Geometry of strings and branes

Halbersma, Reinder Simon

Chapter 4

Brane world scenarios

In the previous two chapters, we described various aspects of the correspondence between gravity and gauge theories. The material in the final two chapters concerns the structure of conformal supergravity and its couplings to matter. This chapter aims to provide the connection between these two, at first sight, rather different subjects.

We will start with giving a short review of two old problems, the hierarchy problem and the cosmological constant problem, and we will indicate how recent developments have shed some new light on these subjects. In particular, we will show how brane world scenarios, such as those of Randall and Sundrum [138, 139], provide a new framework in which various phenomenological aspects of elementary particle physics can be treated. The various techniques which were introduced in the previous chapters, in particular the geometrical properties of supergravity brane solutions, can be re-used in this description.

Attempts to supersymmetrize the brane world scenarios and to embed them in a natural way into string theory have so far met with many obstacles. In order to be able to address this in detail, the structure of five-dimensional supergravity and the various possible couplings to supersymmetric matter need to be taken into account. These problems will be the topic of the following two chapters where they will be studied in a superconformal context.

4.1 Fine-tuning problems

Two longstanding problems in theoretical physics, the hierarchy problem and the cosmological constant problem, are fine-tuning problems. In both cases, there are two fundamental scales, an experimentally observed scale and a theoretically expected scale, which are many orders of magnitude apart. We will first briefly review these problems.

4.1.1 The hierarchy problem

The electroweak scale is defined to be the energy scale in the Standard Model description of elementary particle physics at which the electromagnetic interaction unifies with the weak interaction. Since the Higgs-particle hypothesized to be responsible for the breaking of the electroweak $\mathrm{U}(1)_Y \times \mathrm{SU}(2)_\mathrm{L}$ gauge symmetry into the the electromagnetic gauge symmetry $\mathrm{U}(1)_\mathrm{em}$ has not been observed yet, an accurate number for this energy scale is not available. Instead, we will take the upper-limit[1] of 10^3 GeV, or 10^{-19} m.

The Planck energy scale is theoretically calculated to lie at $\lambda_\mathrm{Planck} = \sqrt{\frac{\hbar c}{G}} = 10^{19}$ GeV or at 10^{-35} m. At the Planck scale, a theory of Quantum Gravity should be revealed, and it is hoped that the gravitational interaction unifies with the remaining three interactions described by the Standard Model. The hierarchy of sixteen orders of magnitude between these two scales, and in particular the difficulty in explaining the radiative stability of electroweak scale masses in a theory with the Planck scale as the fundamental scale is called the hierarchy problem.

There is a sharp distinction between the two scales: the electroweak interactions have been (or will be in the near future) accurately probed up to scales of λ_weak, but the gravitational interaction has only been probed for distances up to the sub-millimeter range [140]. This opens up the possibility for a qualitatively different picture of gravity already far below the Planck scale. In particular, it is possible that new gravitational effects might appear already just above the electroweak scale. Indeed, in recent years, there was speculation on the existence of TeV scale strings [141].

A related direction that explored was the possible relation between the hierarchy problem and the existence of n extra compact dimensions [142–144]. The essential feature of these models is that in the higher-dimensional theory there is only a single scale: the four-dimensional weak scale λ_weak. The size of the extra dimensions generates the hierarchy between the weak scale and the Planck scale in four dimensions: from the standard relation (1.48) between the gravitational couplings in theories related by dimensional reduction, we find

$$\lambda^2_\mathrm{Planck} = \lambda^{n+2}_\mathrm{weak} \left(\frac{R}{\hbar c}\right)^n . \tag{4.1}$$

The characteristic radius R of such extra dimensions is then given by

$$R = \frac{\hbar c}{\lambda_\mathrm{weak}} \left(\frac{\lambda_\mathrm{Planck}}{\lambda_\mathrm{weak}}\right)^{\frac{2}{n}} . \tag{4.2}$$

Taking $n = 2$, we find $R \approx 2$ mm, whereas $n = 3$ yields $R \approx 9.3$ nm. However, recent measurements have verified that the gravitational interactions follows Newton's law for distances down to 0.2 mm [140]. In order to explain the hierarchy problem, one needs at least

[1] This is one order of magnitude larger than the expected Higgs-mass of around 10^2 GeV, for our discussion this will not make a difference

three nanometer-sized, or more and even smaller, extra dimensions: this will make it harder to measure possible deviations from standard gravitational physics at such distances.

