A_{Q_2} \bar{x}_{Q_2} \end{bmatrix} = \begin{bmatrix} w_{P_1} \\ w_{Q_2} \\ w_{Q_1} \\ \bar{w}_{Q_2} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ L_{Q_1}\alpha \\ L_{Q_2}\beta \end{bmatrix} \tag{A.3}$$

Note that since $(w_{P_1}, w_{Q_2}, w_{Q_1}, \bar{w}_{Q_2}) \in S_I$, $(w_{P_1}, \bar{w}_{Q_2}, w_{Q_1}, -w_{Q_2}) \in \bar{S}_I$. Hence

$$\begin{bmatrix} A_{P_1} x_{P_1} \\ A_{Q_2} \bar{x}_{Q_2} \\ A_{Q_1} x_{Q_1} \\ -A_{Q_2} x_{Q_2} \end{bmatrix} = \begin{bmatrix} w_{P_1} \\ \bar{w}_{Q_2} \\ w_{Q_1} \\ -w_{Q_2} \end{bmatrix} + \begin{bmatrix} 0 \\ L_{Q_2}\beta \\ L_{Q_1}\alpha \\ 0 \end{bmatrix} \tag{A.4}$$

Since $S_I$ is a simulation relation, there exists for every $x \in \mathrm{im} G_{Q_2}$ an element $(0, x, \tilde{x}_{Q_1}, \tilde{x}_{Q_2}) \in S_I$ such that

$$\begin{bmatrix} 0 \\ x \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ x \\ \tilde{x}_{Q_1} \\ \tilde{x}_{Q_2} \end{bmatrix} + \mathrm{im} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ L_{Q_1} & 0 \\ 0 & L_{Q_2} \end{bmatrix} \tag{A.5}$$

Therefore, (A.4) can be rewritten as

$$\begin{bmatrix} A_{P_1}x_{P_1} \\ A_{Q_2}\bar{x}_{Q_2} \\ A_{Q_1}x_{Q_1} \\ -A_{Q_2}x_{Q_2} \end{bmatrix} = \underbrace{\begin{bmatrix} w_{P_1} \\ \bar{w}_{Q_2} \\ w_{Q_1} \\ -w_{Q_2} \end{bmatrix}}_{\in \bar{S}_I} + \underbrace{\begin{bmatrix} 0 \\ L_{Q_2}\beta \\ \tilde{x}_{Q_1} \\ \tilde{x}_{Q_2} \end{bmatrix}}_{\in S_I} + \mathrm{im}\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ L_{Q_1} & 0 \\ 0 & L_{Q_2} \end{bmatrix} \tag{A.6}$$

$$\in S_I + \bar{S}_I + \mathrm{im}\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ L_{Q_1} & 0 \\ 0 & L_{Q_2} \end{bmatrix}$$

which proves that condition (ii) in Theorem 3.4 is also fulfilled. Condition (i) is also fulfilled due to $S_I$ being a simulation relation. Indeed,

$$\mathrm{im}\begin{bmatrix} G_{P_1} & 0 \\ 0 & G_{Q_2} \\ G_{Q_1} & 0 \\ o & G_{Q_2} \end{bmatrix} + \mathrm{im}\begin{bmatrix} L_{P_1} & 0 \\ 0 & L_{Q_2} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \subset S_I + \mathrm{im}\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ L_{Q_1} & 0 \\ 0 & L_{Q_2} \end{bmatrix} \tag{A.7}$$

$$\subset S_I + \bar{S}_I + \mathrm{im}\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ L_{Q_1} & 0 \\ 0 & L_{Q_2} \end{bmatrix}$$

Moreover, since $S_I$ is a full simulation relation, $\Pi_{X_{P_1}X_{Q_2}}S_I = \Pi_{X_{P_1}X_{Q_2}}(S_I + \bar{S}_I) = \mathcal{X}_{P_1} \times \mathcal{X}_{Q_2}$ and thus $S_I + \bar{S}_I$ is a full simulation relation of $\Sigma_{P_1}\|\Sigma_{Q_2}$ by $\Sigma_{Q_1}\|\Sigma_{Q_2}$.

