Radium Ion Spectroscopy

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Chapter 2

Atomic Parity Violation and Standard Model

2.1 Parity Violation

2.1.1 The Discrete Symmetries

There are three discrete symmetries in the description of the SM. They are the parity (P) symmetry, the time reversal (T) symmetry, and the charge conjugation (C) symmetry. The parity symmetry was introduced by Wigner in 1927 to explain the selection rules observed in atomic transitions [34]. Parity is the operation of space reflection, where all spatial coordinates $\vec{r} \equiv (\vec{x}, \vec{y}, \vec{z})$ are reversed through the origin. Mathematically, it is represented by the operator $\hat{P}$ which transforms

$$\hat{P}\ \vec{r} = -\vec{r}$$
$$\hat{P}\ \psi(\vec{r}, t) = \psi(-\vec{r}, t).$$

In quantum mechanics the parity operator has a quantum number associated with it. Since $\hat{P}^2\psi = \psi$ the parity operator $\hat{P}$ has two eigenvalues: +1 and -1 corresponding to even and odd parity eigenstates respectively.

The time reversal symmetry was also introduced by Wigner in 1932 [35]. The time reversal symmetry transformation is represented by an operator $\hat{T}$ which reverses the time coordinate

$$\hat{T}\ t = -t$$
$$\hat{T}\ \psi(\vec{r}, t) = \psi(\vec{r}, -t).$$

The time reversal operation relates the initial and final states in quantum mechanics and it has no quantum number associated with it.

The charge conjugation symmetry was introduced by Kramers in 1937 following the formulation of the Dirac equation and the development of quantum field theory [36]. It is often called particle-antiparticle conjugation [37]. The charge conjugation symmetry transformation is represented by the operator $\hat{C}$ which reverses the charge

$$\hat{C}\ q \rightarrow -q$$
$$\hat{C}\ \psi(q) \rightarrow \psi(-q).$$

In 1951, Schwinger formulated the CPT theorem which states that the combined CPT operation is a symmetry of any local Lorentz-invariant quantum field theory [38]. The prediction of the SM and confirmation from all experimental observations indicate that all physical laws are invariant under the combined application of all three discrete symmetries ($\hat{C}\hat{P}\hat{T}$) irrespective of their order. Individually, these symmetries are not preserved by nature. Soon after the formulation of CPT theorem, violation of different discrete symmetries were observed after the suggestion to search for it in the weak interaction by Lee and Yang [39]. We will discuss only the violation of parity symmetry in the scope of this thesis.

2.1.2 History of Parity Violation

A physical law conserves the parity symmetry if its form is retained under a parity transformation operation. Parity is conserved if left and right can not be distinguished by performing a physical experiment. It is quite evident that one can not simply apply an abstract transformation operator to the experimental apparatus to get some sense of parity violation. However, one can effectively do this by reversing the parity of the experiment. In order to search for a signature of parity violation, an experiment is performed to measure some physical process and the measurement is repeated with the parity of the experiment reversed. Any discrepancy observed between the two measurements is the basic signature of parity violation and it would imply a fundamental handedness[1] associated with the physical process.

In order to understand the present status of the physics of parity violation, it is illuminating to look at the path of discoveries. Here we present the milestones in the history of significant developments in the field of parity violation.

- **Until 1950**, almost every physicist had a notion that parity is a fundamental symmetry of the universe.

[1]The handedness has also been observed in muon decay.

- **In 1950**, E.M. Purcell and N.F. Ramsey realized that a permanent electric dipole moment (EDM) of a fundamental particle would violate the parity symmetry [40]. This was the beginning of the searches for EDM which have set over the decades strong limits on SM extensions. They also indicated that there was no limit on parity violation for the weak interaction.

- **In 1956**, T.D. Lee and C.N. Yang predicted that the weak force might violate parity symmetry based on their theoretical work on $\tau - \theta$ puzzle [39]. They proposed experiments that could test their prediction. They were later awarded the Nobel Prize for this work. Several experiments were started thereafter.