Another problem of these models is that the hierarchy between the Planck and the weak scale is replaced by the hierarchy between the weak scale and the compactification scale. Even for $n = 6$, representing a ten-dimensional compactification scheme suitable for string theory, this ratio of the weak scale and the compactification scale is still about five orders of magnitude. In the next section, we will see how a specific brane world setup might solve this subtlety in terms of only a single compact dimension.

4.1.2 The cosmological constant problem

The cosmological constant problem is the puzzle that the bound on a cosmological energy scale coming from Hubble's constant is of much smaller value than can be explained by any effective field theory [4].

The cosmological constant can be attributed to a fluid form of matter with a negative pressure that equals minus its density. The associated energy is usually called the vacuum energy density

$$\Lambda \simeq 8\pi G_4 \langle \rho_{\rm vac} \rangle \ . \tag{4.3}$$

On the other hand, the Friedman equations – describing the cosmological evolution of a Robertson-Walker like universe – give a bound on the cosmological constant Λ in terms of the Hubble parameter H_0

$$\Lambda \leq 3H_0^2 \, . \tag{4.4}$$

Hubble's constant measures the relative rate of expansion of the universe; recent astronomical data [145] give the value

$$H_0 \simeq 71 \text{ km s}^{-1}\text{Mpc}^{-1} \, . \tag{4.5}$$

This gives an upper-bound on the energy density of the vacuum

$$\langle \rho_{\rm vac} \rangle \leq 4.0 \cdot 10^{-47} \text{ GeV}^4 \, . \tag{4.6}$$

A naive quantum field theory calculation of summing the zero-point energies of all normal modes of some field of mass m up to a cut-off $\lambda_{\rm cut-off} \gg m$ yields a vacuum energy [4]

$$\begin{aligned} \langle \rho_{\rm vac} \rangle &\simeq \int_0^{\lambda_{\rm cut-off}} \frac{4\pi k^2 {\rm d}k}{(2\pi)^3} \cdot \frac{1}{2}\sqrt{k^2+m^2} \\ &\approx \frac{\lambda^4_{\rm cut-off}}{16\pi^2} \, . \end{aligned} \tag{4.7}$$

Depending on one's confidence in field theory, one can take the cut-off at either the weak scale or at the Planck scale. In these cases, the value of the vacuum energy-density would be 56, respectively 121 orders of magnitude above any reasonable cosmological value

$$\langle \rho_{\rm vac} \rangle_{\rm weak} \approx 6.3 \cdot 10^9 \text{ GeV}^4 \, , \quad \langle \rho_{\rm vac} \rangle_{\rm Planck} \approx 1.3 \cdot 10^{74} \text{ GeV}^4 \, . \tag{4.8}$$

The only viable field theory reason for why the cosmological constant is very small, is that it should be exactly zero through some sort of cancellation mechanism. Supersymmetry is a candidate for such a mechanism since there are fermionic and bosonic contributions to the zero-point energies which can cancel each other. However, since supersymmetry is broken at low-energy scales, the cosmological constant problem is shifted to the problem of finding a mechanism for breaking supersymmetry that protects a vanishing energy from blowing up. At present, no such mechanism has been found. In the next section, we will see how this might nevertheless be circumvented.

4.2 The Randall-Sundrum scenarios

In 1999, Randall and Sundrum published two papers in which they studied three-brane solutions in a five-dimensional Anti-de-Sitter space [138, 139]. We will now briefly review these papers. The Randall-Sundrum (RS) scenarios are by no means the only brane world models: we refer to the reviews [146, 147] for more information on this subject.

