

## University of Groningen

### Aspects of algorithmic algebra

Vidunas, Raimundas

**IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.**

*Document Version*

Publisher's PDF, also known as Version of record

*Publication date:*  
1999

[Link to publication in University of Groningen/UMCG research database](#)

*Citation for published version (APA):*

Vidunas, R. (1999). *Aspects of algorithmic algebra: differential equations and splines*. s.n.

**Copyright**

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license. More information can be found on the University of Groningen website: <https://www.rug.nl/library/open-access/self-archiving-pure/taverne-amendment>.

**Take-down policy**

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

*Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.*

**Aspects of  
Algorithmic Algebra:  
Differential Equations and  
Splines**



Rijksuniversiteit Groningen

**Aspects of Algorithmic Algebra:  
Differential Equations and Splines**

**Proefschrift**

ter verkrijging van het doctoraat in de  
Wiskunde en Natuurwetenschappen  
aan de Rijksuniversiteit Groningen  
op gezag van de  
Rector Magnificus, dr. D.F.J. Bosscher,  
in het openbaar te verdedigen op  
maandag 18 oktober 1999  
om 16.00 uur

door

**Raimundas Vidūnas**  
geboren op 16 januari 1972  
te Lazdijai (Litouwen)

**Promotor:** Prof. dr. M. van der Put  
**Co-promotor:** Dr. G. Vegter

ISBN 90-367-1144-4

# Contents

<b>1</b>	<b>Introduction</b>	<b>5</b>
1.1	Differential equations . . . . .	5
1.2	Splines . . . . .	10
<b>2</b>	<b>Differential equations with one singular point</b>	<b>13</b>
2.1	Introduction . . . . .	13
2.2	Functors associated to the problem . . . . .	16
2.3	Smoothness and irreducibility . . . . .	20
2.4	$\mathbf{S}_{n,d}$ with $n = 2$ . . . . .	28
2.5	Projection to the coefficients of $v$ . . . . .	34
<b>3</b>	<b>Order two differential equations with Galois group <math>\mathbb{G}_m</math></b>	<b>41</b>
3.1	Introduction . . . . .	41
3.2	Local solutions of the Riccati equation . . . . .	43
3.3	The families of differential equations . . . . .	47
3.3.1	Dimension and smoothness . . . . .	50
3.3.2	Counting equations . . . . .	53
3.4	Differential equations with $r \in C(x)$ . . . . .	56
<b>4</b>	<b>Pull-backs of differential equations</b>	<b>65</b>
4.1	Differential equations with the Galois group $\mathbb{D}_\infty$ . . . . .	66
4.2	Algebraic solutions of differential equations . . . . .	71
4.2.1	Basic transformations . . . . .	72
4.2.2	Darboux curves of Fuchsian equations with three poles . . . . .	74
4.2.3	Classification . . . . .	76
4.2.4	Pull-back morphisms . . . . .	78
4.3	An order three example . . . . .	83
<b>5</b>	<b>Computing bivariate splines</b>	<b>85</b>
5.1	Introduction . . . . .	85
5.2	Basic representations of $C^r(\hat{\Delta})$ . . . . .	87
5.3	Computing splines on special triangulations . . . . .	94
5.4	Computation results . . . . .	99

<b>6 Geometrically continuous surface complexes</b>	<b>103</b>
6.1 Introduction . . . . .	103
6.2 $CG^1$ surface complexes . . . . .	105
6.2.1 Some terminology and notation . . . . .	105
6.2.2 Geometrically continuous glueing along two edges . . . . .	106
6.2.3 $CG^1$ glueing at a vertex . . . . .	110
6.2.4 Geometrically continuous surfaces . . . . .	114
6.2.5 Examples . . . . .	120
6.3 Geometrically continuous Bézier complexes . . . . .	126
6.3.1 Polynomial functions on triangles . . . . .	127
6.3.2 Splines on Bézier complexes . . . . .	129
6.4 Dimension of $CG^1$ spline spaces . . . . .	132
6.4.1 $CG^1$ splines on two triangles . . . . .	132
6.4.2 Separation of vertices . . . . .	135
6.4.3 Splines on general $CG^1$ Bézier surfaces . . . . .	139
6.4.4 More examples . . . . .	143
<b>Bibliography</b>	<b>151</b>
<b>Samenvatting</b>	<b>155</b>
<b>Acknowledgements</b>	<b>157</b>