Hankel norm approximation for infinite-dimensional systems

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Stellingen
behorende bij het proefschrift

Hankel norm approximation for infinite-dimensional systems

van
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1. Suppose
   (a) $\Omega = \{\zeta + i\omega \mid a < \zeta < b,\ \omega \in \mathbb{R}\}$, $\overline{\Omega} = \{\zeta + i\omega \mid a \leq \zeta \leq b,\ \omega \in \mathbb{R}\}$,
   (b) $K : \overline{\Omega} \to \mathbb{C}^{\mathtt{p} \times \mathtt{m}}$ is continuous on $\overline{\Omega}$,
   (c) K is analytic in Ω,
   (d) $\|K(s)\| \leq B$ for all $s \in \Omega$ and for some fixed $B < \infty$.
   If $M(\zeta) = \sup\{\|K(\zeta + i\omega)\| \mid \omega \in \mathbb{R}\}$ for $a \leq \zeta \leq b$, then we have
   $$M(\zeta)^{b-a} \leq M(a)^{b-\zeta} M(b)^{\zeta - a} \text{ for all } a < \zeta < b.$$
   Chapter 2.

2. If $a_1 \geq a_2 \geq \ldots \geq 0$ and $\sum_{\mathtt{n}=1}^{\infty} a_{\mathtt{n}} < \infty$, then $\lim_{\mathtt{n} \to \infty} \mathtt{n}\, a_{\mathtt{n}} = 0$.
   Chapter 3.

3. Suppose that A is the infinitesimal generator of a strongly continuous semigroup $\{T(t)\}_{t \geq 0}$ on the Hilbert space X and $C \in \mathcal{L}(X, Y)$. If
   (a) $Q = Q^* \in \mathcal{L}(X)$ satisfies $A^*Qx + QAx = -C^*Cx \quad \forall x \in D(A)$,
   (b) A is normal,
   (c) C has finite rank and
   (d) $\Sigma(A, -, C)$ is exponentially detectable,
   then Q is such that $\sigma(Q) \cap \mathbb{C}_0^- = \sigma_p(Q) \cap \mathbb{C}_0^-$ and $\nu(Q) = \pi(A)$, that is, the number of negative eigenvalues of Q is equal to the number of eigenvalues A in the open right half-plane.
   Chapter 5.

4. Let Ω be an open subset of $\mathbb{R}^{\mathtt{m}}$. If $p \in \mathbb{C}[\xi, \eta_1, \ldots, \eta_{\mathtt{m}}] \setminus \mathbb{C}[\eta_1, \ldots, \eta_{\mathtt{m}}]$, then the behaviour corresponding to p is time-autonomous with respect to $\mathcal{C}^{\infty}(\mathbb{R}, \mathcal{E}'(\Omega))$.

A.J. Sasane, E.G.F. Thomas and J.C. Willems. Time-autonomy versus time-controllability. Submitted for publication to the *Systems and Control Letters*.

5. Let Q be an $\mathtt{n} \times \mathtt{n}$ nonnegative real symmetric matrix, and consider the quadratic form $\mathtt{q}(x) = x^T Q x$ for $x \in \mathbb{R}^{\mathtt{n}}$. Let $\mathtt{C}$ be the hypercube $[-1,1]^{\mathtt{n}}$. Then $\max_{x \in \mathtt{C}} \mathtt{q}(x)$ is at least as large as the trace of Q.

6. A triangle is divided by its three medians into six smaller triangles. The circumcenters of these smaller triangles lie on a circle.

   10830, Problems and Solutions, *The American Mathematical Monthly*, Page 863, November 2000.

7. Let $\mathcal{H}$ be an infinite-dimensional closed subspace of $L_2[0,1]$. Then $\mathcal{H}$ contains a function that is essentially discontinuous, meaning that there is no continuous function g on $[0,1]$ equal to f almost everywhere. This conclusion is not true if g is required to be continuous only on $(0,1]$.

   10668, Problems and Solutions, *The American Mathematical Monthly*, Page 465, May 1998 and Pages 966-967, December, 1999.

8. $\tan\left(\frac{\pi}{2001}\right)$ is irrational.

9. The relation $\tau = 1 + \frac{1}{\tau}$ shows that when a square is removed from one end of a golden rectangle[1], the remaining rectangle is again golden. Circular arcs, inscribed in the successive squares give rise to a composite spiral of quite pleasing proportions.

   Cover.

10. Stellingen are overrated.

[1]a rectangle whose sides are in the ratio $\tau : 1$