### Geometry of Strings and Branes

Foar Heit en Mem

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#### Rijksuniversiteit Groningen

### Geometry of Strings and Branes

#### Proefschrift

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### Introduction

Elementary particle physics aims to describe the fundamental constituents of Nature and their interactions. Experiments indicate that elementary particles fall into two classes: leptons, containing among others the electron and the neutrino; and quarks, which form the building blocks of protons and neutrons. The four known forces between these building blocks of matter are the gravitational, the electromagnetic, the weak, and the strong interaction.

At small length scales, the gravitational interaction is many orders of magnitude weaker than all the other forces<sup>1</sup>, and it can therefore safely be neglected. The remaining three interactions of elementary particles can be described by an elegant theory called the Standard Model. This theory is a gauge theory: it has an internal local symmetry group in which each interaction is described by the exchange of gauge fields. These gauge fields are called the photon, the W-bosons and Z-boson, and the gluons for the electromagnetic, the weak, and the strong interaction, respectively. Gauge fields are different from matter particles in several aspects: the former fall into the class of bosons, particles with integer spin and commuting statistics; the latter are called fermions, particles with half-integer spin and anti-commuting statistics. It can be shown that internal symmetry groups, such as those of the Standard Model, cannot mix bosons with fermions [1].

Microscopic physics is described by quantum mechanics, which can be seen as a deformation of classical dynamics. It has several non-intuitive properties: one cannot simultaneously measure all observables with infinite accuracy, and many quantities can only be expressed in terms of probabilities. The Standard Model is quantum mechanically completely consistent, and the theory is in excellent agreement with experiments.

At macroscopic scales, the interactions of the Standard Model are virtually absent: the strong interaction is confined to small distances; the weak interaction has an exponential decay with distance; and although the electromagnetic force has an infinite range, all large configurations of matter are approximately electrically neutral. Hence, the gravitational interaction becomes the dominant force at large length scales.

Gravity is described by the theory of General Relativity. The basic ingredients of General Relativity are that space and time merge into a spacetime, that matter induces a curved geom-

<sup>&</sup>lt;sup>1</sup>The ratio of the gravitational and the electric force between a proton and an electron is  $10^{-40}$ .

etry on spacetime, and that this geometry in turn determines the dynamics of matter. One can also try to cast General Relativity in the form of a gauge theory: in this case a gauge theory of spacetime symmetries, known as general coordinate transformations, rather than internal symmetries. The corresponding gauge field in this case is called the graviton. General Relativity is a purely classical theory. It successfully explains physics in the range of terrestrial to cosmological length scales.

However, this split of physics into the macroscopic theory of General Relativity and the microscopic Standard Model is not without caveats, because General Relativity has some peculiar properties. First of all, it turns out that certain solutions to the classical field equations, known as black holes, have as a generic feature the occurrence of spacetime singularities [2] around which the gravitational field becomes infinitely large. This undermines the reason for ignoring gravitational interactions in elementary particle physics, and it becomes necessary to treat the gravitational field quantum mechanically.

Most of these spacetime singularities are predicted not to be directly observable. Instead, they are conjectured always to be hidden behind event horizons – surfaces from which not even light can return. Singularities are therefore thought not to be directly observable. However, the quantum mechanical behavior of elementary particles around such event horizons is problematic, since the one-way nature of event-horizons interferes with the probabilistic interpretation of quantum mechanics. This gives rise to information paradoxes [3].

Although the energy scales necessary to probe microscopic gravitational effects are not easily obtained in laboratory experiments, they did occur in the early universe. In order to develop good cosmological models, it is therefore necessary to have a description of gravity at small length scales. As a final remark, there is the related problem of the cosmological constant, a parameter in General Relativity for which the Standard Model predicts a value many orders of magnitude larger than the value inferred from astronomical observations [4].

To solve the problems sketched above, it is necessary to construct a theory of quantum gravity. To see what problems can arise in quantizing gravity, it is instructive to compare electromagnetism and gravity since at the classical and semi-classical level there are many parallels between the two interactions, as we have summarized in table 1. They both share a characteristic long range force, although gravity can never be repulsive. Both interactions also fit into a relativistic framework, and covariant field equations for both theories were found by Maxwell, and by Einstein, respectively. Both actions are invariant under local symmetries. For electromagnetism, these symmetries form the group of phase transformations, known as U(1); for General Relativity, they form the group of general coordinate transformations. There is one particular classical effect of the gravitational interaction that has not yet been observed directly: namely the radiation of gravitational waves<sup>2</sup>, the gravitational counterpart of optics.

