CHAPTER ONE
ZENO: PARTS, WHOLES AND LIMITS

§ 0. Introduction

Those among Zeno's paradoxes in which he aims to demonstrate the impossibility of motion and the unity of 'what is' belong to the most famous arguments in the history of philosophy. Almost every discussion of the problem of continuity and infinite divisibility starts with a sketch of the paradox of the Runner, a paradox of division, or the paradox of the Arrow. Zeno was the first to broach the subject. Before he presented these arguments, people may have assumed that magnitudes are infinitely divisible, or indeed they may not have thought about the matter at all – we shall never know.[1] It is therefore fitting that this study should start with a careful analysis of the reasoning behind those of Zeno's arguments which are somehow related to matters of infinite divisibility. In accordance with the aims formulated in my Introduction, I hope to come up with a set of principles which would explain both the problems raised in these arguments and the details of argument and vocabulary of each of them separately.

The chapter is organized as follows. After a short discussion of Zeno's purposes, I shall start with the analysis of his arguments, begining with the argument of the Arrow, partly because it brings several themes together, and partly because it is clear and interpretatively well established – sufficiently well established, at least, to provide solid ground for further analyses. From then on, the discussion will be ordered thematically, focusing on ever more complicated principles and ideas as they are expressed and developed in Zeno's arguments. Thus almost everything we know about Zeno will be taken

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[1] Anaxagoras probably assumed the possibility of an infinite division (see Simplicius, *In Physica* 164.17-18 = H. Diels and W. Kranz, *Die Fragmente der Vorsokratiker* (Zürich and Berlin, 1964) [DK] fr. 59 B3), but I am inclined to follow those scholars who argue that Anaxagoras (500 – 428 BC) responded to Zeno's arguments, despite the fact that Zeno's dates are somewhat later (ca. 490 – ca. 440?). This would be easily explained on the assumption that Zeno wrote his work as a young man, say around 470-465 (Plato, *Parmenides* 128d6-8 = DK 29 A11). See e.g. G.S. Kirk, J.E. Raven and M. Schofield, *The Presocratic Philosophers* (2nd ed.; Cambridge, 1983) [KRS] 360-362; M. Schofield, *An Essay on Anaxagoras* (Cambridge, 1980) 81-82, and R.D. McKirahan Jr., *Philosophy Before Socrates. An Introduction with Texts and Commentary* (Indianapolis, 1994) 201-218. For perhaps a bit more than healthy scepticism, however, see J. Barnes, *The Presocratic Philosophers* (2nd ed.; London and New York, 1982) 307-308.
Perhaps needless to say, I reject any account according to which Zeno formulated his paradoxes in response to mathematical developments associated with the Pythagoreans. Cf. e.g. G. Vlastos, 'Raven's *Pythagoreans and Eleatics*', in: *Studies in Greek Philosophy* I *The Presocratics* (Princeton, 1995) 180-188 [also in: R.E. Allen and D.J. Furley (eds.), *Studies in Presocratic Philosophy* II *The Eleatics and Pluralists* (London, 1975) 166-176] and G.E.L. Owen, 'Zeno and the Mathematicians', in: Allen and Furley, *Studies* 143-165, at 153-155.

into account.[2] The only major exception is the fourth paradox of motion, also known as the Stadium or the Moving Rows.[3] The reason for ignoring it is that I am not convinced it involves issues of continuity rather than merely the problem of relative motion.[4] If I am right, it loses its relevance for present purposes.

§ 1. Purposes

There are two issues which have to be clarified before we can go on to the arguments themselves. They concern Zeno's purpose in stating them. Firstly, did he think about them as a tightly knit set of arguments, possibly related as horns of a dilemma, or are they more loosely related? Secondly, what exactly was his target in what must have constituted the largest group of arguments, the paradoxes of plurality?

Though I hope to show that Zeno was rather consistent in his use of a small set of basic principles and ideas, one should not assume that he was consciously thinking of his paradoxes as developing in a variety of ways a limited set of assumptions. What they have in common is merely their goal to present problems for those who hold on to the common-sense idea that there is a plurality of objects moving around. In the two paradoxes of plurality we know (among them the paradox of division sketched in the Introduction),[5] the starting hypothesis that there is a plurality of objects – expressed as: 'If there are many' – is reduced to absurdity, either through the unacceptibility of the conclusion of an individual line of argument, or through an inconsistency between the con-

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2 Perhaps this is the place to express my scepticism with regard to the authenticity of the spatial version of the Arrow, at least in so far as it is ascribed to Zeno, e.g. by Diogenes Laertius 9.72 (= DK 29 B4). On this point I follow the general opinion among scholars; see e.g. Barnes, *Presocratic Philosophers* 276, and especially D.N. Sedley, 'Diodorus Cronus and Hellenistic Philosophy', *Proceedings of the Cambridge Philological Society* N.S. 23 (1977) 74-120, at 84 and note 55.

3 'The fourth [argument against motion] is the one about the equal masses moving in a stadium from the opposite side past equal [masses], the former from the end of the stadium, the latter from the middle, with equal speed. In this argument it follows, he thinks, that half the time is equal to its double. There is a fallacy in that he assumes that an equal magnitude moves with an equal speed over an equal period of time, the one past a magnitude in motion, the other past a magnitude at rest. But that is false. For example, let the stationary equal masses be indicated by AA, the ones indicated by BB those starting from the middle, being equal to them in number and magnitude, and the ones indicated by CC those starting from the end, being equal to them in number and magnitude, as well as being equally fast as the Bs. It follows then that the first B is at the same time at the last [C] as the first C [is at the last B], as they are moving past each other. And it follows that the C has traversed past all, while the B has traversed past half the number. Therefore the time is half. For each of them both is past each over an equal time. It also follows that the first B has gone past all the Cs, since the first C and the first B are at the opposite last masses, because both get past the As over an equal time. This is then the argument, and it follows on the basis of the fallacy pointed out.' (*Physica* 6.9; 239b33-240a18 = DK 29 A28)

4 Accounts of the Moving Rows on which it does involve issues of continuity, are offered by, among others, Owen, 'Zeno' 150-151, Lee, *Zeno* 83-102 and Caveing, *Zénon* 105-117. Also the interpretation of J. Immerwahr, 'An Interpretation of Zeno's Stadium Paradox', *Phronesis* 23 (1978) 22-26, relates it to issues of divisibility, though in a somewhat different way. Sceptical are Barnes, *Presocratic Philosophers* 285-294, KRS 274-276, and G. Vlastos, 'Zeno of Elea', in: *Studies* 241-263, at 254-255 [originally in: P. Edwards (ed.), *The Encyclopedia of Philosophy* VIII (New York, 1967) 369-379, at 374]; cf. Makin, 'Zeno' 851.