- **In 1957**, C. Wu et al. performed a significant variation of the experiment proposed by Lee and Yang which led to the first observation of parity violation in nuclear β-decay [41]. Many experiments followed thereafter.

- **In 1959**, Zeldovich speculated that neutral currents might be the cause that leads to parity violating effects in atomic systems and not just in nuclear β-decay [42]. This speculation created an interest among experimentalists, though it was apparent that the effects would be immeasurably small at that time.

- **In 1967**, S.L. Glashow, S. Weinberg, and A. Salam proposed the electroweak theory which unifies electromagnetism with the weak force [1–3]. This unified gauge theory predicts the existence of a massive Z^0 boson which is responsible for the parity violating weak interaction effects.

- **In 1974**, M.A. Bouchiat and C.C. Bouchiat realized that parity violation effects are stronger in heavy atomic systems and the strength of the effects scales with the cube of atomic number (Z^3 law) [43]. This triggered the beginning of a new era of experiments to search for neutral current effects in atomic systems. Since then, there have been several groups pursuing experimental measurements of atomic parity violation in heavy atomic systems [22–24, 30–33, 44–50, 50–53].

2.1.3 Electroweak Unification

The long range electromagnetic interaction in atomic systems is mediated by the exchange of a massless photon (γ). This interaction is parity conserving and is described by the laws of Quantum Electrodynamics (QED). The short range weak interaction is mediated by the exchange of a weak vector boson. This interaction is parity violating. The electroweak theory claims that all electromagnetic phenomena and all weak phenomena are results of the manifestation of one universal electroweak interaction between spin 1/2 quarks (up, down, top, bottom, charm, strange) and leptons (electron, muon, tau, neutrinos) mediated by four spin 1 bosons: two charged ones (W^+ and W^-) and two neutral ones (W^0 and B^0). These four particles are the eigenstates of weak interaction. However, they are not the mass eigenstates. The two charged vector bosons (W^+ and W^-) cannot mix because of their electric charge while the neutral bosons (W^0 and B^0) mix in two orthogonal linear combinations. In one linear combination, the photon remains massless and the coupling strength is given by the charge of the electron. The consistency of the theoretical framework required the existence of a massive second neutral boson Z^0 ($\sim$ 91 GeV) which was predicted by the electroweak theory. The mixing of the photon (γ) and the Z^0 boson (*cf.* Fig. 2.1) is described by a single fundametral parameter, the weak mixing angle or the Weinberg angle (θ_W). The mass eigenstates in terms of the linear combinations of neutral bosons are written as

$$|\gamma\rangle = \sin\theta_W |W^0\rangle + \cos\theta_W |B^0\rangle \tag{2.1}$$

$$|Z^0\rangle = \cos\theta_W |W^0\rangle - \sin\theta_W |B^0\rangle. \tag{2.2}$$

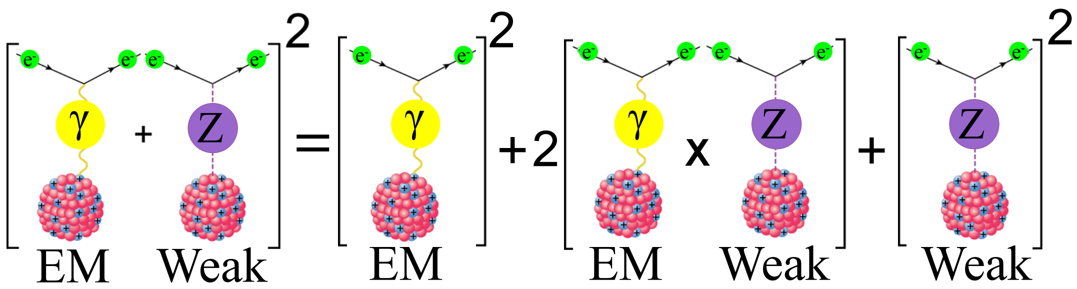

Fig. 2.1: Electrons in an atom interact with the nucleus through the electromagnetic force (EM) via the exchange of massless photons (γ). The weak force (Weak) is mediated by Z^0 bosons. The weak effects by themselves are too small to be seen directly. This figure provides a pictorial representation of the quantum interference between the electromagnetic and weak processes. The sum of the amplitudes squared enables to observe the effect of Z^0 boson exchange. Adapted from [54].