4.2.1 Two-brane setup

The first paper of Randall and Sundrum [138] discusses a two-brane setup also known as the RS1-scenario. They discussed two three-branes in a five-dimensional Anti-de-Sitter spacetime for which the radial coordinate first compactified to a circumference of $2r_c$, and then was acted upon by a $S^1/\mathbb{Z}_2$ orbifold projection.

$$r \equiv r + 2r_c \,, \quad r \equiv -r \,. \tag{4.9}$$

The two three-branes are located at the two fixed-points $r = 0$ and $r = r_c$ of the $\mathbb{Z}_2$-reflection. The brane located at the origin $r = 0$ is called the Planck brane (or hidden brane in [138]), the brane located at the edge $r = r_c$ is called the Standard Model (SM) brane (or visible brane in [138]). It is at this latter brane where our four-dimensional physics takes place. We have displayed this setup (including the mirror-brane at $r = -r_c$) in figure 4.1.

An action which has this setup as a solution is given by

$$S = \frac{1}{2\kappa^2} \int \mathrm{d}^5 x \, \sqrt{|g_{(5)}|} \left(R_{(5)} - 2\Lambda \right) + S_{\mathrm{Planck}} + S_{\mathrm{SM}} \,. \tag{4.10}$$

The actions for the Planck and the Standard Model brane consist of a tension part and higher order corrections that will not be specified further

$$S_{\mathrm{Planck}} = \int \mathrm{d}^4 x \, \sqrt{|g_{(4)}|} \left(\mathcal{L}_{\mathrm{Planck}} - T_{\mathrm{Planck}} \right) \,, \tag{4.11}$$

$$S_{\mathrm{SM}} = \int \mathrm{d}^4 x \, \sqrt{|g_{(4)}|} \left(\mathcal{L}_{\mathrm{SM}} - T_{\mathrm{SM}} \right) \,. \tag{4.12}$$

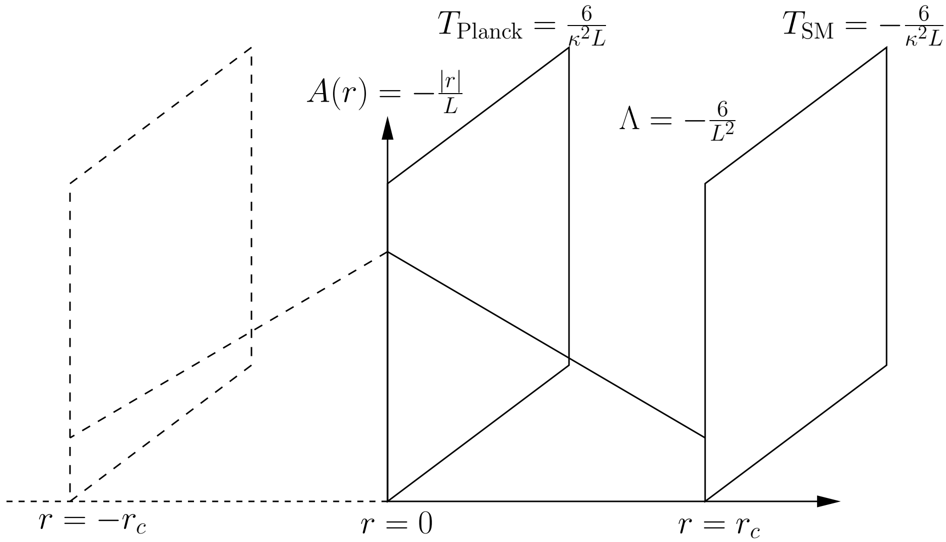

Figure 4.1: The two-brane Randall-Sundrum setup.