## A.2. Proof of Lemma 3.12

Again, the statement will be proved only for $S_I^{\mathrm{sym}}$. Condition (i) and fullness of $S_I^{\mathrm{sym}}$ follow from fullness of $S_I$. Condition (iii) holds true since by interchanging the components, still $C_{Q_2}x_{Q_2} = C_{Q_2}\bar{x}_{Q_2}$ as well as $H_{Q_2}x_{Q_2} = H_{Q_2}\bar{x}_{Q_2}$. Finally, condition (ii) is proved analogously to (A.6) observing that for every $(x_{P_1}, \bar{x}_{Q_2}, x_{Q_1}, x_{Q_2}) \in \tilde{S}_I$, there exists a $(x_{P_1}, x_{Q_2}, x_{Q_1}, \bar{x}_{Q_2}) \in S_I$ for which

$$\begin{bmatrix} A_{P_1}x_{P_1} + B_{P-1}C_{Q_2}x_{Q_2} \\ A_{Q_2}x_{Q_2} + B_{Q_2}C_{P_1}x_{P_1} \\ A_{Q_1}x_{Q_1} + B_{Q_1}C_{Q_2}\bar{x}_{Q_2} \\ A_{Q_2}\bar{x}_{Q_2} + B_{Q_2}C_{Q_1}x_{Q_1} \end{bmatrix} = \underbrace{\begin{bmatrix} w_{P_1} \\ w_{Q_2} \\ w_{Q_1} \\ \bar{w}_{Q_2} \end{bmatrix}}_{\in S_I} + \mathrm{im}\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ L_{Q_1} & 0 \\ 0 & L_{Q_2} \end{bmatrix} \tag{A.8}$$

Furthermore, since $S_I$ is a simulation relation,

$$\operatorname{im}\begin{bmatrix} 0 \\ L_{Q_2} \\ 0 \\ 0 \end{bmatrix} \subset S_I + \operatorname{im}\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ L_{Q_1} & 0 \\ 0 & {}_{Q_2} \end{bmatrix} \tag{A.9}$$

and therefore

$$\begin{aligned}\begin{bmatrix} A_{P_1} & B_{P_1}C_{Q_2} & 0 & 0 \\ B_{Q_2}C_{P_1} & A_{Q_2} & 0 & 0 \\ 0 & 0 & A_{Q_1} & B_{Q_1}C_{Q_2} \\ 0 & 0 & B_{Q_2}C_{Q_1} & A_{Q_2} \end{bmatrix} \tilde{S}_I \subset \\ S_I + \tilde{S}_I + \operatorname{im}\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ L_{Q_1} & 0 \\ 0 & L_{Q_2} \end{bmatrix}\end{aligned} \tag{A.10}$$

## A.3. Proof of Lemma 3.13

Again, we will only prove the first half of the lemma. Since $S_I$ is a full simulation relation, it holds that for every $(0, x)$ there exists $x_{Q_1}, x_{Q_2}$ such that $(0, x, x_{Q_1}, x_{Q_2}) \in S_I$ with $x_{Q_1} \in \ker C_{Q_1} \cap \ker H_{Q_1}$. If we take $x \in \ker C_{Q_2} \cap \ker H_{Q_2}$ then also $x_{Q_2} \in \ker C_{Q_2} \cap \ker H_{Q_2}$. Then $(0, x_{Q_2}, x_{Q_1}, -x) \in \bar{S}_I$ and therefore

$$\begin{bmatrix} 0 \\ x \\ x_{Q_1} \\ x_{Q_2} \end{bmatrix} - \begin{bmatrix} 0 \\ x_{Q_2} \\ x_{Q_1} \\ -x \end{bmatrix} = \begin{bmatrix} 0 \\ x - x_{Q_2} \\ 0 \\ x + x_{Q_2} \end{bmatrix} \in S_I + \bar{S}_I \tag{A.11}$$

Moreover, $(0, x + x_{Q_2}, 0, x - x_{Q_2}) \in (S_I + \bar{S}_I)^{\text{sym}}$ and by the subspace property also

$$\begin{bmatrix} 0 \\ x - x_{Q_2} \\ 0 \\ x + x_{Q_2} \end{bmatrix} + \begin{bmatrix} 0 \\ x + x_{Q_2} \\ 0 \\ x - x_{Q_2} \end{bmatrix} = 2\begin{bmatrix} 0 \\ x \\ 0 \\ x \end{bmatrix} \in (S_I + \bar{S}_I)^{\text{sym}} \tag{A.12}$$

## A.4. Proof of Theorem 3.14

Firstly, it is easy to see that $S$ indeed defines a linear subspace.
Secondly, we have to show that it defines a simulation relation of $\Sigma_{P_1} \| \Sigma_{P_2}$ by