5 For some discussion about the number of arguments (allegedly forty – see DK 29 A15), see the evaluations of Proclus' reports (in his commentary on Plato's *Parmenides*) in: J. Dillon, 'Proclus and the Forty Logoi of Zeno', *Illinois Classical Studies* 11 (1986) 25-41 and especially H. Tarrant, 'More on Zeno's *Forty Logoi*', *Illinois Classical Studies* 15 (1990) 23-37. From Plato (*Parmenides* 127e1-3) we know that Zeno's book started with an argument for the contradictory conclusion that the many are both like and unlike.

clusions of two parallel lines of argument, both apparently meant as equally acceptable.[6] In the four paradoxes of motion the absurdity of the conclusions casts doubt on the possibility of motion.[7] Of the remaining two arguments known to us, the paradox of the millet-seed, that the tiniest part of a millet-seed should cause some noise when falling, because the whole of millet-seeds does so, could very well belong to the paradoxes of plurality or be related to one of them; and the paradox of place attacks a concept which is intimately connected with at least the possibility of motion, since motion implies first being at one place and then at another.[8] The impression given by the arguments we know is confirmed by the report provided by Plato in his *Parmenides*, where he gives Zeno the following account of their purpose:

> This work is really some kind of support for Parmenides' argument, against those who try to make fun of it [by saying] that if [everything] is one, many ridiculous things happen as a consequence of the argument, and which are inconsistent with it. This book then contradicts those who posit many things and pays them back in return with the same consequences, and more, as it wants to make clear that their hypothesis: 'if there are many' would suffer even more ridiculous things than the one that [everything] is one, if someone were to examine it sufficiently.[9]

Earlier on he has agreed to the following description of his arguments:

> Is this then what your arguments are aimed at – at nothing else than contending, against everything being said, that there are not many? And only that you take each of your arguments to be a proof of, so that you also deem to provide as many proofs that there are not many as you have written arguments.[10]

So as each argument separately is sufficient proof that 'there are not many', there is unlikely to be some kind of grand argumentative design, at least not among the arguments based on the hypothesis that there are many.

As far as the second question is concerned, it was Philoponus' view that the pluralism attacked by Zeno is the common-sense pluralism that there are many ordinary objects:

> [T]hose who introduce a multitude, used to do so confidently on the basis of its obviousness (ἐκ τῆς ἐναργείας): there is a horse and a human being and each of the particulars, the assembly of which makes a multitude. Zeno wanted to demolish this obviousness sophistically.[11]

As a matter of fact, however, Zeno's arguments do not seem to need more than what one might call 'bare objects', objects which are nothing more than an amount of matter with a

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6 According to Plato (*Parmenides* 127e1-2) Zeno did not think of his arguments as presenting a dilemma, from which of course an escape is always possible by opting for one of the horns, but as actually proving, on the hypothesis that there are many, that opposites are true of one thing; cf. Simplicius, *In Physica* 139.5-7 = DK 29 B2.

7 There may have been more, as one gathers from Aristotle's restrictive formulation: 'There are four arguments about motion by Zeno presenting difficulties to those who try to solve them.' (*Physica* 6.9; 239b9-11)

8 Though it is quite possible that Zeno's paradox of place belongs to the paradoxes of plurality; see K. Algra, *Concepts of Space in Greek Thought* (Leiden, 1995) 50, especially note 61.

9 128c6-d6

10 127e7-128a1

11 *In Physica* 42.18-21

certain shape.[12] This is especially clear in those cases where Zeno's reasoning resembles that employed in the paradoxes of motion, for in the latter objects do not play a part, let alone ordinary objects; there are merely the spatial and temporal equivalents of bare objects: distances or periods.[13] What is more, one argument, the second half of the second paradox of plurality, where it is argued that between any two objects there is a third (see § 4.2) requires that these objects be bare objects. For why should there be a third object between, say, a book and a table? But also in this respect Zeno is in the company of Parmenides, who argues for the unity of what is, as follows:

> Nor is it divided, since it all exists alike
> nor is it more here – which would prevent it from holding together –
> nor less there, but is all full of being.
> So it is all continuous: for what is, draws near to what is.[14]

This argument does not work for books and tables: if they are drawn near to each other, they still do not form one continuous whole.[15] On the other hand, if these books and tables are considered as mere lumps of matter with a certain shape, there is a strong intuition that they do form a continuous lump of matter.

This does not mean, though, that Philoponus' report is incorrect, for it is quite plausible that Zeno, oblivious of the distinction between ordinary objects and bare objects, *thought* he was arguing against this so-called 'simple pluralism', the view that the world contains more than one thing, and these are ordinary things.[16] The fact, however, that he at least implicitly adopted this reductive account of objects is crucial in explaining why he could come up with arguments involving problems of continuity and infinite divisibility. For it is in the case of bare objects that these problems come to the fore, as they, being bare, lack the internal structure which marks them off from other entities and blocks their random divisibility, while on the other hand, being objects, they should be separate and unified entities. The two most important principles I will ascribe to Zeno in this chapter express these two opposing demands.