We will try the following Ansatz for the solution

$$ds^2 = e^{2A(r)} \eta_{\mu\nu} dx^\mu dx^\nu + dr^2 \,. \tag{4.13}$$

The equations of motion for this Ansatz are given by a generalization of (1.67) and (2.53)

$$\begin{aligned} 3A''(r) &= -\kappa^2 \left(T_{\rm Planck}\, \delta(r) + T_{\rm SM}\, \delta(r - r_c) \right) \,, \\ 6A'(r)^2 &= -\Lambda \,. \end{aligned} \tag{4.14}$$

In order to generate the appropriate delta-functions, $A(r)$ has to depend on the absolute value of the radial coordinate. Using (4.9), this gives the following expressions

$$A(r) = -\frac{|r|}{L} \,, \quad \Lambda = -\frac{6}{L^2} \,, \quad T_{\rm Planck} = -T_{\rm SM} = \frac{6}{\kappa^2 L} \,. \tag{4.15}$$

We will now look at the fluctuations around this solution. First, we note that the off-diagonal fluctuations of the metric correspond to those isometries of the five-dimensional space that are broken by the three-branes. Hence, these Kaluza-Klein vectors $A_\mu(x^\mu)$ are massive and can be ignored in a linearized analysis.

The remaining fluctuations are described by a symmetric tensor $h_{\mu\nu}(x^\mu)$ and a scalar field $T(x^\mu)$. The tensor $h_{\mu\nu}(x^\mu)$ generates four-dimensional gravity, and the effective gravitational action can be obtained from substituting (4.13) and (4.15) into the action (4.10) and integrating out the radial coordinate

$$\begin{aligned}\frac{1}{16\pi G_5}\int \mathrm{d}^5x\,\sqrt{|g_{(5)}|}R_{(5)} &= \frac{1}{16\pi G_5}\int_{-r_c}^{r_c}\mathrm{d}r\,e^{-2|r|/L}\int \mathrm{d}^4x\,\sqrt{|g_{(4)}|}R_{(4)} \\ &\equiv \frac{1}{16\pi G_4}\int \mathrm{d}^4x\,\sqrt{|g_{(4)}|}R_{(4)}\,. \qquad (4.16)\end{aligned}$$

From this, we deduce that the effective four-dimensional gravitational constant depends only weakly on r_c

$$\begin{aligned}G_4 &= \frac{G_5}{L}\left(1-e^{-2r_c/L}\right) \\ &\approx \frac{G_5}{L}\,, \quad r_c \gg L\,. \qquad (4.17)\end{aligned}$$

The second observation is that the effective four-dimensional metric, describing the gravitational fluctuations, is given by the metric localized on the Planck brane

$$\begin{aligned}g^{(4)}_{\mu\nu}(x^\mu) &= g^{\mathrm{Planck}}_{\mu\nu}(x^\mu) \\ &\equiv g^{(5)}_{\mu\nu}(x^\mu, r=0) \\ &= \eta_{\mu\nu}+h_{\mu\nu}(x^\mu)\,. \qquad (4.18)\end{aligned}$$

It is with respect to this metric that physical quantities will have to be measured with. If we postulate that the Standard Model matter content is located on the brane at $r = r_c$, then all terms, in particular the mass terms, in the Lagrangian $\mathcal{L}_{\mathrm{SM}}$ will couple to the metric

$$\begin{aligned}g^{\mathrm{SM}}_{\mu\nu}(x^\mu) &\equiv g^{(5)}_{\mu\nu}(x^\mu, r=r_c) \\ &= e^{-2r_c/L}g^{\mathrm{Planck}}_{\mu\nu}(x^\mu)\,. \qquad (4.19)\end{aligned}$$

From this we deduce that a mass m_0 in $\mathcal{L}_{\mathrm{SM}}$ corresponds to a physical mass m which is shifted by an exponential factor

$$m = m_0\,e^{-r_c/L}\,. \qquad (4.20)$$

So we see that the energy scales on both branes are related by an exponential factor which gives a method of solving the hierarchy problem. In particular, if $r_c \approx 35L$ then we can obtain TeV scale masses on the Standard Model brane from Planck scale masses on the hidden Planck brane.