$\Sigma_{Q_1} \| \Sigma_{Q_2}$. Take any $(x_{P_1}, x_{P_2}, x_{Q_1}, x_{Q_2}) \in S$. Then there exist $(x_{P_1}, x_{Q_2}, x_{Q_1}, \bar{x}_{Q_2}) \in (S_I + \bar{S}_I)^{\text{sym}}$ and $(x_{Q_1}, x_{P_2}, \bar{x}_{Q_1}, x_{Q_2}) \in (S_{II} + \bar{S}_{II})^{\text{sym}}$ with the property that

$$C_{P_1} x_{P_1} = C_{Q_1} x_{Q_1} = C_{Q_1} \bar{x}_{Q_1}, C_{P_2} x_{P_2} = C_{Q_2} x_{Q_2} = C_{Q_2} \bar{x}_{Q_2} \tag{A.13}$$

and

$$H_{P_1} x_{P_1} = H_{Q_1} x_{Q_1} = H_{Q_1} \bar{x}_{Q_1}, H_{P_2} x_{P_2} = H_{Q_2} x_{Q_2} = H_{Q_2} \bar{x}_{Q_2} \tag{A.14}$$

so that condition (iii) of Theorem 3.4 is already fulfilled. To show that condition (i) also holds, take first any $d_{P_1} \in \operatorname{im} L_{P_1}$. Since $(S_I + \bar{S}_I)^{\text{sym}}$ is a simulation relation, there exist $x_{Q_i} \in \operatorname{im} L_{Q_i}, i = 1, 2$ such that

$$\begin{bmatrix} d_{P_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} d_{P_1} \\ 0 \\ x_{Q_1} \\ x_{Q_2} \end{bmatrix} + \operatorname{im} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ L_{Q_1} & 0 \\ 0 & L_{Q_2} \end{bmatrix} \tag{A.15}$$

with $(d_{P_1}, 0, x_{Q_1}, x_{Q_2}) \in (S_I + \bar{S}_I)^{\text{sym}}$. Since $x_{Q_1} \in \operatorname{im} G_{Q_1}$ and $(S_{II} + \bar{S}_{II})^{\text{sym}}$ is also a full simulation relation, there exist $\bar{x}_{Q_1} \in \operatorname{im} L_{Q_1}$ and $\bar{x}_{Q_2} \in \operatorname{im} L_{Q_2} \cap \ker C_{Q_2} \cap \ker H_{Q_1}$ such that $(x_{Q_1}, 0, \bar{x}_{Q_1}, \bar{x}_{Q_2}) \in (S_{II} + \bar{S}_{II})^{\text{sym}}$. By Lemma 3.13, there exists an element $(0, \bar{x}_{Q_2}, 0, \bar{x}_{Q_2}) \in (S_I + \bar{S}_I)^{\text{sym}}$ and therefore

$$\begin{aligned} \begin{bmatrix} d_{P_1} \\ 0 \\ x_{Q_1} \\ x_{Q_2} \end{bmatrix} + \begin{bmatrix} 0 \\ \bar{x}_{Q_2} \\ 0 \\ \bar{x}_{Q_2} \end{bmatrix} = \begin{bmatrix} d_{P_1} \\ \bar{x}_{Q_2} \\ x_{Q_1} \\ x_{Q_2} + \bar{x}_{Q_2} \end{bmatrix} &\in (S_I + \bar{S}_I)^{\text{sym}}, \\ \begin{bmatrix} x_{Q_1} \\ 0 \\ \bar{x}_{Q_1} \\ \bar{x}_{Q_2} \end{bmatrix} &\in (S_{II} + \bar{S}_{II})^{\text{sym}} \\ \Longrightarrow \begin{bmatrix} d_{P_1} \\ 0 \\ x_{Q_1} \\ \bar{x}_{Q_2} \end{bmatrix} &\in S \end{aligned} \tag{A.16}$$

for any $d_{P_1} \in \operatorname{im} L_{P_1}$ with $x_{Q_1} \in \operatorname{im} L_{Q_1}$ and $\bar{x}_{Q_2} \in \operatorname{im} L_{Q_2} \cap \ker C_{Q_2} \cap \ker H_{Q_2}$. Hence,

$$\operatorname{im} \begin{bmatrix} L_{P_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} \subset S + \operatorname{im} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ L_{Q_1} & 0 \\ 0 & L_{Q_2} \end{bmatrix} \tag{A.17}$$

By the same arguments one can also show that

$$\operatorname{im}\begin{bmatrix} 0 \\ L_{P_2} \\ 0 \\ 0 \end{bmatrix} \subset S + \operatorname{im}\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ L_{Q_1} & 0 \\ 0 & L_{Q_2} \end{bmatrix} \tag{A.18}$$

Similarly, consider any

$$\begin{bmatrix} g_{P_1} \\ 0 \\ g_{Q_1} \\ 0 \end{bmatrix} \in \operatorname{im}\begin{bmatrix} G_{P_1} \\ 0 \\ G_{Q_1} \\ 0 \end{bmatrix} \subset (S_I + \bar{S}_I)^{\text{sym}} + \operatorname{im}\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ L_{Q_1} & 0 \\ 0 & L_{Q_2} \end{bmatrix} \tag{A.19}$$