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12 One may even point to a word indicative of their bareness: ὄγκοι, as used by Aristotle in his report of the fourth paradox of motion and translated there by 'masses' (see note 3), at least if one wants to follow Lee, *Zeno* 84, and M. Caveing, *Zénon d'Élée. Prolégomènes aux doctrines du continu. Étude historique et critique des Fragments et Témoignages* (Paris, 1982) 107-109, and accepts that the word goes back to Zeno.

13 The only argument ascribed to Zeno which does require ordinary objects, is mentioned by Philoponus, immediately after the passage quoted above, in *In Physica* 42.24-28. It concerns the argument that each of the units of the hypothesized multitude is not one, because it has many properties. However, as H.D.P. Lee, *Zeno of Elea. A Text, with Translation and Notes* (Cambridge, 1936) 27-29, already remarks, this argument is certainly not Zeno's.

14 DK 28 B8.22-25; translation, with one small adaptation, taken from KRS 250-251. That this is meant to be an argument for the unity of what is, appears from DK 28 B8.5-6.

15 With M. Schofield, 'Did Parmenides Discover Eternity?', *Archiv für Geschichte der Philosophie* 52 (1970) 113-135, but against a host of other scholars, I assume that Parmenides is here talking about spatial continuity. Cf. S. Makin, *Indifference Arguments* (Oxford, 1993) 28-32, whose account, despite protestations of neutrality, seems much more natural if it implies that Parmenides is concerned with spatial continuity.

16 Only understood in this way I can agree with S. Makin, 'Zeno of Elea', in: E. Craig (ed.), *The Routledge Encyclopedia of Philosophy* IX (London, 1998) 843-853, at 844, who, taking his lead from Philoponus, argues that 'simple pluralism' was Zeno's target. But Zeno can only be successful in so far as he reduces the ordinary objects of simple pluralism to bare objects. It is therefore misleading to argue, as Makin does, that the atomists were forced by Zeno's arguments to go beyond the simple pluralism of common sense. As I will show, they remain close to the Zenonian scheme and merely explicate and develop his conception of bare objects.

§ 2. Starting off: the Arrow

The argument I shall start with is perhaps the most elegant: the Arrow. We have only one source for it, Aristotle's brief and somewhat cryptic account in *Physica* 6.9; 239b5-7. As it stands, the text reads:

> And Zeno argues fallaciously. For if everything is always at rest or in motion when it is over against what is equal, and the thing flying is always in the now, the flying arrow is, he claims, motionless.
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> Ζήνων δὲ παραλογίζεται· εἰ γὰρ αἰεί, φησίν, ἠρεμεῖ πᾶν ἢ κινεῖται ὅταν ᾖ κατὰ τὸ ἴσον, ἔστιν δ' αἰεί τὸ φερόμενον ἐν τῷ νῦν, ἀκίνητον τὴν φερομένην εἶναι ὀϊστόν.

Thus we seem to have two premisses and the conclusion:

(1) Whenever something is over against what is equal, it is at rest or in motion.
(2) The flying arrow is always in the now.
∴ (c) The flying arrow is motionless.

Though this can hardly be called an argument, the general idea seems to be clear enough.[17] We should understand 'over against what is equal' as 'occupying a place equal to its own size' and then argue that being over against what is equal somehow brings with it being at rest. Moreover, we should do something with (2) in order to get closer to a structured argument. This can be done by supplying:

(3) In the now everything is over against what is equal.[18]

For not only does (3) provide a conceptual link between (2) and (1), it also gets one to:

(4) The flying arrow is at rest or in motion in the now.

And (4) contains as a part the proposition that is needed in order to reach (c), namely:

(5) The flying arrow is at rest in the now.

For with (5) one would have established at least a kind of immobility for the arrow, and thus have something to go on if one wants to reach the conclusion (c) that the arrow is motionless.

Now two questions present themselves. First, how are we to make the step from (4) to (5)? And second, how are we to reach (c) on the basis of (5)? In § 2.1 I shall confine

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17 As may also appear from the consensus among previous interpretations, despite all their, sometimes significant, differences.

18 Some scholars, e.g. Lee, *Zeno* 52, emend (2) in such a way as to get (3): by adding κατὰ τὸ ἴσον after ἐν τῷ νῦν (which phrase actually occurs in one late manuscript and is perhaps read by Themistius). W.D. Ross, *Aristotle's* Physics*. A Revised Text with Introduction and Commentary* (Oxford, 1936) 658 is prepared to emend thus, but thinks it is not necessary, as κατὰ τὸ ἴσον can be understood in (2); cf. Caveing, *Zénon* 97-98. The obvious gain of such transformations of (2) into (3) is that one does not have to supply such an important premiss any more, but one also loses something. For as it stands, (2) seems to give some kind of ontological priority to the now, which disappears by turning it into (3). On this, see below.

myself to discussing the second question, because it is this question which is most relevant to the present enterprise, but also because I am not absolutely sure whether I am able to defend the text as it stands – an attempt will be found in § 2.2. The difficulty with the transmitted text is the following. Of course it is possible to get from (4) to (5): on the basis of (3) one could argue that there is no movement in the now and that therefore one half of the disjunction in (4) may be struck, so that it follows that the flying arrow is at rest in the now. But why do this in such a circuitous way, going from (1) and (3) to (4) and then, using (3) again, to (5)? If Zeno wanted this paradox to be effective, his premisses had better have some appeal, but what could that possibly be in the case of (1)? It is quite plausible to say that something is at rest when it is over against what is equal, but to say that it can also be in motion seems somewhat strange.

To get rid of this problem, numerous emendations have been proposed. Here I accept the one which, also under the influence of Ross' edition, is most commonly adopted[19]: deleting ἢ κινεῖται,[20] so that instead of (1) one has:

(1_{em})Everything is at rest whenever it is over against what is equal.