Some problems with the above model are that the fine-tuning between the weak scale and the Planck scale is replaced by the fine-tuning between the Anti-de-Sitter radius L and the brane separation r_c. This is related to the problem of treating the scalar field $T(x^\mu)$ that describes the relative motion between the branes. For consistency reasons, this so-called radion has to be a massive field with the correct expectation value in order to maintain stability of the solution.

Another problem is that the Standard Model brane has a negative tension which makes it quite hard to embed the whole setup into string theory in a natural way. We refer to the literature [146, 147] for a more thorough discussion of these subtleties.

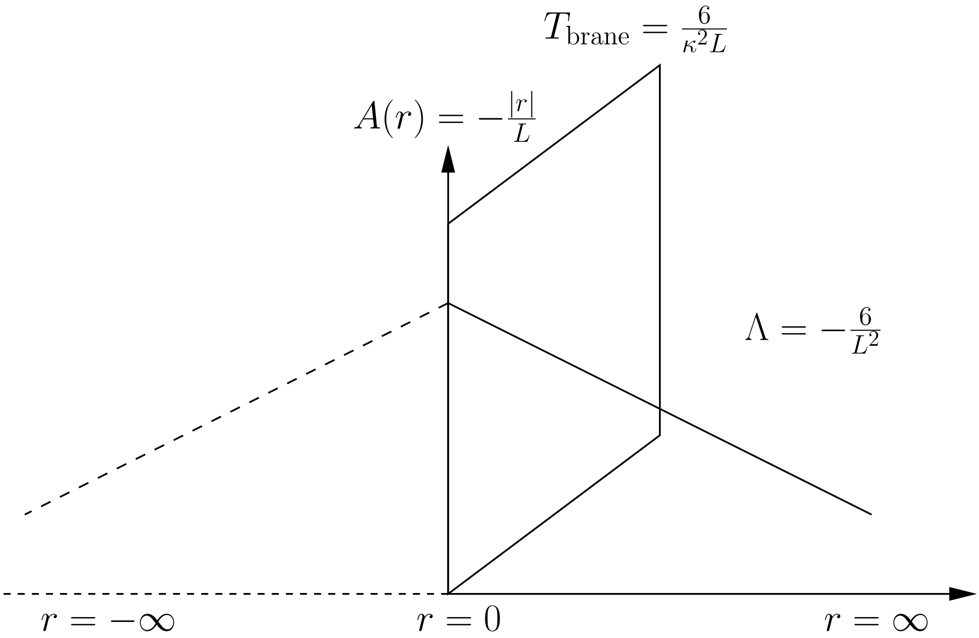

Figure 4.2: The single-brane Randall-Sundrum setup.

4.2.2 Single-brane setup

In the previous section, we made the observation that the four-dimensional Newton's constant (4.17) does not depend on the brane separation if r_c is large compared to the Anti-de-Sitter radius L. We also noted that the relevant four-dimensional metric was equal to the metric located on the brane in the origin, the Planck brane. This suggests that gravity might be effectively localized on the Planck brane.

This reasoning led Randall and Sundrum to consider a modified scenario in which the Standard Model brane was pushed to infinity [139]. The resulting one-brane scenario (or RS2-scenario[2]) is shown in figure 4.2.

The action supporting this configuration is given by

$$S = \frac{1}{2\kappa^2} \int \mathrm{d}^5 x \sqrt{|g_{(5)}|} \left(R_{(5)} - 2\Lambda\right) - T_{\text{brane}} \int \mathrm{d}^4 x \sqrt{|g_{(4)}|} \,. \tag{4.21}$$

We will again look for solutions which preserve four-dimensional Poincaré invariance

$$\mathrm{d}s^2 = e^{2A(r)} \eta_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}r^2 \,. \tag{4.22}$$

[2]The number here indicates chronology, not the number of branes.