From (A.19) it follows that there exists an element $(x_{P_1}, 0, x_{Q_1}, x_{Q_2}) \in (S_I + \bar{S}_I)^{\text{sym}}$ such that

$$\begin{bmatrix} g_{P_1} \\ 0 \\ g_{Q_1} \\ 0 \end{bmatrix} = \begin{bmatrix} x_{P_1} \\ 0 \\ x_{Q_1} \\ x_{Q_2} \end{bmatrix} + \operatorname{im}\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ L_{Q_1} & 0 \\ 0 & L_{Q_2} \end{bmatrix} \tag{A.20}$$

with $x_{Q_2} \in \operatorname{im} L_{Q_2} \cap \ker C_{Q_2} \cap \ker H_{Q_2}$ . Since $(S_{II} + \bar{S}_{II})^{\text{sym}}$ is full, there exists an element $(x_{Q_1}, 0, \bar{x}_{Q_1}, \bar{x}_{Q_2}) \in (S_{II} + \bar{S}_{II})^{\text{sym}}$ such that $\bar{x}_{Q_2} \in \operatorname{im} L_{Q_2} \cap \ker C_{Q_2} \cap \ker H_{Q_2}$. Lemma 3.13 ensures that there also exists an element $(0, \bar{x}_{Q_2}, 0, \bar{x}_{Q_2}) \in (S_I + \bar{S}_I)^{\text{sym}}$ since $x_{Q_2}, \bar{x}_{Q_2}$ and therefore $\bar{x}_{Q_2} \in \operatorname{im} L_{Q_2} \cap \ker C_{Q_2} \cap \ker H_{Q_2}$ as well as

$$\begin{bmatrix} x_{P_1} \\ 0 \\ x_{Q_1} \\ x_{Q_2} \end{bmatrix} + \begin{bmatrix} 0 \\ \bar{x}_{Q_2} \\ 0 \\ \bar{x}_{Q_2} \end{bmatrix} = \begin{bmatrix} x_{P_1} \\ \bar{x}_{Q_2} \\ x_{Q_1} \\ \bar{x}_{Q_2} + x_{Q_2} \end{bmatrix} \in (S_I + \bar{S}_I)^{\text{sym}} \tag{A.21}$$

Therefore, there exists an element $(x_{P_1}, 0, x_{Q_1}, \bar{x}_{Q_2}) \in S$ with $\bar{x}_{Q_2} \in \operatorname{im} L_{Q_2}$ such that

$$\begin{bmatrix} g_{P_1} \\ 0 \\ g_{Q_1} \\ 0 \end{bmatrix} = \begin{bmatrix} x_{P_1} \\ 0 \\ x_{Q_1} \\ \bar{x}_{Q_2} \end{bmatrix} + \operatorname{im}\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ L_{Q_1} & 0 \\ 0 & L_{Q_2} \end{bmatrix} \tag{A.22}$$

which proves that

$$\operatorname{im}\begin{bmatrix} G_{P_1} \\ 0 \\ G_{Q_1} \\ 0 \end{bmatrix} \subset S + \operatorname{im}\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ L_{Q_1} & 0 \\ 0 & L_{Q_2} \end{bmatrix} \tag{A.23}$$

Similarly, one can show that

$$\operatorname{im}\begin{bmatrix} 0 \\ G_{P_2} \\ 0 \\ G_{Q_2} \end{bmatrix} \subset S + \operatorname{im}\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ L_{Q_1} & 0 \\ 0 & L_{Q_2} \end{bmatrix} \tag{A.24}$$

and therefore condition (i) in Theorem 3.4 is completely fulfilled.
As to condition (ii), take any $(x_{P_1}, x_{P_2}, x_{Q_1}, x_{Q_2}) \in S$. Since $(S_i + \bar{S}_i)^{\text{sym}}, i = 1, 2$ are simulation relations, there exist $(x_{P_1}, x_{Q_2}, x_{Q_1}, \bar{x}_{Q_2}), (v_{P_1}, v_{Q_2}, v_{Q_1}, \bar{v}_{Q_2}) \in (S_I + \bar{S}_I)^{\text{sym}}$ and $a, b \in \mathbb{R}$ such that

$$\begin{bmatrix} A_{P_1} x_{P_1} + B_{P_1} C_{Q_2} x_{Q_2} \\ A_{Q_2} x_{Q_2} + B_{Q_2} C_{P_1} x_{P_1} \\ A_{Q_1} x_{Q_1} + B_{Q_1} C_{Q_2} \bar{x}_{Q_2} \\ A_{Q_2} \bar{x}_{Q_2} + B_{Q_1} C_{Q_1} x_{Q_1} \end{bmatrix} = \begin{bmatrix} v_{P_1} \\ v_{Q_2} \\ v_{Q_1} \\ \bar{v}_{Q_2} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ L_{Q_1} a \\ L_{Q_2} b \end{bmatrix} \tag{A.25}$$