The quantum mechanical motion of particles in the background of classical force fields is sometimes called first quantization. For the electromagnetic force, this was studied in the first few decades of the twentieth century during which in particular the nature of black

<sup>&</sup>lt;sup>2</sup>Indirect evidence for gravitational waves comes from the rotation time decay of binary star systems [5].

Process	Electromagnetism	Gravity
Force	$F_{\rm el} = \frac{q_1 q_2}{r^2}$	$F_{\rm gr} = -\frac{Gm_1m_2}{r^2}$
Relativistic	$\partial^{\nu} F_{\nu\mu} = J_{\mu}$	$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}$
Classical action	$\mathcal{L}_{\rm em} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$	$\mathcal{L}_{\mathrm{GR}} = \frac{1}{16\pi G} \sqrt{ g } R$
Symmetry	$\mathrm{U}(1)$	General coordinate transf.
Radiation	Optics	Gravitational waves
Spectrum	Black body	Hawking radiation
Phenomenology	H-atom spectral lines	Black hole entropy
Microscopic origin	Energy levels	Density of states

Table 1: (Semi-)classical electromagnetism versus gravity.

body radiation and the origin of the energy levels of the hydrogen atom were clarified. In the last few decades of the last century, the quantum mechanical behavior of particles in gravitational fields has been clarified: in particular, the process of Hawking radiation [6] and the microscopic origin [7] of entropy [8,9] for certain classes of black holes were discovered.

To continue the discussion of quantum gravity, it is more useful to compare the gravitational with the weak or the strong interaction, as we have summarized in table 2, since electromagnetism has no self-interactions at the quantum level, in contrast to the other three interactions. For the electromagnetic interaction, one can apply quantization methods to the classical action  $\mathcal{L}_{em}$  given in table 1, but this procedure fails for the action of General Relativity since it has an energy-dependent coupling constant G – this makes the theory nonrenormalizable.

Some progress towards solving this non-renormalizability problem was obtained by the discovery of supergravity in 1976 [10]. Supergravity is a modified version of General Relativity having spacetime symmetries as well as internal symmetries. A characteristic property of this so-called supersymmetry is that it mixes bosons with fermions [11]. In chapter 5, we will be more precise about the structure of supersymmetry and its cousin conformal supersymmetry. Although supergravity is better behaved at high energies than General Relativity, it is still non-renormalizable. The best one can hope for is that supergravity is a low-energy effective description of a theory of quantum gravity. This is rather similar to the situation concerning the weak interaction where Fermi's theory of beta-decay is also a non-renormalizable theory, but it can be seen to arise from the Standard Model.

In order to go beyond the low-energy effective description of a theory, a prescription for calculating scattering amplitudes at higher energies is necessary. For the strong interaction, this so-called S-matrix theory was developed during the nineteen sixties, and it uses

Process	Weak or strong interaction	Gravity
Low-energy theory	Fermi's theory of beta-decay	Supergravity
Scattering amplitudes	S-matrix theory:	Perturbative string theory:
	Feynman graphs	Riemann surfaces
Classical action	Standard Model:	String field theory:
	$\mathcal{L}_{\rm SM} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a + \dots$	$\mathcal{L}_{SFT} = \frac{1}{2}\Psi \star Q\Psi + \dots$
Symmetry	$\mathrm{SU}(2)$ or $\mathrm{SU}(3)$	Unknown
Solitonic solutions	Monopoles	Branes
Duality	Electric/magnetic charges	Strong/weak coupling
Quantization method	BRS-method	<b>BV</b> -formalism

Table 2: Quantizing the weak or strong interaction versus gravity.

a perturbative expansion over Feynman graphs in order to calculate amplitudes. The precise prescription is fixed by a Lagrangian formulation. In the case of the strong interaction, as well as the electroweak interactions, all the Feynman rules can be derived from the Lagrangian of the Standard Model.

