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## Hankel norm approximation for infinite-dimensional systems

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Stellingen  
behorende bij het proefschrift

Hankel norm approximation for infinite-dimensional systems

van  
Amol Jagannath Sasane

1. Suppose

- (a)  $\Omega = \{\zeta + i\omega \mid a < \zeta < b, \omega \in \mathbb{R}\}$ ,  $\bar{\Omega} = \{\zeta + i\omega \mid a \leq \zeta \leq b, \omega \in \mathbb{R}\}$ ,
- (b)  $K : \bar{\Omega} \rightarrow \mathbb{C}^{p \times m}$  is continuous on  $\bar{\Omega}$ ,
- (c)  $K$  is analytic in  $\Omega$ ,
- (d)  $\|K(s)\| \leq B$  for all  $s \in \Omega$  and for some fixed  $B < \infty$ .

If  $M(\zeta) = \sup\{\|K(\zeta + i\omega)\| \mid \omega \in \mathbb{R}\}$  for  $a \leq \zeta \leq b$ , then we have

$$M(\zeta)^{b-a} \leq M(a)^{b-\zeta} M(b)^{\zeta-a} \text{ for all } a < \zeta < b.$$

Chapter 2.

2. If  $a_1 \geq a_2 \geq \dots \geq 0$  and  $\sum_{n=1}^{\infty} a_n < \infty$ , then  $\lim_{n \rightarrow \infty} n a_n = 0$ .

Chapter 3.

3. Suppose that  $A$  is the infinitesimal generator of a strongly continuous semigroup  $\{T(t)\}_{t \geq 0}$  on the Hilbert space  $X$  and  $C \in \mathcal{L}(X, Y)$ . If

- (a)  $Q = Q^* \in \mathcal{L}(X)$  satisfies  $A^*Qx + QA x = -C^*Cx \quad \forall x \in D(A)$ ,
- (b)  $A$  is normal,
- (c)  $C$  has finite rank and
- (d)  $\Sigma(A, -, C)$  is exponentially detectable,

then  $Q$  is such that  $\sigma(Q) \cap \mathbb{C}_0^- = \sigma_p(Q) \cap \mathbb{C}_0^-$  and  $\nu(Q) = \pi(A)$ , that is, the number of negative eigenvalues of  $Q$  is equal to the number of eigenvalues  $A$  in the open right half-plane.

Chapter 5.

4. Let  $\Omega$  be an open subset of  $\mathbb{R}^m$ . If  $p \in \mathbb{C}[\xi, \eta_1, \dots, \eta_m] \setminus \mathbb{C}[\eta_1, \dots, \eta_m]$ , then the behaviour corresponding to  $p$  is time-autonomous with respect to  $\mathcal{C}^\infty(\mathbb{R}, \mathcal{E}'(\Omega))$ .

A.J. Sasane, E.G.F. Thomas and J.C. Willems. Time-autonomy versus time-controllability. Submitted for publication to the *Systems and Control Letters*.

5. Let  $Q$  be an  $n \times n$  nonnegative real symmetric matrix, and consider the quadratic form  $q(x) = x^T Q x$  for  $x \in \mathbb{R}^n$ . Let  $C$  be the hypercube  $[-1, 1]^n$ . Then  $\max_{x \in C} q(x)$  is at least as large as the trace of  $Q$ .
6. A triangle is divided by its three medians into six smaller triangles. The circumcenters of these smaller triangles lie on a circle.  
10830, Problems and Solutions, *The American Mathematical Monthly*, Page 863, November 2000.
7. Let  $\mathcal{H}$  be an infinite-dimensional closed subspace of  $L_2[0, 1]$ . Then  $\mathcal{H}$  contains a function that is essentially discontinuous, meaning that there is no continuous function  $g$  on  $[0, 1]$  equal to  $f$  almost everywhere. This conclusion is not true if  $g$  is required to be continuous only on  $(0, 1]$ .  
10668, Problems and Solutions, *The American Mathematical Monthly*, Page 465, May 1998 and Pages 966-967, December, 1999.
8.  $\tan\left(\frac{\pi}{2001}\right)$  is irrational.
9. The relation  $\tau = 1 + \frac{1}{\tau}$  shows that when a square is removed from one end of a golden rectangle<sup>1</sup>, the remaining rectangle is again golden. Circular arcs, inscribed in the successive squares give rise to a composite spiral of quite pleasing proportions.  
Cover.
10. Stellingen are overrated.

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<sup>1</sup>a rectangle whose sides are in the ratio  $\tau : 1$