To compensate, however, for the ease with which this emendation is accepted, I shall discuss the issue in § 2.2, where I shall try to make as strong as possible a case for the manuscripts' text, which, moreover, will not depend on any philosophical analysis of Zeno's argument.

§ 2.1. Argument

Forward then with the Arrow: how to reach (c)? It may seem very easy to do so: by combining (2), that the arrow is always in the now, with (5), that it is then at rest, we get the conclusion that the arrow is always at rest. As far as it goes, there is nothing wrong with this, but it is important to get clear about what it actually means. To that purpose, I shall confront Zeno's argument with an objection raised by some scholars, but which is based on a distinction between periods and nows going back to Aristotle.

Immediately preceding his report of the Arrow, Aristotle makes the following remarks:

> In the now [something moving] is always over against something, in the sense of being, but yet it is not at rest (ἐν δὲ τῷ νῦν ἔστιν μὲν ἀεὶ κατά τι μὲν ὄν, οὐ μέντοι ἠρεμεῖ). For it is neither possible to be in motion nor to be at rest in the now. Rather, though (μὲν) it is correct to say that in the now [something] is not in motion and is over against something, really (δ') it is not possible that it is over against something in a [period of] time, resting. For it follows [then] that what is in flight is at rest.[21]

Aristotle wants to block here the inference from the absence of movement in the now to being at rest in the now. He does so by denying that something is either in motion or at rest in the now, for then he can maintain that there is some third state between them, neither

19 E.g. G. Vlastos, 'A Note on Zeno's Arrow', in: *Studies* 205-218, at 205 [originally in: *Phronesis* 11 (1966) 3-18, at 3]; J. Lear, 'A Note on Zeno's Arrow', *Phronesis* 26 (1981) 91-104, at 91; Owen, 'Zeno' 157; KRS 272-273; Barnes, *Presocratic Philosophers* 276-277; and R.D. McKirahan Jr., 'Zeno', in: A.A. Long (ed.), *The Cambridge Companion to Early Greek Philosophy* (Cambridge, 1999) 134-158, at 151; cf. Caveing, *Zénon* 95-98.

20 Ross, Physics ad locum and 657-658, drawing support from Themistius.

21 *Physica* 6.8; 239a35-b4

being in motion nor being at rest, but just being over against something, which only applies in the now. Being in motion and being at rest he thus understands in terms of traversing or not traversing a distance *over a period*.[22] So in a sense he should be perfectly willing to accept being at rest in the now, if 'being at rest' would merely mean 'not traversing a distance', without further qualification. That is, he should be willing to accept (5) if it means:

(5a) The flying arrow does not traverse a distance in the now.

For from what he has said, we can only deduce that he does not accept (5) if it means:

(*5b) The flying arrow does not traverse a distance over a period, in the now,

since over the period in which the now is situated it is the case that the arrow traverses a distance. Similarly he would be obliged to reject (1_{em}) if it meant:

(*1_{em}b) Everything does not traverse a distance over a period, whenever it is over against what is equal,

but not if it meant:

(1_{em}a) Everything does not traverse a distance whenever it is over against what is equal.

Of course (*5b) and (*1_{em}b) are not well-formed formulae, but I use them to bring out the connection between Aristotle's remarks and a distinction brought to bear on the issue by some contemporary scholars. They argue that there is a sense in which (1) is false, for surely something can be in motion in the now, even though there is no movement in the now: in the now something can be *in the process of* traversing a certain distance, even though it does not actually traverse a distance in the now.[23] Thus they grant Zeno (1_{em}a) and (5a), but deny him:

(1_{em}b) Everything is not in the process of traversing a distance, whenever it is over against what is equal,

and:

(5b) The flying arrow is not in the process of traversing a distance, in the now.

Now the point of this distinction is that it is (5b), and therefore also (1_{em}b), which Zeno needs to establish a paradoxical conclusion. For the conclusion that the flying arrow is motionless can also be read in two ways, either as:

(c_a) The flying arrow does not traverse a distance in the now,

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22 Cf. *Physica* 6.8; 239a26: 'Being at rest is being in the same [place] over a period ..'

23 Cf. Barnes, *Presocratic Philosophers* 279-283; Owen, 'Zeno' 157-162.

or as:

(c_b) The flying arrow does not traverse a distance over a period.

We can then set out the logical relations between the possible readings of (c) on the one hand and (5) on the other in a table:

(5) / (c)	(5a)	(5b)
(c_a)	(5a) = (c_a)	(5b) implies (c_a)
(c_b)	no immediate relation	(5b) implies (c_b)

The problem for Zeno is that in order to have a real paradox he must take (c_b) to be his conclusion, for there does not seem to be anything paradoxical about (c_a). That is problematic, for as we saw, on the objection sketched above, Zeno is only entitled to (5a).

I do not think that this objection would cut any ice with Zeno, but answering it will make us understand better the idea behind Zeno's argument. As a first step towards explaining how Zeno thought to conclude (c_b) from (5a), we should redescribe this transition by ascribing to him a principle which says that what is true of the arrow at every now is true of it for the whole period. Since the relevant properties true of the arrow at every now and for the whole period may, in the case of 'being in motion' and 'being at rest', and 'moving' and 'resting', be taken as contradictory properties, or, as in the case of 'traversing a distance' and 'not traversing a distance', certainly are contradictory, we may state this principle as follows:

(N) For every now *n* in a period *T* a is *P* in *n*, if and only if a is *P* over *T*.

for some property *P*. According to this principle, which one might call the Now-principle, the situations at the level of all the nows and at the level of whole periods will mirror each other.[24]

From the objection based on the distinction between 'being in the process of doing ϕ' and 'doing ϕ', it is clear that there are instances of (N) which are not true. What, then, is the appeal of (N)?