If the mediating particle is a charged vector boson (W^+ or W^-), the interaction is called *charged current interaction*, e.g., nuclear β-decay. If the mediating particle is a neutral vector boson (Z^0), the interaction is called *neutral current interaction*. The existence of Z^0 boson and the neutral current interaction was confirmed by indirect observation of neutral current processes [55, 56] and later by direct observation of the Z^0 resonance [57, 58]. Over the years the accelerator based high energy experiments have confirmed the predictions of the electroweak theory with great precision.

The weak mixing angle θ_W connects two independent coupling constants of the electroweak theory. One is the electric charge (e), the coupling constant of the electromagnetic interaction. The other is g_W, the coupling constant of the weak interaction. These two coupling constants are connected to the Weinberg angle via the relation

$$\sin^2\theta_W = \frac{e^2}{g_W^2}. \tag{2.3}$$

Since electroweak theory is a quantum field theory, these two coupling constants vary with the energy at which they are measured due to radiative corrections. The prediction of the electroweak theory for the running of Weinberg angle is shown in Fig. 2.2.

It shows that $\sin^2\theta_W$ first decreases by about $\sim 3\%$ from low energy to the mass of the Z^0 boson. This is caused by the creation and annihilation of quark-antiquark pairs resulting in the vacuum polarization which shields the interacting particles, causing g_W to increase. At very high energy (beyond 100 GeV) the $W^{\pm}$ pairs start to dominate vacuum polarization resulting in anti-shielding, causing g_W to decrease and $\sin^2\theta_W$ to increase. This prediction of the SM is confirmed by several experiments and several experiments are underway.

2.1.4 Experiments Worldwide

Here we restrict ourselves to the APV experiments aiming for a precise determination of Weinberg angle. A number of such experiments have been performed at different energy scales, each of which has contributed an experimentally measured value of the Weinberg angle. These experiments along with the planned experiments are listed in Table 2.1.

The e^+e^- experiment performed at LEP, CERN (LEP 1 in Fig. 2.2) yielded a result that defines θ_W at the Z^0 pole. This is the most precise measurement performed at high energy frontier. Interestingly the leptonic (0.23113 ± 0.00021) and the hadronic (0.23222 ± 0.00027) measurements of $\sin^2\theta_W$ differ by 3.2σ [62]. Two other experiments at this energy scale were performed at SLC (SLAC) and