as well as $(x_{Q_1}, x_{P_2}, \bar{x}_{Q_1}, x_{Q_2}), (w_{Q_1}, w_{P_2}, \bar{w}_{Q_1}, w_{Q_2}) \in (S_{II} + \bar{S}_{II})^{\text{sym}}$ and $l, m \in \mathbb{R}$ such that

$$\begin{bmatrix} A_{Q_1} x_{Q_1} + B_{Q_1} C_{P_2} x_{P_2} \\ A_{P_2} x_{P_2} + B_{P_2} C_{Q_1} x_{Q_1} \\ A_{Q_1} \bar{x}_{Q_1} + B_{Q_1} C_{Q_2} x_{Q_2} \\ A_{Q_2} x_{Q_2} + B_{Q_1} C_{Q_1} \bar{x}_{Q_1} \end{bmatrix} = \begin{bmatrix} w_{Q_1} \\ w_{P_2} \\ \bar{w}_{Q_1} \\ w_{Q_2} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ L_{Q_1} l \\ L_{Q_2} m \end{bmatrix} \tag{A.26}$$

Because of (A.13), observe that $v_{Q_2} = w_{Q_2} + L_{Q_2} m$ and $v_{Q_1} + L_{Q_1} a = w_{Q_1}$. Furthermore, we know that there exists an element $(0, L_{Q_2} m, L_{Q_1} c, L_{Q_2} d) \in (S_I + \bar{S}_I)^{\text{sym}}$ with $L_{Q_1} c \in \ker C_{Q_1} \cap \ker H_{Q_1}$ and similarly, $(L_{Q_1} a, 0, L_{Q_1} n, L_{Q_2} p) \in (S_{II} + \bar{S}_{II})^{\text{sym}}$ with $L_{Q_2} p \in \ker C_{Q_2} \cap \ker H_{Q_2}$. With Lemma 3.13, also $(0, L_{Q_2} p, 0, L_{Q_2} p) \in (S_I + \bar{S}_I)^{\text{sym}}$ and $(L_{Q_1} c, 0, L_{Q_1} c, 0) \in (S_{II} + \bar{S}_{II})^{\text{sym}}$. Hence,

$$\begin{aligned} \begin{bmatrix} v_{P_1} \\ v_{Q_2} \\ v_{Q_1} \\ \bar{v}_{Q_2} \end{bmatrix} &= \begin{bmatrix} v_{P_1} \\ v_{Q_2} - L_{Q_2} m - L_{Q_2} p \\ v_{Q_1} - L_{Q_1} c \\ \bar{v}_{Q_2} - L_{Q_2} d - L_{Q_2} p \end{bmatrix} + \begin{bmatrix} 0 \\ L_{Q_2}(m + p) \\ L_{Q_1} c \\ L_{Q_2}(d + p) \end{bmatrix} \\ &= \begin{bmatrix} v_{P_1} \\ w_{Q_2} - L_{Q_2} p \\ v_{Q_1} - L_{Q_1} c \\ \bar{v}_{Q_2} - L_{Q_2}(d + p) \end{bmatrix} + \begin{bmatrix} 0 \\ L_{Q_2}(m + p) \\ L_{Q_1} c \\ L_{Q_2}(d + p) \end{bmatrix} \end{aligned} \tag{A.27}$$

and

$$\begin{bmatrix} w_{Q_1} \\ w_{P_2} \\ \bar{w}_{Q_1} \\ w_{Q_2} \end{bmatrix} = \begin{bmatrix} w_{Q_1} - L_{Q_1}a - L_{Q_1}c \\ w_{P_2} \\ \bar{w}_{Q_1} - L_{Q_1}n - L_{Q_1}c \\ w_{Q_2} - L_{Q_2}p \end{bmatrix} + \begin{bmatrix} L_{Q_1}(a+c) \\ 0 \\ L_{Q_1}(n+c) \\ L_{Q_2}p \end{bmatrix} \tag{A.28}$$

$$= \begin{bmatrix} v_{Q_1} - L_{Q_1}c \\ w_{P_2} \\ \bar{w}_{Q_1} - L_{Q_1}(n+c) \\ w_{Q_2} - L_{Q_2}p \end{bmatrix} + \begin{bmatrix} L_{Q_1}(a+c) \\ 0 \\ L_{Q_1}(n+c) \\ L_{Q_2}p \end{bmatrix}$$