To answer this question, we have to start with finding out what distinguishes the instances for which (N) is true from those for which it is not. The former are of the type 'being in the process of doing ϕ': something can be in the process of doing ϕ both in the

[24] It is assumed that either a is *P* at every now or a is not *P* at every now. That ascribing (N) to Zeno amounts to a mere redescription of his move from (5a) to (c_b) becomes clear if we apply it to 'being at rest' in the sense of 'not traversing a distance'. This would give him:

(N_1) For every now *n* of a period *T* a does not traverse a distance in *n*, if and only if a does not traverse a distance over *T*,

which can be abbreviated as:

(N_1) (5a) if and only if (c_b).

now and over a period. The reason why in those cases (N) is true, however, is not that over the period *as a whole* something is in the process of doing ϕ, but that it is in *every now* in that period that it is in the process of doing ϕ. So though (N) itself, with its biimplication, gives no precedence to either nows or periods, it is in fact the level of the nows which is explanatorily basic – at least in the cases of the type 'being in the process of doing ϕ'. On the other hand, (N) is not generally true for instances of the type 'doing ϕ', that is, in the case something *does* ϕ.[25] And the reason why it is not, is precisely that in those cases something does ϕ over a period as a whole, without it being the case that it does ϕ in the now. From the explanatory point of view, the whole period is basic.

All this suggests that according to a conception on which nows are basic and periods somehow consist of nows, (N) is bound to come out true, since there is no conceptual space for instances falsifying (N). And it is not difficult to recognize that Zeno has indeed adopted such a conception. In premiss (2), that the flying arrow is always in the now, he confines reality to the now; it is in the now when things are.[26,27] Moreover, this premiss also suggests the ultimate source of the idea that the now is basic: the tensed conception of time according to which it consists of past, present and future; of these three parts, the past in itself is not any more, but derives its reality from having been present, and the future is not yet, but derives its reality from the fact that it will be present. Thus time consists of nothing but nows, the past ones, the future ones and the present one.[28] That it is this conception which underlies premiss (2) becomes clear if we would replace 'the now' with the more neutral 'instant',[29] so that we would get:

(2*) The flying arrow is always in an instant.

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25 I add the qualification 'generally' because for actions denoted by verbs which do not imply some accomplishment realized over a period of time, (N) is true.

26 The only viable reading of 'is' in (2), it seems to me, is the existential reading; it is impossible to understand the expression 'being in the now' in a quasi-local sense. For why then would it be incorrect to say that the flying arrow is always in a period? If I am at home, I am also in my home-town, so if I am in the now, I am also in some period in which the now is situated. To make (2) completely unambiguous, we could reformulate it as:

(2_{clear}) Every time at which the flying arrow exists, is the now.

27 Lear, 'Zeno's Arrow', followed by KRS 273-274, wants to read a similar idea into (2), by interpreting (2) as:

(2_{Lear}) The flying arrow is always *so* in the now.

What he is in fact doing is combining the supposition of (2), that the arrow is flying, and what in (2) is said about the arrow, that it is always in the now, into one statement about the arrow. There are two reasons why I think we should not adopt this reading. First, why would Aristotle or Zeno have expressed himself so obscurely? Second, we do not need it, for as it stands, (2) suggests an even more plausible thesis than (2_{Lear}), a thesis, moreover, from which (2_{Lear}) follows.

28 Cf. the explanation given by M. Dummett, 'Is Time a Continuum of Instants?', *Philosophy* 75 (2000) 497-515, at 501. This link between the thesis that everything has to exist and happen in the now and the thesis that time consists of nows is missed by F.A. Shamsi, 'A Note on Aristotle, *Physics* 239b5-7: What Exactly Was Zeno's Argument of the Arrow?', *Ancient Philosophy* 14 (1994) 51-72, at 63-64 and again 66, where he argues against the interpretations of Lear, 'Zeno's Arrow' and KRS 273-274, which stress the importance of the present in Zeno's argument. For Shamsi does so on the basis of the incorrect consideration that in that case Aristotle's diagnosis that Zeno falsely assumes that time consists of nows (239b8-9), would not make sense.

29 This is not to say that 'the now' is an ambiguous term in Zeno's argument, standing both for the present (instant) and the instant in general, under which all instants fall. For as premiss (2) says, the flying arrow is *always*, that is, in the past, in the present, and in future, in the now, that is, in the present instant. 'The now' is here not any more ambiguous than 'the present (instant)'. Only when the term 'the now' is detached from its conceptual background in the tensed conception of time, and starts to function as a mere boundary between periods – as in Aristotle –, is an ambiguity introduced.

Phrased thus, the premiss is not by any means as compelling as in the original form, for with (2*) one immediately starts wondering why the flying arrow cannot be over a period.

Given this conception of time as consisting of nows, it is only natural that Zeno adopts (N) without appropriate restrictions, so that it also applies to 'traversing a distance'. For when does the arrow move otherwise if not in the now? If it is only over a period as a whole that it can move, motion is not a real phenomenon, since reality is enclosed in the now, that is, in each of the nows separately. Conversely, if the arrow does not move in the now, it does not really move over a period either.

Having thus established the conceptual grounds allowing Zeno to pass from (5a) to (c_b), we can also see what is wrong with the objection raised that Zeno should base his conclusion (c_b) on (5b), that the flying arrow is in the now not in the process of traversing a distance. According to Zeno's conception of time, time, and therefore also the periods within it, consists of nows. Earlier I invoked this conception to explain Zeno's adherence to (N), for if a period consists of nows, a phenomenon present over a period must be present in each of its nows, at least if that phenomenon is to have reality. Thus if motion and rest are to be real phenomena, they must be present in the now, without any reference to anything beyond the now. But motion or rest in the sense of being in the process of doing ϕ can only be present in the now if they are also present in it in the sense of doing ϕ, because 'being in the process of doing ϕ' is a derived notion, referring to 'doing ϕ'.[30] Therefore there is no place for (5b) to play a part in Zeno's argument.

