# Hankel norm approximation for infinite-dimensional systems

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# Standard notation

Here we list some standard notation that we use in the thesis. Non standard symbols which are defined in the thesis can be looked up in the Index.

**Set theory**

1. $\mathbb{N}$: the set of natural numbers $\{1, 2, 3, \ldots\}$.
2. $\complement\, S$: complement of the set $S$.

**Linear algebra**

Throughout this thesis, unless otherwise specified, when considering a vector space, the underlying field is $\mathbb{C}$. In a matrix, blank entries are always to be understood to be zeros.

1. $\dim(\mathcal{V})$, where $\mathcal{V}$ is a finite-dimensional vector space: The dimension of $\mathcal{V}$.
2. $\ker(T)$ and $\operatorname{ran}(T)$, where $T : \mathcal{V}_1 \to \mathcal{V}_2$ is a linear transformation from the vector space $\mathcal{V}_1$ to the vector space $\mathcal{V}_2$: the null space and the range of $T$, respectively.

**Linear analysis**

1. $\mathcal{L}(\mathcal{H}_1, \mathcal{H}_2)$, where $\mathcal{H}_1$ and $\mathcal{H}_2$ are Hilbert spaces: the Banach space of all bounded linear operators with the norm $\|T\| = \sup_{(0\neq)x\in\mathcal{H}_1} \frac{\|Tx\|_{\mathcal{H}_2}}{\|x\|_{\mathcal{H}_1}}$.
2. $\rho(A)$, $\sigma(A)$, $\sigma_p(A)$, $\sigma_c(A)$, where $A : D(A)\,(\subset \mathcal{H}) \to \mathcal{H}$ is a closed linear operator, and $\mathcal{H}$ is a Hilbert space: the resolvent set, spectrum, point spectrum and continuous spectrum, respectively, of $A$. If $D(A)$ is dense in $\mathcal{H}$, then $A^*$ denotes the adjoint of $A$.

3. $\mathcal{C}(X_1, X_2)$, where $X_1$ and $X_2$ are topological spaces: the space of continuous maps from $X_1$ to $X_2$. If $X_2$ is $\mathbb{C}$, we simply write $\mathcal{C}(X_1)$ instead of $\mathcal{C}(X_1, \mathbb{C})$.
4. $\ell_2(\mathbb{N})$ $(\ell_2(\mathbb{Z}))$: the Hilbert space of square summable complex sequences indexed by $\mathbb{N}$ (respectively $\mathbb{Z}$), with the inner product
$$\langle a, b\rangle_2 = \sum_{n=1}^{\infty} a_n \overline{b_n} \quad \left(\text{respectively } \langle a, b\rangle_2 = \sum_{n=-\infty}^{\infty} a_n \overline{b_n}\right).$$
5. $\ell_1$: the Banach space of summable complex sequences indexed by $\mathbb{N}$ with the norm
$$\|a\|_1 = \sum_{n=1}^{\infty} |a_n|.$$
6. $\ell_\infty$: the Banach space of complex sequences indexed by $\mathbb{N}$ and bounded in absolute value with the norm
$$\|a\|_\infty = \sup_{n\in\mathbb{N}} |a_n|.$$
7. $L_2^{\text{loc}}(\Omega, X)$, where $\Omega(\subset \mathbb{R})$ is measurable and $X$ is a Banach space:
$$\left\{ f : \Omega \to X \;\middle|\; \begin{array}{l} f \text{ is Bochner-Lebesgue measurable and} \\ \int_K \|f(\omega)\|^2 d\omega < \infty \text{ for any compact set } K(\subset \Omega) \end{array} \right\}.$$
$L_p(\Omega, X)$, where $1 \le p < \infty$:
$$\left\{ f : \Omega \to X \;\middle|\; \begin{array}{l} f \text{ is Bochner-Lebesgue measurable and} \\ \int_\Omega \|f(\omega)\|^p d\omega < \infty \end{array} \right\}$$
with the norm $\|f\|_p = \left(\int_\Omega \|f(\omega)\|^p d\omega\right)^{\frac{1}{p}}$.If $X$ is $\mathbb{C}$, we simply write $L_p(\Omega)$ instead of $L_p(\Omega, \mathbb{C})$.
If $p = 2$ and $X$ is a Hilbert space, then it is also a Hilbert space with the inner product $\langle f, g\rangle_2 = \int_\Omega \langle f(\omega), g(\omega)\rangle d\omega$.
$L_\infty(\Omega, X)$:
$$\left\{ f : \Omega \to X \;\middle|\; \begin{array}{l} f \text{ is Bochner-Lebesgue measurable and} \\ \operatorname{esssup}_{\omega\in\Omega} \|f(\omega)\| < \infty \end{array} \right\}$$
with the norm $\|f\|_\infty = \operatorname{esssup}_{\omega\in\Omega} \|f(\omega)\|$.
8. $H_\infty(X)$, where $X$ is a Banach space:
$$\left\{ f : \mathbb{C}_0^+ \to X \mid f \text{ is analytic and } \sup_{s\in\mathbb{C}_0^+} \|f(s)\| < \infty \right\}.$$