Domain-Walls and Gauged Supergravities

T.C. de Wit

For my family
In loving memory of my father

The work described in this thesis was performed at the Center for Theoretical Physics in Groningen, with support from the “Stichting voor Fundamenteel Onderzoek der Materie" (FOM).

Printed by Universal Press - Science Publishers / Veenendaal, The Netherlands.
Typeset in Times using LaTeX 2_ε.

Rijks*universiteit* Groningen

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Proefschrift

ter verkrijging van het doctoraat in de
Wiskunde en Natuurwetenschappen
aan de Rijks*universiteit* Groningen
op gezag van de
Rector Magnificus, dr. F. Zwarts,
in het openbaar te verdedigen op
maandag 13 oktober 2003
om 14.15 uur

door

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geboren op 13 februari 1977
te Waspik

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ISBN-nummer: 90-367-1867-8

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Introduction

Since the birth of particle physics, with the discovery of the electron by Thomson in 1897, much progress has been made in explaining the observable phenomena in nature. In order to explain the properties of particles at the (sub)atomic scale quantum mechanics was developed around the nineteen-twenties. Based on experiments it was realized that all particles in nature have a fundamental property called "spin", the value of which divides them into two classes: bosons and fermions, each with distinct properties. Somewhat earlier, in 1905, Einstein proposed his theory of special relativity, which radically changed our notions of space and time; it showed how both concepts are intricately connected. A combination of special relativity and quantum mechanics finally led to the Standard Model around 1970, which quite successfully describes the interactions between the elementary particles that form the building blocks of all observable matter in the universe. There are three fundamental forces incorporated in the Standard Model: the electromagnetic, the weak, and the strong force. Here the concept of gauge symmetry plays an important role. By making this symmetry local, i.e. introducing coordinate dependent transformation parameters, spin 1 gauge bosons are introduced that mediate the force between two particles. The best known example is the photon that causes an electromagnetic field between two charged particles, causing them to attract or repulse. Similarly, the additional fundamental forces are carried by W/Z bosons and gluons respectively. The Standard Model has been verified to great precision, nevertheless there are some discrepancies. First of all there is the Higgs boson which is responsible for giving masses to the other fundamental particles, but still has not been found.[1] Secondly, the Standard Model contains nineteen fine-tuned parameters – e.g. corresponding to masses of elementary particles – that cannot be theoretically predicted, and is not a fundamental theory.

Another major achievement of 20th century theoretical physics was Einsteins theory of general relativity, dealing with the fourth fundamental force: gravity. The theory was constructed in 1914 in an attempt to implement special relativity into Newtonian gravity and further improved our knowledge about space and time. Some of its successes were the predictions of small deviations of planetary orbits and the deflection of light from heavy objects. More speculative predictions are black holes and gravitational radiation, which both only have been verified indirectly. Furthermore, predictions could be made regarding the evolution of our universe. Although this theory was capable of explaining the interactions between massive objects at relatively large length scales, something goes wrong when trying to describe gravity at small scales where quantum effects become important. Considering that the gravitational force is extremely

[1] There is hope that the new LHC accelerator, due 2006, will provide conclusive experimental proof of its existence.

force	mediating particle	acts on	range	relative strength
strong nuclear force	gluon	quarks	nuclear distances	20
electromagnetism	photon	charged particles	infinite	1
weak nuclear force	W- or Z-boson	quarks and leptons	nuclear distances	10^{-7}
gravity	graviton?	massive particles	infinite	10^{-36}

Table 1: The four fundamental forces. The relative strengths are based on two interacting up-quarks separated by a distance of 10^{-18} m [1].

weak compared to the other three fundamental forces at small scales, see table 1, it is not strange that general relativity theory has only been tested up to approximately 1 mm. An attempt to describe gravity by using similar quantization techniques as used for the Standard Model failed. The theory suffered from infinities since the gravitational coupling constant $\kappa = 8\pi G/c^4$ is not dimensionless and is therefore unsuitable for performing perturbation expansions, which are common in particle physics. The typical length scale where our classical ideas of gravity and space-time lose their validity is given by the Planck length:

$$\ell_P = \sqrt{\frac{hG}{c^3}} \approx 4.1 \cdot 10^{-35} \text{ m}, \tag{1}$$

with h Planck's constant, G Newton's gravitational constant, and c the velocity of light.

Summarizing, at both ends of the scale spectrum two quite successful theories were obtained, that did not seem to be compatible. These arguments show the need for a theory of "quantum gravity", that can handle all four fundamental forces simultaneously. The quest for this unified theory has been the main target for the research done in high energy physics during the last twenty years.

A partial success was reached in 1976 by the discovery of supergravity; an extension of general relativity theory that behaved better at high energies, i.e. the infinities were partially cancelled. The crucial ingredient here was "supersymmetry", a symmetry between bosons and fermions, that predicts that for every boson in nature there exists a corresponding fermionic particle, and vice versa. The gauge theory of supersymmetry is given by supergravity. The spin 2 gauge boson responsible for mediating the gravitational force is called the graviton. Its supersymmetric partner is the so-called gravitino. In order to measure these particles energies would be needed that are way out of the range of our present (and future) accelerators.