We are now in a better position to understand the summary of Zeno's reasoning given at the beginning of this sub-section. At first sight it looks like a kind of *modus ponens*:

(2) The arrow is always in the now
(5) In the now the arrow is at rest
∴ (c) The arrow is always at rest,

where 'always' in (2) and (c) has the same meaning, indicating a generalization. On my interpretation, however, the meaning of 'always' in (2) comes closer to 'only': it is only in the now that something exists. The chain of reasoning which leads to the generalization of the rest in the now to the rest over the whole period of the flight, is left implicit: since reality is confined to the now, this whole period consists of nothing more than the nows making it up, so that everything true over a period should be based on what is true in each now; as there is no movement in the now, there is no movement over the whole period either. We cannot escape from this argument by claiming that there is motion in the now in the sense of being in the process of moving, as this would require motion over a period as prior. We cannot escape from it either by turning motion into nothing more than being at different places at different nows, while leaving intact Zeno's conception of periods as being nothing more than the nows making it up.[31] For that would amount to conceding to Zeno that motion is not a real phenomenon. There are only two ways of escape. The first is to deny the ontological priority of the now, so that not only the past and the future are just

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30 Cf. Makin, 'Zeno' 851.

31 As seems to be the standard response to the Arrow; see e.g. White, *Continuous* 4-5 and 69, B. Russell, 'The Problem of Infinity Considered Historically' [Lecture 6 from *Our Knowledge of the External World* (New York, 1929) 182-198], in: W.C. Salmon (ed.), *Zeno's Paradoxes* (Indianapolis and New York, 1970) 45-58, at 51, and Salmon, *Space, Time and Motion* 41.

as real as the present, but also they do not have to be reduced to past nows and future nows.[32] The second way is to give up the idea that the now is without duration, so that (3) is not true.[33] What Zeno has succeeded in establishing is that the natural conception of time consisting of nows which are taken to be durationless, does not go together with motion as a real phenomenon.[34]

§ 2.2. Text

Now that we have come to grips with the argument of the Arrow, we are ready to take a closer look at the text of the manuscripts and see whether it can be saved as it stands. According to the manuscripts, Aristotle's report of Zeno's Arrow in *Physica* 6.9; 239b5-7 is as follows:

> Ζήνων δὲ παραλογίζεται· εἰ γὰρ αἰεί, φησίν, ἠρεμεῖ πᾶν ἢ κινεῖται ὅταν ᾖ κατὰ τὸ ἴσον, ἔστιν δ' αἰεί τὸ φερόμενον ἐν τῷ νῦν, ἀκίνητον τὴν φερομένην εἶναι ὀϊστόν.

If we take this report as giving us two premisses and a conclusion, we have to suppose that Zeno argued thus:

(1) Whenever something is over against what is equal, it is at rest or in motion.
(2) The flying arrow is always in the now.
(3) In the now everything is over against what is equal.
(4) Therefore (from (3) and (1)) the flying arrow is at rest or in motion in the now.
(5) But (from (3) and (4)) the flying arrow is at rest in the now.
(c) Therefore (from (5) and (2)) flying arrow is motionless and at a standstill.

Because this seems a needlessly complex argument and because it is difficult to account for the appeal of (1) as a premiss, I accepted for § 2.1 the most commonly adopted emendation of this report, the deletion of ἢ κινεῖται. This is certainly not the only possible emendation. The alternatives fall into two groups. The first group, to which the accepted emendation (αi) belongs as well, contains three others:

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32 Thus this goes further than just giving up the A-series conception of time, for often philosophers who reject the priority of the A-series, still maintain the conception of time as consisting of instants. It seems a plausible conjecture that these philosophers, while overtly rejecting the priority of the A-series, tacitly retain it.

33 This way out is taken by Dummett, 'Does Time'.

34 One might think that the distinction between a standard and a non-standard sense of sizelessness and durationlessness, as drawn with the help of non-standard analysis, *viz.* between standard points and instants, on the one hand, and infinitesimal distances and periods, on the other, would provide a refutation of the argument itself. For then an arrow in flight would in the now still be over against what is equal in the standard sense, yet move over an infinitesimal distance. Thus (3) would turn out true in the standard sense, but false in the non-standard sense. On the other hand, only something over against what is equal in the non-standard sense is really at rest, so that (1_{em}) would be false in the sense required to arrive at Zeno's paradoxical conclusion (see M.J. White, 'Zeno's Arrow, Divisible Infinitesimals, and Chrysippus', *Phronesis* 27 (1982) 239-254). This solution, however, is only a more sophisticated version of the objection discussed in the main text based on the distinction between motion in the sense of doing ϕ and motion in the sense of being in the process of doing ϕ, since in the mathematics the infinitesimals needed to formulate the non-standard sense of being over against what is equal, are defined in terms of real lengths.

(αii) οὐ κινεῖται instead of ἢ κινεῖται[35]
(αiii) μὴ κινεῖται instead of ἢ κινεῖται[36]
(αiv) καὶ μὴ κινεῖται instead of πᾶν ἢ κινεῖται[37]

The other group leave the clause ἢ κινεῖται intact, but inserts a clause before ὅταν:

(βi) ἠρεμεῖ δὲ
(βii) οὐδὲν δὲ κινεῖται
(βiii) καὶ μὴ κινεῖται
(βiv) οὐ κινεῖται δὲ[38]

If there is to be an emendation, I prefer not (αi), but (αiii) or (αiv), because they stay close to the text we have. (That I have not adopted one of these two for the analysis of § 2.1, is because neither has been proposed in the literature.) In this sub-section, though, I shall discuss some possible drawbacks of the two groups of emendations and then present a defence of the text as given by the manuscripts.