A corresponding formalism yielding scattering amplitudes for gravity involves the concept of strings: i.e. at small length scales, particles are postulated to be tiny vibrating strings. The motivation is that the spectrum of a closed string contains the graviton, the gauge field for gravity. Since strings sweep out worldsheets rather than worldlines, as particles do, the idea of Feynman graphs has to be extended to surfaces. It was shown in the nineteen eighties that a perturbative expansion over Riemann surfaces gives quantum mechanically consistent scattering amplitudes.

The string theory analog of the Standard Model was developed in the nineteen eighties, this goes under the name of string field theory. In this theory, one single string field describes all string vibrations simultaneously. For the simplest models of perturbative string theory, it can be shown that the corresponding string field theory yields the same answers for scattering amplitudes, but for more complicated perturbative string theories, there are technical complications in constructing the corresponding string field theories.

The fields in the Lagrangian of the Standard Model can be rotated by two- or threedimensional unitary matrices, in which case the gauge group is called SU(2) or SU(3), respectively. Since matrices do not commute, such theories are called non-Abelian gauge theories. The quantization of the classical action of an interaction is often called second quantization, and for the weak and the strong interactions this can be consistently done using the methods of BRS-quantization [12, 13]. The symmetry groups of string field theories are much larger and much more complicated than the gauge groups of the Standard Model, and in many cases not known explicitly. This means that traditional methods of quantization fail, and one needs to use more sophisticated methods such as the BV anti-field formalism [14]. Just as the quantization of the weak interaction required more sophisticated tools than the quantization of electromagnetism, it seems also likely that the quantization of gravity will require new methods in this area.

Gauge theories often have solitons – solutions of the classical field equations with finite energy. In modified theories of the weak interaction there are for example magnetic monopoles. The presence of such magnetic monopoles can imply that there is a duality between electric and magnetic charges. Such dualities are powerful symmetries, since they often relate separate regimes of a given theory. String theory has higher-dimensional solitonic solutions called branes<sup>3</sup>. In string theory, there is also a number of dualities, such as dualities between strongly and weakly coupled regimes of different versions of string theory. In all of these dualities, branes play an essential role. The overall framework of string theory and branes is called M-theory, where the M can mean anything ranging from Mystery to Membrane, according to taste. It is not clear yet whether strings are the fundamental degrees of freedom of quantum gravity, or if there is perhaps a formulation in terms of branes.

The organization of this thesis is as follows. We will start in chapter 1 with a more elaborate treatment of the string theory framework, including the basic features of string theory and supergravity, as well as the various dualities and brane solutions of these theories. In chapter 2, we will describe the AdS/CFT correspondence – a recently discovered duality between theories of gravity in Anti-de-Sitter spacetimes and conformal quantum field theories. This is a remarkable duality, because several quantities within quantum gravity can be expressed in terms of concepts known from quantum field theory. A central theme in the AdS/CFT correspondence is a special brane solution of string theory: the D3-brane.

In chapter 3, we will present our results [15] that show how this duality can be extended to a duality between gravity in more general curved spacetimes called domain-walls and more general quantum field theories – the DW/QFT correspondence. In particular, we will discuss a large class of brane solutions that includes the D3-brane. After choosing a suitable coordinate frame, the so-called dual frame, we will study the near-horizon geometry of these brane solutions of supergravity, and we will analyze what kinematical information can be extracted from the dual field theories.

The domain-walls that appear in the analysis mentioned above describe spaces that are separated into several domains by a boundary surface – the domain-wall. Across such domain-walls, physical quantities can change their values in a discontinuous fashion. Domain-walls that have such discontinuities are sometimes called "thin" domain-walls. On the other hand, domain-walls that can be interpreted as smooth interpolations between different supergravity vacua go under the name of "thick" domain-walls. At the end of chapter 3, we will explain how these thick domain-walls have the interpretation of renormalization group flows in their

<sup>&</sup>lt;sup>3</sup>Compare 0,1,2, ... many with particle, string, membrane, ... brane.

dual quantum field theories.

Domain-wall spacetimes have attracted renewed attention recently: they are a member of the class of brane world scenarios. In chapter 4, we will describe such brane world scenarios in more detail: the basic idea is that our four-dimensional universe is actually a hypersurface within a five-dimensional supergravity theory. The size of the extra fifth dimension transverse to the so-called brane world can be used to gain insight in the origin of some unnatural properties of four-dimensional physics. For instance, the so-called Randall-Sundrum scenarios were used to obtain a better understanding of the cosmological constant problem, as well as the unnatural ratio of the strength of the gravitational force and the remaining three interactions, the so-called hierarchy problem.