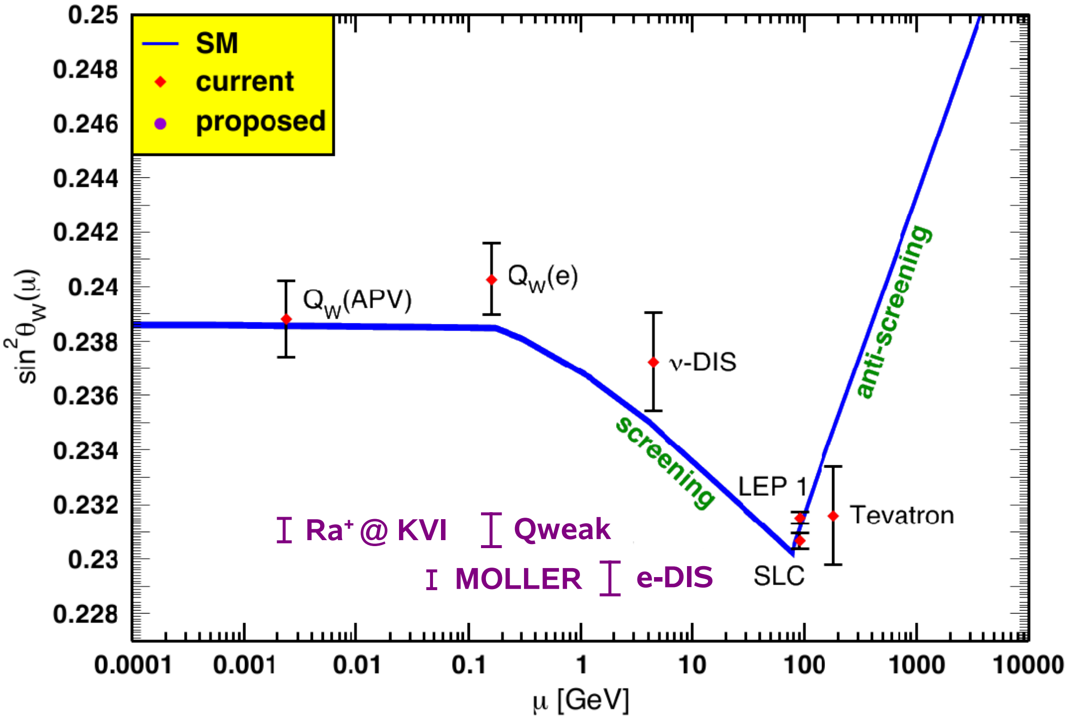

Fig. 2.2: The running of the γ-Z^0 mixing angle from low atomic energies to the high energy scales. The blue trace is the prediction of the SM. The data points in red color are the completed or ongoing experiments. The data points in magenta color are proposed experiments with the anticipated error bars. Adapted from [59].

Tevatron (Fermilab) [63].

The NuTeV experiment at Fermilab (ν-DIS in Fig. 2.2) determined the Weinberg angle by looking at the scattering of the beams of neutrinos and antineutrinos off a proton target. The interaction between the (anti)neutrinos and the quarks can be either a charged or a neutral weak current. By determining the ratio of the charged to neutral current cross sections for either the neutrino or antineutrino beam, a value of the Weinberg angle was extracted. The final report was published in 2002 with a value for $\sin^2\theta_W$ that differed from the SM prediction by 3σ [61].

The E158 experiment at SLAC (Q_W(e) in Fig. 2.2) determined the weak charge of the electron by looking at the amount of parity violation in electron-electron scattering. The measurement was performed by accelerating polarized electrons to ~ 50 GeV and scatter them off other electrons in a liquid hydrogen target. Since the Z boson couples differently to left handed electrons than to

Table 2.1: Recent APV experiments from low energy to high energy scales. Experiments marked with an asterisk have been proposed. The last column indicates the level of precision that has been achieved or anticipated.

Measurement	Laboratory	Observable	Precision
Cs APV	Q_W(APV)@JILA Boulder	Q_W(Cs)	0.35% [22]
ee scattering	E158@SLAC	Q_W(e)	0.5% [60]
ν-DIS	NuTeV@Fermi Lab	-	0.7% [61]
e^+e^- at Z^0 pole	LEP@CERN	-	0.07% [62]
ep scattering*	Qweak@J-Lab	Q_W (p)	0.3%
ee scattering*	Moller@J-Lab	Q_W (e)	0.1%
e-DIS*	J-Lab	Q_W (e)	0.45%
Fr APV*	Legnaro	Q_W(Fr)	-
Ba^+ APV*	Seattle	$Q_W(Ba^+)$	0.5%
Yb^+ APV*	Los Alamos	$Q_W(Yb^+)$	0.1%
Ra^+ APV*	Groningen	$Q_W(Ra^+)$	3%

right handed electrons, the polarized electron beam is scattered differently depending on its polarization. This leads to a parity violating asymmetry. The determination of parity violation leads to the determination of the Weinberg angle. The first data was collected in the summer of 2004 [64] and the final report was published a year later [60].