Thus, (A.25) can be rewritten as

$$\begin{bmatrix} A_{P_1}x_{P_1} + B_{P_1}C_{Q_2}x_{Q_2} \\ A_{Q_2}x_{Q_2} + B_{Q_2}C_{P_1}x_{P_1} \\ A_{Q_1}x_{Q_1} + B_{Q_1}C_{Q_2}\bar{x}_{Q_2} \\ A_{Q_2}\bar{x}_{Q_2} + B_{Q_1}C_{Q_1}x_{Q_1} \end{bmatrix} = \begin{bmatrix} v_{P_1} \\ w_{Q_2} - L_{Q_2}p \\ v_{Q_1} - L_{Q_1}c \\ \bar{v}_{Q_2} - L_{Q_2}(d+p) \end{bmatrix} + \begin{bmatrix} 0 \\ L_{Q_2}(m+p) \\ L_{Q_1}(c+a) \\ L_{Q_2}(d+p+b) \end{bmatrix}$$

and similarly, (A.26) becomes

$$\begin{bmatrix} A_{Q_1}x_{Q_1} + B_{Q_1}C_{P_2}x_{P_2} \\ A_{P_2}x_{P_2} + B_{P_2}C_{Q_1}x_{Q_1} \\ A_{Q_1}\bar{x}_{Q_1} + B_{Q_1}C_{Q_2}x_{Q_2} \\ A_{Q_2}x_{Q_2} + B_{Q_1}C_{Q_1}\bar{x}_{Q_1} \end{bmatrix} = \begin{bmatrix} v_{Q_1} - L_{Q_1}c \\ w_{P_2} \\ \bar{w}_{Q_1} - L_{Q_1}(n+c) \\ w_{Q_2} - L_{Q_2}p \end{bmatrix} + \begin{bmatrix} L_{Q_1}(a+c) \\ 0 \\ L_{Q_1}(n+c+l) \\ L_{Q_2}(p+m) \end{bmatrix}$$

Consequently, there exists an element $(v_{P_1}, w_{P_2}, v_{Q_1} - L_{Q_1}c, w_{Q_2} - L_{Q_2}p) \in S$ such that

$$\begin{bmatrix} A_{P_1}x_{P_1} + B_{P_1}C_{P_2}x_{P_2} \\ A_{P_2}x_{P_2} + B_{P_2}C_{P_1}x_{P_1} \\ A_{Q_1}x_{Q_1} + B_{Q_1}C_{Q_2}x_{Q_2} \\ A_{Q_2}x_{Q_2} + B_{Q_1}C_{Q_1}x_{Q_1} \end{bmatrix} = \begin{bmatrix} v_{P_1} \\ w_{P_2} \\ v_{Q_1} - L_{Q_1}c \\ w_{Q_2} - L_{Q_2}p \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ L_{Q_1}(a+c) \\ L_{Q_2}(p+m) \end{bmatrix}$$

which concludes the proof for $S$ being a simulation relation of $\Sigma_{P_1}\|\Sigma_{P_2}$ by $\Sigma_{Q_1}\|\Sigma_{Q_2}$.

Thirdly, it has to be shown that $S$ as defined in (3.16) is full, i.e. for any $(x_{P_1}, x_{P_2})$ there has to exist a $(x_{Q_1}, x_{Q_2})$ such that $(x_{P_1}, x_{P_2}, x_{Q_1}, x_{Q_2}) \in S$. Since $(S_I + \bar{S}_I)^{\text{sym}}$ is a full simulation relation, there exists for every $(x_{P_1}, x_{Q_2})$ a $(\bar{x}_{Q_1}, \bar{x}_{Q_1})$ such that $(x_{P_1}, x_{Q_2}, \bar{x}_{Q_1}, \bar{x}_{Q_1}) \in (S_I + \bar{S}_I)^{\text{sym}}$. Moreover, since also $(S_{II} + \bar{S}_{II})^{\text{sym}}$ is full, there exists for an arbitrary $x_{P_2}$ and the given $\bar{x}_{Q_1}$ a $(\hat{x}_{Q_1}, \hat{x}_{Q_2})$ such that $(\bar{x}_{Q_1}, x_{P_2}\hat{x}_{Q_1}, \hat{x}_{Q_2}) \in (S_{II} + \bar{S}_{II})^{\text{sym}}$. Fullness of $(S_I + \bar{S}_I)^{\text{sym}}$ also ensures that there exists an element $(0, \hat{x}_{Q_2}, \tilde{x}_{Q_1}, \tilde{x}_{Q_2}) \in (S_I + \bar{S}_I)^{\text{sym}}$ with $\tilde{x}_{Q_1} \in \ker C_{Q_1} \cap \ker H_{Q_1}$. By Lemma 3.13, however, an element

