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Aspects of algorithmic algebra

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Stellingen

behorende bij het proefschrift

Aspects of Algorithmic Algebra:
Differential Equations and Splines

van

Raimundas Vidūnas

1. For a non-negative integer n the differential Galois group of the equation

$$x y'' - (x + n) y' + n y = 0$$

is the multiplicative group \mathbb{G}_m . A general solution of this equation is

$$y = C_1 e^x + C_2 \left(1 + x + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} \right).$$

Recall that the power series of e^x at $x = 0$ is $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

This example illustrates a quite common feature of second order differential equations over $C(x)$ with the differential Galois group \mathbb{G}_m . Namely, with the exception of few easy examples, such an equation has regular singular points where the power series of (any) two independent solutions with rational logarithmic derivatives, truncated up to some (positive, possibly quite big) order, are equal or proportional.

(Chapter 3 of this thesis.)

2. The figure on the front cover of this thesis illustrates the fact that it is possible to construct a CG^1 geometrically continuous surface using polynomial triangular patches of degree two. The torus there is “made” of 32 triangular Bézier patches. A way to obtain this picture is to subdivide the rectangle $[-\pi, \pi] \times [-\pi, \pi]$ into 32 triangles by lines $x, y \in \{0, \pm\frac{\pi}{2}\}$ and $x + y \in \{0, \pm\frac{\pi}{2}, \pm\pi, \pm\frac{3\pi}{2}\}$, and take the following parametrization of a torus by this rectangle:

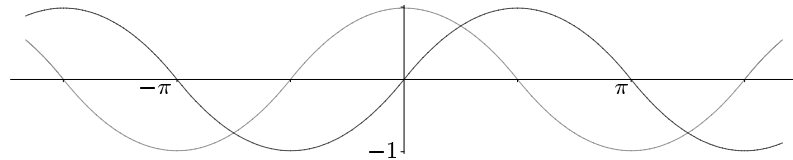
$$(X, Y, Z) = (a C\cos(x) + C\cos(x + y), a S\sin(x) + S\sin(x + y), S\sin(y)).$$

Here $a = 3$, and $S\sin(t)$ and $C\cos(t)$ are periodic functions on \mathbb{R} defined as follows:

$$S\sin(t) := \begin{cases} \frac{2}{5} (t - k\pi) ((k + 1)\pi - t), & \text{if } t \in [k\pi, (k + 1)\pi] \text{ for even } k \in \mathbb{Z}, \\ \frac{2}{5} (t - k\pi) (t - (k + 1)\pi), & \text{if } t \in [k\pi, (k + 1)\pi] \text{ for odd } k \in \mathbb{Z}. \end{cases}$$

and $C\cos(t) := S\sin(t + \frac{\pi}{2})$. By the construction of these functions, the restriction of the given parametrization to each of the mentioned triangles is given by polynomial functions of degree 2. One can check that the 32 triangular patches are C^1 regular. They join with CG^1 continuity because $S\sin(t)$ and $C\cos(t)$ are C^1 functions on \mathbb{R} .

As a matter of fact, the functions $S\sin(t)$ and $C\cos(t)$ approximate the trigonometric functions $\sin(t)$ and $\cos(t)$ quite well. The approximation error is less than 0.05 everywhere. Here are the graphs of these functions:



(Compare with figure 1.1 and example 6.31 in the thesis.)

3. Een oplossing van de Hurwitz differentiaalvergelijking

$$y''' + \frac{7x-4}{x(x-1)}y'' + \frac{2592x^2 - 2963x + 560}{252x^2(x-1)^2}y' + \frac{57024x - 40805}{24696x^2(x-1)^2}y = 0.$$

is $y = \frac{d\phi(t)}{dt} \sqrt{4t^4 - 21t^2 + 28} (2t - 3)$, waarbij

$$\phi(z) = \frac{z^3 (3z^2 - 7)^3 (2z^2 - 7z + 7)^3 (11z^2 - 35z + 28)^3}{1728 (z^3 - 7z + 7)^7}$$

en t een wortel van $\phi(t) - x = 0$ is.

(Zie sectie 4.3 van dit proefschrift.)

4. Een *reken-reeks* van lengte n is een rij (a_0, a_1, \dots, a_n) van positieve gehele getallen met de volgende eigenschappen:

(a) $a_0 = 1$.

(b) Voor $i = 1, \dots, n$ is $a_i \in \{x + y, x - y, xy, x/y\}$, waarbij x en y in de verzameling $\{a_0, \dots, a_{i-1}\}$ van al “berekende” getallen zitten.

De *complexiteit* van een (positief) geheel getal B is de minimale lengte van een reken-reeks (a_0, a_1, \dots, a_n) met $a_n = B$.

Laat B_k het kleinste getal zijn met complexiteit k . In zekere zin, bestaat de getalreeks (B_1, B_2, \dots) uit de “lastigst” te berekenen getallen. Deze reeks begint met:

$$2, 3, 5, 7, 13, 41, 113, 311, 1821.$$

Verrassend genoeg, is het laatste getal geen priemgetal.

5. *Douter de tout ou tout croire, ce sont deux solutions également commodes, qui l'une et l'autre nous dispensent de réfléchir.*

(Henri Poincaré, *La science et l'hypothèse*, 1902)

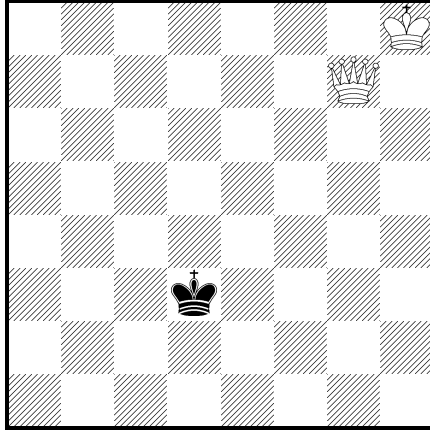
Alles betwijfelen en alles geloven zijn twee even gemakkelijke oplossingen, die ons beide ontslaan van nadenken.

(Vertaling van W.A. Verloren van Themaat)

Op het eind van de twintigste eeuw is deze stelling nog net zo relevant.

6. Het verschijnen van sommige mathematische objecten (zoals logaritmische spiraal, Fibonacci getalreeks, fractalen) in de levende natuur kan mogelijk verklaard worden uit het feit dat deze objecten gedefiniëerd kunnen worden door een kleine hoeveelheid informatie, ook in de genetische code.

7. Dit is een (schaak)stelling:



In deze stelling heeft wit 10 zetten nodig om schaakmat te geven. In alle andere stellingen (afgezien van de symmetrieën van het schaakbord) met dezelfde drie schaakstukken zijn 9 zetten altijd genoeg om schaakmat te bereiken.
(Zie ook *D.Levy, M.Newboorn, How computers play chess, pg.143, Computer Science Print, New York, 1991.*)

8. The Year 2000 problem: an end of the world as we know it.

9. Deze stelling is vol van leegte.