Several experiments are planned to be performed in the medium energy sector. The Qweak experiment at the Jefferson Lab (J-Lab) aims to measure the weak charge of the proton Q_W(p) by elastic electron-proton scattering. The Moller experiment and e-DIS experiment at J-Lab aim to measure the weak charge of electron Q_W(e).

In the lower energy sector, there exists only one measurement which determined the weak nuclear charge with a sub-percent accuracy [22]. The result of this measurement agrees to better than one standard deviation with the prediction of the SM (*cf.* Fig. 2.2). This level of agreement is a consequence of tremendous theoretical effort that was pursued to achieve the level of accuracy called for by the experimental result [29, 65]. Over the years the theoretical prediction has relaxed towards the experimental value about a decade after the final experimental number. Then it becomes desirable to perform an independent high precision measurement of the weak charge in Cs or in a different system using a completely different experimental concept. Several groups are pursuing this goal. For cesium (Z=55), a new measurement was performed by the ENS-Paris group [30] and plans exist at SUNY, Stony Brook. At INFN, Legnaro an experiment is planned to measure the weak charge for francium (Z=87) in a magneto optical trap (MOT). At Seattle an experiment to measure the weak charge for Ba^+(Z=56) is underway. At Los Alamos an experiment is planned to measure the weak charge for Yb^+ (Z=70). At KVI, we are preparing a competitive APV experiment using a single trapped ionic radium (Z=88).

2.2 Parity Violation Experiments

2.2.1 Parity Violation in Atomic Systems

Parity violation in atomic systems is caused by the interaction of electrons with the quarks in the nucleus through the exchange of Z^0 bosons. This interaction can be depicted using a Feynman diagram (Fig. 2.3). In the left diagram the parity violation of an electron due to a t-channel interaction with a Z^0 boson inside the nucleus is shown. In the right diagram it is shown that the weak interactions can be treated in a contact potential approximation, with G_F being the point contact coupling constant. It is related to the underlying physical weak force coupling constant g_W as $G_F \propto g_W$ [66].

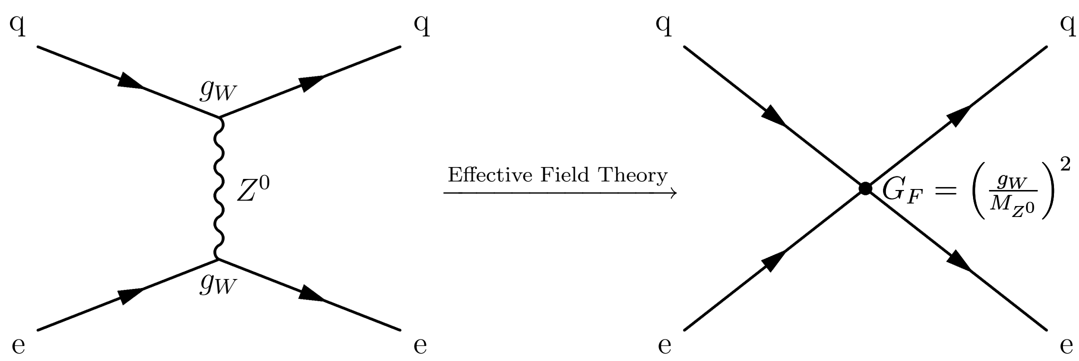

Fig. 2.3: Feynman diagrams depicting atomic parity violation. The left diagram shows the interaction of an electron (e) with a quark (q) with an exchange of Z^0 boson. The right diagram shows the coupling of four fermions in a contact potential approximation using effective field theory.

The dominant contribution to atomic parity violation comes from the exchange of the Z^0 between the electrons and the quarks in the nucleus, where the

Z^0 couples to the spin of the electron and to the weak charge of the quarks. As a result of coherent addition of the weak charges of each quark, the nucleus acquires a weak nuclear charge (Q_W). This weak interaction introduces a non-zero off-diagonal matrix element in the energy eigenstate representation of $\hat{P}$. As a result, the energy eigenstates will not have a well defined parity. A state of well defined parity gets a tiny admixture from opposite parity states and becomes a state of ill defined parity. This mixing is maximum between the states for which the overlap of electron wave functions at the nucleus is maximum. Far away from the nucleus, the exchange of Z^0 bosons between the electrons and the quarks is suppressed because of the large mass of Z^0.