The effect of the (α)-group of emendations is to replace (1) with:

(p) Everything is at rest whenever it is over against what is equal,

where 'being at rest', except in the case of (αi), is specified as involving 'not being in motion'. The problem one could have with such an emendation is the following. The basis of Zeno's paradox is (3), that in the now the arrow is over against what is equal. How should one understand this premiss in order to agree with it? It seems to me that one can only understand the predicate 'being κατὰ τὸ ἴσον' if it implies a contrast with 'being κατὰ τὸ ἄνισον'. This contrast can be given sense if we assume that it is the same as that between presence and absence of movement.[39] But then (3) would already express the absence of movement in the now. Moreover, in § 2.1 we have seen that 'being at rest' in (1_{em}), that is, (p), and (5) should be interpreted in terms of not traversing a distance, that is, absence of movement. Why then go from (3) to (5) with the help of (p), if (3) already says the same as (5)? It seems as if (p) as a premiss is superfluous. This point becomes all the more forceful if we follow Aristotle in using the predicate 'motionless' in the conclusion of the Arrow.

The (β)-group of emendations also run into trouble on this account, as they too yield (p). But in addition they give us:

(q) Everything is either at rest or in motion.

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35 Ross, Physics ad locum and 658, though his reference to *Physica* 8.8; 263b30 for the construction is rather unfortunate, since there there is no asyndetic οὐ to be found.

36 Not found, though if (αii) is allowed, then (αiii) should be allowed as well, since the negation μή instead of οὐ is possible in conditional clauses like these; cf. e.g. *Physica* 6.10; 241a15.

37 Not found, but perhaps not too far-fetched. Just as in e.g. *Physica* 6.8; 239a27-29 a subject like 'something' can be understood.

38 The proposals are by Lachelier, Diels, Cornford and Emminger respectively. For a very extensive discussion of most proposals, including references, see Shamsi, 'Note on Aristotle'. He does not mention Diels, for whose emendation see: Lee, *Zeno* 52-53 and 78-83, and Caveing, *Zénon* 97.

39 Also in other interpretations one encounters this idea, e.g. McKirahan, 'Zeno' 151.

But what is the function of (q) in the argument? Simplicius, though reading the same text as the manuscripts, might be taken to suggest an answer: to argue from absence of movement to being at rest.[40] But again Zeno does not need this step, as he only needs to establish that the arrow is motionless. The only alternative answer I can think of, is that (q) forms a contrast with an immediate implication of the conclusion (c) that there is an arrow which is both flying and at a standstill – but that would be rather meagre for a serious premiss.

These are the disadvantages of the two groups of emendations. Perhaps they do not seem too bothersome, but they would become more so if we could defend the text of the manuscripts. To such a defence I now turn.

The basic idea of my defence is that we should read (1) in the context of Aristotle's discussion of the Arrow, including the last lines of the previous chapter of *Physica* 6. My conjecture is that (1) as such was never formulated by Zeno, but is used by Aristotle to represent two propositions presumably used by Zeno, but repugnant to Aristotle himself.

As a first step towards seeing the point of that conjecture, we have to consider the idea expressed by (1), and its relation to propositions (3) and (4). Taken as a fully relevant proposition, (1) says that the mere fact that something is over against what is equal, does not determine whether it is at rest or in motion; an arrow which is flying will be over against what is equal just as much as an arrow which is not flying, but at a standstill. Thus the following diagram represents the idea expressed by (1):

being at rest or being in motion
↑
being over against what is equal

In this diagram the arrow indicates temporal coincidence ('whenever').

However, not for every kind of temporal coincidence is (1) true. The underdeterminatation expressed in (1) applies only to being over against what is equal *in the now*, as opposed to over a period (for something over against what is equal over a period can only be at rest). This is in fact what is pointed out in (3): if one wants to state when it is that being over against what is equal applies in such a way that both rest and motion can be temporally coinciding with it, then one should say that it is in the now as opposed to over a period. 'Whenever' does not range over periods, but only over nows. The combination of (1) and (3) can be represented then as follows:

in the now, not over a period	{	being at rest or being in motion ↑ being over against what is equal

In this diagram the combination of 'being over against what is equal' with 'in the now, not over a period' represents (3), the right half (1), as above, and the combination of 'being at rest or being in motion' with 'in the now, not over a period' may be said to represent what is said about the arrow in (4). The important point to bear in mind is that this scheme is

[40] *In Physica* 1011.25: 'What is not in motion is at rest, since everything is either in motion or at rest.'

completely determined by any two of these three propositions in that if two propositions are true, the third one is also true.

The second step is taken by considering Aristotle's remarks immediately before the report of the Arrow. Let me quote them again:

> (3*) In the now [something moving] is always over against something (κατά τι), in the sense of being – but it is not at rest. For (not-4) it is neither possible to be in motion nor to be at rest in the now. Rather, though it is correct to say that in the now [something] is not in motion and is over against something, really it is not possible that it is over against something in a [period of] time, resting. For it follows [then] that what is in flight is at rest.[41]

We see here Aristotle referring, in denial of course, to (4) and stating a proposition (3*) closely resembling (3). The difference, however, is telling: by replacing κατὰ τὸ ἴσον with κατά τι, Aristotle gets rid of the implicit reference to the idea that movement causes something to be κατὰ τὸ ἄνισον.

Now it is my suggestion that in his report of the Arrow, Aristotle combines (3) and (4) – which together completely determine the scheme represented above – into (1). His reason for doing so is that he wants to separate his real diagnosis and solution of the Arrow, given after the report,[42] from his earlier denial of (4) and his objection against the inference of (5) from (3). With these two points Aristotle expresses a desire to stick to a use of 'being at rest' as only applying over periods.[43] However, since these two points thus are in fact only a matter of a hygienic use of language, to be adopted in order to avoid confusion and mistakes, they are not really relevant for solving Zeno's paradox. (As I shall show in Chapter Three § 5.2.3.2, Aristotle thinks that the unhygienic acceptance of (4) and of the inference of (5) from (3) stem from the same source, namely the idea that time is composed of nows.) So ascribing (3) and (4) to Zeno would be harmless, as we can still block the argument if they were granted to him.