$(\tilde{x}_{Q_1}, 0, \tilde{x}_{Q_1}, 0)$ is contained in $(S_{II} + \bar{S}_{II})^{\text{sym}}$. Hence

$$\begin{bmatrix} x_{P_1} \\ x_{Q_2} \\ \bar{x}_{Q_1} \\ \bar{x}_{Q_2} \end{bmatrix} + \begin{bmatrix} 0 \\ \hat{x}_{Q_2} - x_{Q_2} \\ \tilde{x}_{Q_1} \\ \tilde{x}_{Q_2} \end{bmatrix} = \begin{bmatrix} x_{P_1} \\ \hat{x}_{Q_2} \\ \bar{x}_{Q_1} + \tilde{x}_{Q_1} \\ \bar{x}_{Q_2} + \tilde{x}_{Q_2} \end{bmatrix} \in (S_I + \bar{S}_I)^{\text{sym}}$$

$$\begin{bmatrix} \bar{x}_{Q_1} \\ x_{P_2} \\ \hat{x}_{Q_1} \\ \hat{x}_{Q_2} \end{bmatrix} + \begin{bmatrix} \tilde{x}_{Q_1} \\ 0 \\ \tilde{x}_{Q_1} \\ 0 \end{bmatrix} = \begin{bmatrix} \bar{x}_{Q_1} + \tilde{x}_{Q_1} \\ x_{P_2} \\ \hat{x}_{Q_1} + \tilde{x}_{Q_1} \\ \hat{x}_{Q_2} \end{bmatrix} \in (S_{II} + \bar{S}_{II})^{\text{sym}}$$

from which the element

$$\begin{bmatrix} x_{P_1} \\ x_{P_2} \\ \hat{x}_{Q_1} + \tilde{x}_{Q_1} \\ \hat{x}_{Q_2} \end{bmatrix} \in S$$

can be constructed for any $(x_{P_1}, x_{P_2})$.

# B

# Completeness of circular assume-guarantee reasoning for linear systems

## B.1. Proof of Theorem 3.20

We will only prove that $S_I$ is a full simulation relation; the result for $S_{II}$ follows by symmetry. Observe first that $S_I$ indeed defines a linear subspace and is non-empty. To show that $S_I$ is a simulation relation of $\Sigma_{P_1}\|\Sigma_{Q_2}$ by $\Sigma_{Q_1}\|\Sigma_{Q_2}$, note first that by construction

$$S_I \subset \ker \begin{bmatrix} C_{P_1} & 0 & -C_{Q_1} & 0 \\ 0 & C_{Q_2} & 0 & -C_{Q_2} \\ H_{P_1} & 0 & -H_{Q_1} & 0 \\ 0 & H_{Q_2} & 0 & -H_{Q_2} \end{bmatrix}$$

Furthermore, since for every $x_{Q_2} \in \mathrm{im} L_{Q_2}$ $(0, x_{Q_2}, 0, x_{Q_2}) \in S_I$ it holds that

$$\mathrm{im} \begin{bmatrix} 0 \\ L_{Q_2} \\ 0 \\ 0 \end{bmatrix} \subset S_I + \mathrm{im} \begin{bmatrix} 0 \\ 0 \\ 0 \\ L_{Q_2} \end{bmatrix}$$

Moreover, since $S$ is a full simulation relation, for every $g_{P_1} \in \mathrm{im} L_{P_1}$ there exists a $(g_{P_1}, 0, x_{Q_1}, x_{Q_2}) \in S$ such that $x_{Q_i} \in \mathrm{im} L_{Q_i}, i = 1, 2$. But then there also exists an element $(g_{P_1}, x_{Q_2}, x_{Q_2}, x_{Q_2}) \in S_I$ such that for some $\alpha, \lambda$

$$\begin{bmatrix} g_{P_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} g_{P_1} \\ x_{Q_2} \\ x_{Q_1} \\ x_{Q_2} \end{bmatrix} + \begin{bmatrix} 0 \\ L_{Q_2}\lambda \\ L_{Q_1}\alpha \\ L_{Q_2}\lambda \end{bmatrix} = \underbrace{\begin{bmatrix} g_{P_1} \\ 0 \\ x_{Q_1} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ x_{Q_2} + L_{Q_2}\lambda \\ 0 \\ x_{Q_2} + L_{Q_2}\lambda \end{bmatrix}}_{\in S_I}$$

$$+ \begin{bmatrix} 0 \\ 0 \\ L_{Q_1}\alpha \\ 0 \end{bmatrix} \in S_I + \mathrm{im} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ L_{Q_1} & 0 \\ 0 & L_{Q_2} \end{bmatrix}.$$