The weak charge Q_W of the nucleus is expressed as

$$Q_W = (2Z + N)Q_w(u) + (Z + 2N)Q_w(d) \tag{2.4}$$

$$= [Q_w(u) + 2Q_w(d)]N + [2Q_w(u) + Q_w(d)]Z, \tag{2.5}$$

where N is the number of protons and Z is the number of protons inside the nucleus. Using the values of the weak charges of the up (u) and down (d) quarks according to the electroweak theory, the weak nuclear charge takes the form

$$Q_W = -N + (1 - 4\sin^2\theta_W)Z. \tag{2.6}$$

Experimentally, the value of $\sin^2\theta_W$ has been determined to be $\simeq 0.23$ [62]. Thus $Q_W \simeq$ - N + 0.08 Z $\sim$ -N, and hence the weak nuclear charge is roughly proportional to the atomic number Z. The extent to which the admixed opposite parity states overlap at the nucleus is roughly proportional to Z^2. Combining these two facts together leads to a conclusion that the amplitude of mixing effect is proportional to Z^3 [43].

The neutral weak interaction between an atomic electron and the nucleus is described by a zero-range effective Hamiltonian which is added as a perturbation to the usual Coulomb Hamiltonian. The parity violating part of the electron-nucleus interaction Hamiltonian H^{APV} can be expressed as

$$H^{APV} = H_1^{NSI} + H_2^{NSD}, \tag{2.7}$$

where H_1^{NSI} is the nuclear spin independent contribution and H_2^{NSD} is the nuclear spin dependent contribution. H_1^{NSI} is a scalar in the electronic variables. Thus, it is unable to change the angular momentum of the electron. H_1^{NSI} can be expressed as

$$H_1^{NSI} = A_e V_N, \tag{2.8}$$

where A_e is the axial vector component of the electronic neutral weak current and V_N is the vector component of the nucleonic neutral weak current.

H_2^{NSD} is dependent on the nuclear spin and plays a role only in the odd isotopes. It can change the electron angular momentum. H_2^{NSD} actually consists of two parts,

$$H_2^{NSD} = H_{2a}^{NSD} + H_{2b}^{NSD}. \tag{2.9}$$

The first part (H_{2a}^{NSD}) originates from unbalanced nucleon spin and is equal to $V_e A_N$, where V_e is the vector component of the electronic neutral weak current and A_N is the axial vector component of the nucleonic neutral weak current. The second part (H_{2b}^{NSD}) originates from the electromagnetic coupling of the electrons to the nuclear anapole moment. The combined nuclear spin dependent contribution $H_2^{NSD} = H_{2a}^{NSD} + H_{2b}^{NSD}$ is only a few percent of the nuclear spin independent contribution H_1^{NSI}. The study of isotopes with zero and non-zero nuclear spin can distinguish these contributions.

In the non-relativistic limit, H_1^{NSI} can be written as [67]

$$H_1^{NSI} = Q_W \times \frac{G_F}{2\sqrt{2}} \gamma_5 \rho_N(r), \tag{2.10}$$

where Q_W is the weak nuclear charge, G_F is the weak coupling constant or Fermi constant, γ_5 is the Dirac matrix and $\rho_N(r)$ is the nucleon density function. $\rho_N(r)$ is non-zero only for nuclear dimensions and it is essentially a delta function. It can be either the proton density function or the neutron density function and both of them can be assumed to be the same. Nevertheless the nuclear parameter plays a role in the interpretation of the weak interaction results. Hence the parameters such as charge radii, hyperfine structures and lifetimes have to be determined experimentally.