The equations of motion for this Ansatz are analogous to (4.14)

$$\begin{aligned} 3A''(r) &= -\kappa^2 T_{\text{brane}}\,\delta(r)\,, \\ 6A'(r)^2 &= -\Lambda\,. \end{aligned} \tag{4.23}$$

We find the following solution

$$A(r) = -\frac{|r|}{L}\,, \quad \Lambda = -\frac{6}{L^2}\,, \quad T_{\text{brane}} = \frac{6}{\kappa^2 L}\,. \tag{4.24}$$

Next, we want to analyze the effective gravitational dynamics for this solution. A standard Kaluza-Klein reduction over a compact fifth dimension of size R_5 relates Newton's constant G_5 in five dimensions to the four-dimensional gravitational constant G_4 through the volume of the compact dimension (c.f. (1.48))

$$G_4 = \frac{G_5}{R_5}\,. \tag{4.25}$$

For the RS2 scenario, where the fifth dimension is infinite, such a mechanism would imply that the effective gravitational interaction would have a vanishing strength on the brane. However, when we compare (4.25) with (4.17), then we see that something remarkable has happened: the warp-factor in the metric ensures that the infinite fifth dimension effectively behaves as a region of finite size L.

In order to determine the effective four-dimensional action, we substitute the solution given in (4.22) and (4.24) into the action (4.21) and integrate out the radial coordinate to find for the effective four-dimensional action

$$S = \frac{1}{16\pi G_4}\int \mathrm{d}^4x\,\sqrt{|g_{(4)}|}\,\left(R_{(4)} - 2\Lambda_{\text{eff}}\right)\,, \tag{4.26}$$

where the effective cosmological constant Λ_{eff} on the brane actually vanishes

$$\begin{aligned} \Lambda_{\text{eff}} &\equiv \Lambda + \frac{\kappa^2 T_{\text{brane}}}{L} \\ &= 0\,. \end{aligned} \tag{4.27}$$

In the last equality we made once again use of (4.24). This is a nice result: the cosmological constant in the bulk and the tension on the brane cancel each other, and observers on the brane experience a vanishing effective cosmological constant.

However, recent astronomical observations [148,149] indicate that the universe is not only expanding, but that it is actually accelerating. This implies that the cosmological constant is not only very small, but also positive which undermines the relevance of the RS2 model. For more details concerning these problems, we refer to the literature [150].

4.2.3 Localization of gravity on the brane

The most remarkable feature of the Randall-Sundrum brane world is that gravity in the five-dimensional bulk is effectively localized on the four-dimensional brane. This is surprising since no elementary branes have gravitational degrees of freedom on their worldvolume. Instead, the fluctuations around the static solutions are described by scalar, vector or tensor multiplets.

Randall and Sundrum calculated the effective Schrödinger-like equation which the four-dimensional graviton modes have to satisfy and showed that the graviton is indeed localized near the brane. Another check is to calculate the corrections to Newton's law on the brane. In general, one-loop corrections to the graviton propagator induce $1/r^3$ corrections to the gravitational potential [151, 152]

$$V(r) = \frac{G_4 m_1 m_2}{r}\left(1 + \frac{\alpha G_4}{r^2}\right) . \tag{4.28}$$

If only spins ≤ 1 contribute, then the coefficient α is given by the following expression in terms of the numbers N_s of particles of spin s

$$45\pi\alpha = 12N_1 + 3N_{\frac{1}{2}} + N_0 \, . \tag{4.29}$$

In order to calculate these coefficients, we first clarify the interpretation of the RS2 scenario in the AdS/CFT correspondence. Recall that in the Poincaré coordinates for Anti-de-Sitter spacetime the dual CFT is located at $r = -\infty$. The RS2 scenario has a three-brane located in the origin which acts as an UV cutoff in the dual CFT. Moving the brane to $r = -\infty$ removes the cutoff and gives back the Anti-de-Sitter solution without the absolute value function in the exponential.

