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Hankel norm approximation for infinite-dimensional systems

Sasane, Amol Jagannath

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Publisher's PDF, also known as Version of record

Publication date:

2001

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Sasane, A. J. (2001). *Hankel norm approximation for infinite-dimensional systems*. s.n.

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Rijksuniversiteit Groningen

Hankel norm approximation
for infinite-dimensional systems

Proefschrift

ter verkrijging van het doctoraat in de
Wiskunde en Natuurwetenschappen
aan de Rijksuniversiteit Groningen
op gezag van de
Rector Magnificus, dr. D.F.J. Bosscher,
in het openbaar te verdedigen op
vrijdag 21 september 2001
om 14.15 uur

door

Amol Jagannath Sasane

geboren op 28 maart 1976
te Bombay, India.

Promotor: Prof. dr. R.F. Curtain

Co-promotor: dr. H.J. Zwart

Beoordelingscommissie: Prof. dr. D.Z. Arov
Prof. dr. A. Dijksma
Prof. dr. A.C.M. Ran

Acknowledgements

It is a pleasure to express my deep gratitude to my supervisor Ruth Curtain for her untiring and dedicated guidance during the course of this long and chequered thesis. I learnt a lot about systems theory and life under her tutelage, and collaborating with her has been a rewarding and fruitful experience. Also I would like to thank her for trying to get rid of Indianisms from my English.

I wish to thank Professors Thomas Azizov (Voronezh State University, Russia), Aad Dijksma and Erik Thomas (both from the University of Groningen) for their help with my frequent questions in functional analysis. They were always very helpful and listened patiently to my questions and came up with counterexamples to my sometimes over-ambitious conjectures or helped me with proofs in case my claims were true. I am indebted to Professor Erik Thomas for the courses he agreed to give in English and the special topics he taught at the end of the courses upon request. I benefitted immensely from all the discussions I had with him. Thanks also for trusting me with the exercise classes for the course on Distribution Theory. I would like to thank Professor Jan Willems for inspiring and creating an enthusiasm in me for mathematics in general and systems theory in particular. His intuition, insight and perfectionism have been a constant source of inspiration.

Many thanks to the members of the reading committee, Professors Arov (South-Ukrainian Pedagogical Univeristy), Dijksma and Ran (Vrije Universiteit, Amsterdam) and to Professor Hans Zwart (University of Twente, Enschede) for fishing out various typos and several mistakes in the proofs. Their comments greatly improved the thesis.

I would like to thank all the members and visitors of the system theory group in Groningen for their cheerful company: Tommaso Cotroneo, Harish Pillai, Kees Praagman, Paolo Rapisarda, Shiva Shankar, Kiyotsugu Takaba and Harry Trentelman. Special thanks to Madhu Belur for being always helpful with my questions about Unix and for the chats in Hinglish, and to Job Oostveen for showing me the ropes in the world of infinite-dimensions.

Thanks to all my friends in Groningen and elsewhere: in particular, my former house-mate Sergey Kondratyuk (for sharing many a rainy weekend in front of the tv) and Kanat Çamlıbel (for being a Monthly problem solver).

Finally, thank you Atul, Shilpa, Sanjyot, Aai and Daddy for everything.

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