2.2.2 APV: Low Energy Test of the SM

While the strong and electromagnetic interactions are parity even, the weak interaction is parity odd. Hence a measurable parity violating effect in an atomic system can only be explained by the weak interaction. Measuring parity violation via a low energy precision experiment and finding agreement with the SM to a sub-percent accuracy would exclude new bosons with a mass < 1 TeV, which is already better than the capabilities of current high energy experiments. On the other hand, any deviation would indicate "new physics" beyond the SM.

The interface between an atomic parity violation experiment and the SM is the weak nuclear charge, Q_W. The weak charge Q_W is obtained by combining the results of atomic parity violation experiments and relativistic many-body calculations. Thus, a parity violation experiment in conjunction with required atomic theory can determine an experimental value of the weak nuclear charge Q_W, from which the weak mixing angle θ_W can be extracted and compared with the standard model prediction shown in Fig. 2.2.

2.2.3 Principle of APV Experiments

Due to the Z^0 induced mixing of states, there is a small but non-zero parity violating transition amplitude (A^{PV}) between states of the same parity. The amplitude A^{PV} will change sign if parity of the experiment is effectively reversed, for example, by reversing the directions of electric, magnetic, and laser fields [22, 50], while all parity conserving amplitudes will remain unchanged under this reversal. The goal of an APV experiment is to precisely determine this parity violating amplitude A^{PV} which is observed by its interference with either a parity conserving transition amplitude (A^{PC}) or another parity violating transition amplitude that can be induced externally.

The amplitude A^{PC} can be a magnetic dipole (M1) transition amplitude and this is the basis of an optical rotation experiment. The physics underlying the optical rotation experiment is concerned with the rotation of the plane of polarization of light as it passes through a medium of atoms, owing to the intrinsic chirality of atoms arising from the weak interaction. This APV-induced optical rotation violates mirror symmetry. An excellent description of the optical rotation experiments is given in [68]. Optical rotation experiments have been performed on atomic bismuth [69], lead [70], thallium [71] and samarium [72]. APV experiments based on Stark interference aim at measuring the transition rate between two states, for which there is a parity violating transition matrix element [68, 73]. In such experiments the amplitude A^{PV} can be measured by observing its interference with a Stark-induced electric dipole (E1) transition amplitude. Candidates for such an experiment are atomic cesium [22], ytterbium [50], and francium [74]. Alternatively, the amplitude A^{PC} can be chosen as an electric quadrupole (E2) transition amplitude (Fig. 2.1). Candidates for such an experiment are Yb^+, Ba^+, and Ra^+.

Since the parity violating matrix elements are much smaller than the parity conserving matrix elements, the weakly allowed parity conserving transition completely dominates the experiment. In terms of Rabi frequencies the absolute transition rate between the two states is given by,

$$R = \mid \Omega^{\rm PV} + \Omega^{\rm PC} \mid^2 \tag{2.11}$$

$$= |\Omega^{\rm PV}|^2 + 2{\rm Re}\left(\Omega^{\dagger \rm PV}\Omega^{\rm PC}\right) + |\Omega^{\rm PC}|^2 \tag{2.12}$$

The first term may be neglected as it depends quadratically on the small parity violating transition amplitude. The last term depends quadratically on the large parity conserving transition amplitude which dominates the transition rate. This indicates that a direct approach of measuring the absolute strength of parity violating transitions is bound to fail and hence one needs to take an indirect approach by exploiting the interference term, which is linear in parity violating amplitude. This interference term is a pseudoscalar and hence it will change sign upon a coordinate system reversal. Since the last term dominates the transition rate, it must be distinguished from the interference term. This can be done by using the parity violating signature of the interference term which makes it behave differently from the parity conserving term under coordinate system reversals.

Trapped heavy atoms and ions are good systems for the measurement of atomic parity violation. Because of the electric charge an ion can be preferred over an atom, since it is easier to manipulate an ion than an atom. Alkali atoms or alkaline-earth ions with a single valence electrons are preferred for such an experiment.