Hence, from the analogy with the AdS/CFT correspondence, we expect that the theory on the brane is given by $\mathcal{N} = 4$ supersymmetric $\mathrm{SU}(N)$ Yang-Mills theory. This theory has a single vector, four spinors, and six scalars that all transform in the N^2-dimensional[3] adjoint representation

$$(N_1, N_{\frac{1}{2}}, N_0) = (N^2, 4N^2, 6N^2) \, . \tag{4.30}$$

Substituting this into (4.28), we obtain

$$V(r) = \frac{G_4 m_1 m_2}{r}\left(1 + \frac{2N^2 G_4}{3\pi r^2}\right) . \tag{4.31}$$

In order to eliminate N, we recall the relations

$$4\pi g_s N = \left(\frac{L}{\ell_s}\right)^4 , \qquad \frac{1}{16\pi G_{10}} = \frac{2\pi}{g_s^2 (2\pi\ell_s)^8} \, . \tag{4.32}$$

[3]The adjoint representation of $\mathrm{SU}(N)$ has dimension $N^2 - 1$, which scales as N^2 for large N.

The gravitational coupling constants in ten and five dimensions are related to each other by the volume of the five sphere S^5

$$\begin{aligned} G_5 &= \frac{G_{10}}{\Omega_{(5)} L^5} \\ &= \frac{G_{10}}{\pi^3 L^5} \, . \end{aligned} \tag{4.33}$$

Substituting this into (4.31), we finally obtain

$$V(r) = \frac{G_4 m_1 m_2}{r} \left(1 + \frac{2L^2}{3r^2} \right) \, . \tag{4.34}$$

This is the same result as was found in [153] from a canonical one-loop calculation. With this illustrative example, we conclude this section. More discussion on this topic can be found in [139].

4.3 Supersymmetric brane worlds

The brane world scenarios described in the previous section have a very rich structure, and one would like to embed them in a natural way into string theory. As a first step, one would like to have a supersymmetric version of a brane world scenario. It turns out that finding such a simple extension is nontrivial. In this section, we will summarize the current status of the search for a supersymmetric brane world scenario.

4.3.1 Conditions on the scalar potential

As we remarked in the previous chapter, one can make a distinction between “thin” branes and “thick” branes. The “thin” brane approach has as an advantage that it is more similar to the original RS scenario. In [154] a method was developed to formulate supersymmetric theories in the presence of delta function singularities in general, and in the presence of brane sources in particular. A possible embedding of such a supersymmetric RS2 scenario with singular sources into string theory might be a suitable Calabi-Yau compactification of the eleven-dimensional Hořava-Witten model [53].

The “thick” brane approach would be to search for supersymmetric interpolating soliton solutions of five-dimensional supergravity, in much the same way as was described in the previous chapter. For such supersymmetric branes, the amount of supersymmetry preserved on the brane is generically half of the supersymmetry of the bulk theory. In particular, in order to have $\mathcal{N} = 1$ supergravity on the four-dimensional brane, one would have to start with $\mathcal{N} = 2$ supergravity in the five-dimensional bulk.

As we saw in the previous chapter, in order to find interpolating soliton solutions, the critical points of the scalar potential need to be analyzed. The toy model example that we discussed in chapter 3 only had a single scalar field, but the matter multiplets that can couple to five-dimensional models have a set of scalars ϕ^i.

The structure of such supergravity matter-couplings is complicated. A useful tool in studying matter-couplings is the geometry induced by the scalar fields present in the various matter multiplets. These scalars can be viewed as coordinates on a manifold. The extensive mathematical literature on the various manifolds can then be used to analyze the structure of the various matter-couplings. In particular, the generalization of (3.73) in the case of multiple scalar fields ϕ^i is given by

$$V(\phi^i) = \frac{(d-1)^2}{2} g^{ij} \frac{\partial W}{\partial \phi^i} \frac{\partial W}{\partial \phi^j} - \frac{d(d-1)}{4} W(\phi)^2 \,, \tag{4.35}$$

where g^{ij} is the metric on the manifold spanned by the scalars. The flow equations (3.80) are then given by

$$\begin{aligned} \phi^{i\prime}(r) &= \mp (d-1) g^{ij} \frac{\partial W}{\partial \phi^j} \,, \\ A'(r) &= \pm \frac{1}{2} W(\phi^i) \,. \end{aligned} \tag{4.36}$$

For a supersymmetric brane world scenario exhibiting localization of gravity to exist, the following conditions need to be fulfilled.