Clearly, also $\operatorname{im}\begin{bmatrix} 0 & 0 \\ B_{Q_2} & G_{Q_2} \\ 0 & 0 \\ B_{Q_2} & G_{Q_2} \end{bmatrix} \in S_I$. Since $\operatorname{im}\begin{bmatrix} B_{P_1} & G_{P_1} \\ 0 & 0 \\ B_{Q_1} & G_{Q_1} \\ 0 & 0 \end{bmatrix} \in S$, there exists for all $\beta, \delta$ an element $(w_{P_1}, w_{Q_2}, w_{Q_1}, w_{Q_2}) \in S_I$ such that

$$\begin{bmatrix} B_{P_1} & G_{P_1} \\ 0 & 0 \\ B_{Q_1} & G_{Q_1} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \beta \\ \delta \end{bmatrix} = \begin{bmatrix} w_{P_1} \\ w_{Q_2} \\ w_{Q_1} \\ w_{Q_2} \end{bmatrix} + \begin{bmatrix} 0 \\ G_{Q_2}\rho \\ G_{Q_1}\tau \\ G_{Q_2}\rho \end{bmatrix} = \underbrace{\begin{bmatrix} w_{P_1} \\ 0 \\ w_{Q_1} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ w_{Q_2} + G_{Q_2}\rho \\ 0 \\ w_{Q_2} + G_{Q_2}\rho \end{bmatrix}}_{\in S_I} + \begin{bmatrix} 0 \\ 0 \\ G_{Q_1}\tau \\ 0 \end{bmatrix}$$

for some $\rho, \tau$. Finally, for any $(x_{P_1}, x_{Q_2}, x_{Q_1}, x_{Q_2}) \in S_I$ there exists a $(z_{P_1}, z_{Q_2}, z_{Q_1}, z_{Q_2}) \in S_I$ such that

$$\begin{bmatrix} A_{P_1}x_{P_1} + B_{P_1}C_{Q_2}x_{Q_2} \\ B_{Q_2}C_{P_1}x_{P_1} + A_{Q_2}x_{Q_2} \\ A_{Q_1}x_{Q_1} + B_{Q_1}C_{Q_2}x_{Q_2} \\ B_{Q_2}C_{Q_1}x_{Q_1} + A_{Q_2}x_{Q_2} \end{bmatrix} = \begin{bmatrix} z_{P_1} \\ z_{Q_2} \\ z_{Q_1} \\ z_{Q_2} \end{bmatrix} + \begin{bmatrix} 0 \\ G_{Q_2}\mu \\ G_{Q_1}\nu \\ G_{Q_2}\mu \end{bmatrix} = \underbrace{\begin{bmatrix} z_{P_1} \\ 0 \\ z_{Q_1} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ z_{Q_2} + G_{Q_2}\mu \\ 0 \\ z_{Q_2} + G_{Q_2}\mu \end{bmatrix}}_{\in S_I} + \begin{bmatrix} 0 \\ 0 \\ G_{Q_1}\nu \\ 0 \end{bmatrix}.$$

In particular, for any $(0, x, 0, x) \in S_I$ it holds that

$$\begin{bmatrix} B_{P_1}C_{Q_2}x \\ A_{Q_2}x \\ B_{Q_1}C_{Q_2}x \\ A_{Q_2} \end{bmatrix} = \begin{bmatrix} B_{P_1}C_{Q_2}x \\ 0 \\ B_{Q_1}C_{Q_2}x \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ A_{Q_2}x \\ 0 \\ A_{Q_2}x \end{bmatrix} \subset S_I + \operatorname{im}\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ G_{Q_1} & 0 \\ 0 & G_{Q_2} \end{bmatrix}$$

since

$$\begin{bmatrix} B_{P_1}C_{Q_2}x \\ 0 \\ B_{Q_1}C_{Q_2}x \\ 0 \end{bmatrix} \in \operatorname{im}\begin{bmatrix} B_{P_1} \\ 0 \\ B_{Q_1} \\ 0 \end{bmatrix} \subset S_I + \operatorname{im}\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ G_{Q_1} & 0 \\ 0 & G_{Q_2} \end{bmatrix}$$

In order to prove fullness of $S_I$, we first note that for any $x_{Q_2} \in \mathcal{X}_{Q_2}$ there is an element $(0, x_{Q_2}, 0, x_{Q_2}) \in S_I$. Moreover, for any $x_{P_1} \in \mathcal{X}_{P_1}$ there is a $(x_{P_1}, 0, w_{Q_1}, w_{Q_2}) \in S$. Hence there is a $(x_{P_1}, w_{Q_2}, w_{Q_1}, w_{Q_2}) \in S_I$ and therefore also

$$\begin{bmatrix} x_{P_1} \\ 0 \\ w_{Q_1} \\ 0 \end{bmatrix} = \begin{bmatrix} x_{P_1} \\ w_{Q_2} \\ w_{Q_1} \\ w_{Q_2} \end{bmatrix} - \begin{bmatrix} 0 \\ w_{Q_2} \\ 0 \\ w_{Q_2} \end{bmatrix} \in S_I$$

Combining these two results we conclude that for every $(x_{P_1}, x_{Q_2})$ there exists a $(w_{Q_1}, x_{Q_2})$ such that $(x_{P_1}, x_{Q_2}, w_{Q_1}, x_{Q_2}) \in S_I$ which proves fullness.

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