The most promising candidate so far for a theory of quantum gravity is superstring theory. String theory assumes that all particles can be represented by different oscillational modes of a string, with a typical length ℓ_S of the order of the Planck length ℓ_P. One of the modes turns out to be a spin 2 particle, behaving like a graviton. Subsequentially it was found that the low energy limit of superstring theory is given by supergravity. There is an intuitive reason why superstring theory is free from infinities. These infinities usually occur at singular points, however a string moving in space-time sweeps out a two dimensional surface, as opposed to a line in the case of a point particle. Exactly this fact causes the interactions not to take place at one single point, but to spread out over a small area. Intuitively that is the reason for string theory to be free from infinities, which usually occur at singular points.

Unfortunately, this theory also has its disadvantages. String theory is only defined perturbatively, i.e. scattering amplitudes are expressed as an infinite expansion in powers of the string coupling constant g_S, associated with the "Feynman-diagrams" of string theory. The main setback however was apparent when there seemed to exist five different superstring theories, whereas we hoped to obtain one unique theory of quantum theory.

This opinion was drastically changed after the discovery of dualities, that enabled us to relate different energy regimes of different theories. An important role was played by the so-called "brane" solutions of string theory. They are solitonic membrane-like objects that can be seen as higher-dimensional generalizations of strings. The five apparently distinct theories and their brane-solutions seemed to be related by a web of dualities, suggesting that they all represented various limits of one single fundamental theory, called "M-theory". Unfortunately there is not much known about this theory. However, by studying the low energy limits of M-theory and the various dualities between them, hopefully we will get closer to a unified theory.

We will now give a brief description of the topics discussed in this thesis. In chapter 1 we will briefly describe the framework of string theory and supergravity, needed to understand the context of the rest of the thesis. Chapter 2 will provide the motivations for the research described in the remainder of the thesis. The main motivation is the concept of "brane-world scenarios", which assumes that our four-dimensional universe can be represented as a four-dimensional brane-solution in five dimensions. With these types of models several problems in cosmology were tried to be solved, e.g. the cosmological constant problem and the hierarchy problem. The branes used in these models separate space-time into two regions and are called "domain-walls". A supersymmetrized version is not easy to construct; the domain-walls have to satisfy several conditions in order to describe the correct vacuum structure of the five-dimensional space-time. The determination of all possible domain-wall candidates requires a knowledge of matter couplings of five-dimensional supergravity. The scalar fields occurring in such theories can be interpreted as coordinates of a manifold. The potential energy of the scalars is given by the scalar potential, which is a function of all the scalars of the scalar manifold. The vacuum-structure of the five-dimensional space-time is determined by the minima of the scalar potential and the geometry of the scalar manifold.

The five-dimensional matter-coupled supergravity theory is a special case of a "gauged supergravity", i.e. a supergravity theory where one or more global symmetries has been made local. One way of constructing these gauged supergravities is by means of dimensional reduction. One starts with a higher-dimensional supergravity theory and "curls up" some extra dimensions to end up effectively with a supergravity in a lower space-time dimension. An extension of this method is called generalized dimensional reduction; here one uses a symmetry of a theory to obtain masses in lower dimensions. In this case, the symmetry used will appear as a gauged symmetry of the reduced theory. When applied to supergravity one can construct gauged supergravities. A general introduction to this topic is given in chapter 3, after which it is applied to eleven- and ten-dimensional supergravity in chapter 4.

The remaining three chapters 5, 6 and 7 provide another method to obtain gauged supergravities: the three-step superconformal program. We used the program in order to obtain a more general matter coupled five-dimensional $\mathcal{N} = 2$ Poincaré supergravity than currently known in the literature. The space-time symmetries in Poincaré supergravity are given by translations and rotations, which are part of the super-Poincaré group. The conformal program extends this group to the largest group of space-time symmetries, namely the superconformal group. By introducing extra symmetries, the corresponding conformal supergravity will contain more structure and will be easier to analyze.

The first step of the program is given in chapter 5 which describes the construction and gauging of the superconformal algebra in five dimensions, resulting in the so-called "Standard Weyl multiplet" which is the minimal representation of the superconformal algebra containing the graviton. The fields in this multiplet are the superconformal background fields.

The second step will be the subject of chapter 6 where we construct various matter multiplet representations of the superconformal algebra, and determine their actions and supersymmetry transformation rules in a background of the Weyl multiplet fields. We will only consider vector-tensor multiplets and hypermultiplets. Both contain scalars that give rise to interesting geometries on the corresponding scalar manifolds.

The last step is given in chapter 7 where the superconformal algebra is "broken down" to the super-Poincaré algebra by making convenient gauge choices for the non-Poincaré symmetries. This "gauge-fixing" process will produce five-dimensional matter coupled Poincaré supergravity, that can be used for many applications. Finally, in appendices A–C, we give our conventions and some in-depth information about the geometrical properties of quaternionic-like manifolds that are generated by the hypermultiplet couplings.