Supersymmetric versions of such theories have proven to be hard to find. The main obstacle is realizing supersymmetry on the four-dimensional brane world solution: it is related to finding the vacuum structure of the corresponding five-dimensional supergravity theory. This, in turn, requires a detailed knowledge of all possible couplings of five-dimensional matter models to supergravity. The scalar fields of these matter models can be interpreted as coordinates on an abstract space. Many properties of the matter-coupled supergravity theory can then be expressed in terms of the geometrical properties of the corresponding space of scalar fields.

In particular, the scalar fields generate a potential that determines the vacuum structure of the supergravity theory. For supersymmetric brane worlds to exist, this scalar potential needs to possess two different, stable minima that need to satisfy some additional constraints. Moreover, one needs to find a suitable solution that smoothly interpolates between two such minima. Such an analysis, which had been started in the nineteen eighties (albeit for different reasons), has recently been renewed, but still does not encompass the most general fivedimensional matter-coupled supergravity theory.

We will take a systematic approach to construct these five-dimensional matter-couplings. This so-called superconformal program starts from the most general spacetime symmetry group, the group of superconformal transformations, which considerably simplifies the analysis of matter-couplings to supergravity. The different models possessing superconformal symmetry are called multiplets. First of all, there is the so-called Weyl multiplet: this is the smallest multiplet of the superconformal group that possess the graviton. On the other hand, there are the matter multiplets: they interact with the Weyl multiplet that forms a fixed background of conformal supergravity. Matter-couplings to non-conformal supergravity can then be obtained by breaking the conformal symmetries.

In chapter 5, we will present our results [16] on the five-dimensional Weyl multiplets. We will see that there are two versions of this multiplet: the Standard Weyl multiplet and the Dilaton Weyl multiplet. Multiplets similar to the Standard Weyl multiplet also exist in four and six dimensions, but the Dilaton Weyl multiplet had so far only been found in six dimensions. We will use a well-known method to deduce the transformation rules for the different fields: the so-called Noether method. In particular, we will construct the multiplet of conserved Noether currents for the various conformal symmetries. A remarkable detail

is that the current multiplet that couples to the Standard Weyl multiplet contains currents that satisfy differential equations, a mechanism that so far had only been known from tendimensional conformal supergravity.

Our results [17] on five-dimensional superconformal matter multiplets will be presented in chapter 6. We will discuss so-called vector multiplets: these are multiplets that contain the gauge field of the gauge group under which the multiplet transforms. We will analyze vector multiplets that transform under arbitrary transformations of the gauge group: the socalled vector-tensor multiplets. In particular, we will consider representations of the gauge group that are reducible but not completely reducible. This gives rise to previously unknown interactions between vector fields and tensor fields. The conformal symmetries can only be realized on the tensor fields if these satisfy their equations of motion. By dropping the usual restriction that the equations of motion have to follow from an action principle, we can also formulate vector-tensor multiplets with an odd number of tensor fields.

Apart from vector-tensor multiplets, we will also consider hypermultiplets in chapter 6. These multiplets also possess scalar fields but not gauge fields. The scalar fields span a vector space over the quaternions. Realizing the conformal algebra on the scalar fields will induce a non-trivial geometry called hyper-complex geometry on the space of scalars. Similarly as for tensor fields, the superconformal algebra can only be realized on the fields of the hypermultiplet with the use of equations of motion. Also in this case, we will consider equations of motion that do not follow from an action principle. The special cases for which there is an action correspond to hyper-complex manifolds possessing a metric: the so-called hyper-Kähler manifolds. Furthermore, we will analyze the interaction of hypermultiplets with vector multiplets, and we will also make use of the scalar field geometry in this case. At the end of chapter 6, we will give an overview of all the geometrical concepts that we will make use of.

The matter-couplings to conformal supergravity that we will construct in this way can be used as a starting point to construct matter-couplings to non-conformal supergravity. At the end of chapter 6, we will sketch some ingredients of this procedure. Whether the fivedimensional matter-couplings of supergravity that can be obtained in this way will actually modify the vacuum structure in such a way that supersymmetric brane world scenarios can be realized, remains an open question that will have to be answered by future research.