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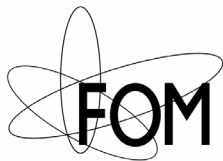
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# Size effects in cellular solids

Cihan Tekoğlu



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for Metals Research**

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