1. The scalar potential $V(\phi^i)$ needs to have two different stable[4] critical points, ϕ^i_1 and ϕ^i_2 corresponding to fixed points of the holographically dual beta-function

$$V'(\phi^i_1) = V'(\phi^i_2) = 0 \,, \quad \beta(\phi^i_1) = \beta(\phi^i_2) = 0 \,. \tag{4.37}$$

2. At these critical points, the value of the scalar potential needs to be equal and negative in order to have an Anti-de-Sitter background with the same cosmological constant on both sides of the brane

$$V(\phi^i_1) = V(\phi^i_2) < 0 \,. \tag{4.38}$$

3. In order to have a decreasing warp factor on both sides of the brane, the holographically dual beta-function needs to have two IR fixed points: i.e its derivative at the critical points needs to be positive

$$\beta'(\phi^i_1) = \beta'(\phi^i_2) > 0 \,. \tag{4.39}$$

4. Finally, one needs to find an explicit solution that interpolates between the two critical points of the scalar potential.

[4]Stable in the sense of satisfying the Breitenlohner-Freedman bound [93] defined in section 2.3.1.

4.3.2 Overview of $\mathcal{N} = 2$ supergravity in $D = 5$

We will now review the status of the search for supersymmetric brane world scenarios in the context sketched above by giving a short overview of the literature on $\mathcal{N} = 2$ supergravity in five dimensions.

The pure, ungauged, $\mathcal{N} = 2$ supergravity in five dimensions was developed in 1981 by Cremmer [155]. A few years later, Günaydin, Sierra and Townsend [156–158] used the $\mathcal{N} = 2$ vector multiplets to construct the gauged $\mathcal{N} = 2$ supergravity. In recent years, the couplings to other matter multiplets have also been constructed. In particular, Günaydin and Zagermann [159–161] constructed the couplings of $\mathcal{N} = 2$ tensor multiplets to supergravity, and Ceresole and Dall'Agata [162–164] did the same for $\mathcal{N} = 2$ hypermultiplets. The total Lagrangian of five-dimensional $\mathcal{N} = 2$ supergravity coupled to an arbitrary number of vector multiplets, tensor multiplets and hypermultiplets was given in [162].

It was shown in [165] that five-dimensional $\mathcal{N} = 2$ supergravity coupled only to vector multiplets could not fulfill the above criteria, since all critical points of the scalar potential were UV fixed points. Shortly thereafter, it was also shown that adding tensor multiplets did not improve the situation [162]. Adding hypermultiplets yields more possibilities: it was even shown that IR fixed points could exist [166]. For vacua having the same value of the cosmological constant, only UV-IR flows have been found so far.

As a final remark, we mention that if a supersymmetric RS scenario will be found, it is still not directly related to string theory. A possible embedding into a higher-dimensional theory might be given by the compactification of eleven-dimensional supergravity on a three-dimensional Calabi-Yau manifold[5] to five-dimensional supergravity coupled to matter. In particular, the number of vector multiplets and hypermultiplets in five-dimensional supergravity realizing a supersymmetric brane world should be related to the Hodge numbers of the Calabi-Yau manifold.

In four dimensions, conformal supergravity has turned out to be a very effective tool in analyzing matter-couplings to ordinary supergravity. The next two chapters will describe the conformal approach to five-dimensional supergravity matter-couplings. In chapter 5, we will introduce conformal supersymmetry and construct the so-called Weyl multiplet: the smallest irreducible supermultiplet containing the graviton. In chapter 6, we will couple vector, tensor and hypermultiplets in a superconformal manner to this Weyl multiplet.

[5] Such complex manifolds have six real dimensions.