This proposal, that we do not need to ascribe (1) to Zeno himself, because it is merely used by Aristotle to refer to the two propositions (3) and (4), has many advantages. While retaining the text of the manuscripts, we do not have to account for the plausibility of (1) any more, since (1) is not a premiss of Zeno's argument. Rather we have (3) and (4), and they have immediate appeal. For if one agrees to (2), according to which reality is confined to the present, one will also agree to (4) if one wants rest and motion to be part of reality. And one accepts (3) as soon as one is impressed by the argu-ment that the now cannot have any duration because then it would contain a part of the past or the future.[44] What is more, we do not have to supply (3) without any direct evi-dence any more; by taking the wider context of Aristotle's report into account, we can point to evidence.

Therefore we may ascribe to Zeno the following simple argument:

(2) The flying arrow is always in the now.

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41 *Physica* 6.8; 239a35-b4

42 'This argument is incorrect, because time is not composed of indivisible nows.' (*Physica* 6.9; 239b8-9) And: '[The conclusion] follows on the basis of the assumption that time consists of nows. For if that is not admitted, there will not be a deduction.' (*Physica* 6.9; 239b31-33)

43 See *Physica* 6.8; 239a26-29. It has to be noted, though, that the whole section (*Physica* 6.8; 239a23-b4) is not very perspicuous. For a discussion, see Chapter Three § 5.2.3.2.

44 For this line of argument, see § 5.2.2.1.

(4) Therefore it is in the now that it moves or rests.
(3) But in the now it is also over against what is equal.
(5) Therefore it does not move in the now, but rests.
(c) Therefore it is motionless and at a standstill during its flight.

§ 3. Parts and wholes

§ 3.1. Looking back to the Arrow

Though in the previous section I have stressed the specific part played by Zeno's conception of time in the argument of the Arrow, it is also possible to recognize a more general pattern in Zeno's reasoning there. This concerns the relation between parts and wholes: the principle (N) according to which what is true at each now within a period is true over the whole period and *vice versa*, is a temporal version of a more general whole-part-principle which says:

(WP) What is true of a whole is true of each of its parts and *vice versa*.

That is:

(WP) For every whole *x*, *x* is P if and only if for every part *y* of *x*, *y* is P.

Of course (WP) is not true of every property P. In order that (WP) be true, P must be a property which a whole and a part of it can possess in the same sense. Let me call such properties 'mass-like properties', because they behave like masses: e.g. each amount of water as a whole as well as every part of it is water.

The source of trouble with (WP) which, as we shall see, Zeno exploits in several of his arguments, is that in itself it does not rule out that what we might call limit entities are parts of a whole. Now this may not be taken to be a problem for all mass-like properties. Take for example colours: it might be meaningful to say that not only a table, but also a single point or a single line is blue.[45] But clearly this is not true if also some kind of additivity is involved with the property. *Moving* is a case in point: an arrow might be said to move over a period or over half that period, but as we saw it is part of Zeno's argument of the Arrow that an arrow does not move in the now. Another example, as we will later see also used by Zeno, would be *having size*. Size and movement, and all other properties which can be measured, involve additivity: normal parts of a whole having such a property have that property as well, but not to the same amount; and the amounts of the parts together make up the amount of the whole. (One could distinguish thus within such measurable properties between a pure mass-like aspect, which is not additive and which all parts have in the same way, and a determinate aspect, determinating the quantity of the part, which is additive.)

Faced with such problems for (WP) on the basis of measurable properties, one might consider ruling out limit entities as proper parts. As we shall see in § 3.3., in one

[45] Though I would say that 'blue' is here ambiguous in the same way as 'being in motion' is between 'actually moving' and 'being in the process of moving'. Thus I would claim that a single point in itself cannot be blue in the primary sense.

argument Zeno does adopt such a conclusion. Why then does he not do so in the Arrow? The reason is obvious: though the principle (N) itself is neutral on the issue of ontological priority, it is underpinned by the priority of the now in relation to a whole period of time. In the context of the Arrow this priority was derived from Zeno's conception of time, not from any view on the relation between wholes and parts in general. But as we again shall see later, on some occasions Zeno assumes that it is generally the case that parts are prior to wholes. It is no surprise then that paradoxical conclusions ensue.

§ 3.2. The argument of the millet-seed

There is even one Zenonian argument which derives a paradoxical conclusion in an argument about parts and wholes without invoking a limit entity. In *Physica* 7.5 Aristotle reports:

> Zeno's argument is not true, that whatever part of a millet-seed makes a sound. For there is no reason why it should not be the case that it does not move the air which a whole bushel moved in falling, not in any period of time (οὐδὲν γὰρ κωλύει μὴ κινεῖν τὸν ἀέρα ἐν μηδενὶ χρόνῳ τοῦτον ὃν ἐκίνησεν πεσὼν ὁ ὅλος μέδιμνος).[46]

Aristotle gives us only the conclusion of the argument and some information about its setting, which involves a whole bushel of millet-seeds falling down. It is obvious that Zeno based his conclusion on the fact that a whole bushel makes a sound, but we do not have any reason to attribute to Zeno anything about sounds consisting in air being moved and so on – all of that seems an Aristotelian development. How exactly Zeno argued for his conclusion is not clear. Simplicius, however, in his commentary on the passage from Aristotle, gives a fuller story:

> By this point he also solves the argument of Zeno the Eleatic, which he presented in questioning Protagoras the sophist. 'Now tell me, Protagoras,' he said, 'does one millet-seed make a sound in falling down, or a ten thousandth part of a millet-seed?' When he said it did not, Zeno said: 'But a bushel of millet-seeds, does it make a sound in falling down or not?' When he said that a bushel did, Zeno replied: 'Why then, is there not a ratio of a bushel of millet-seeds to one millet-seed, or to a ten thousandth part of one?' When he admitted there was, Zeno continued: 'Why then, will not the ratios of the sounds to each other be the same? For as the things making the sounds are, so are also the sound. That being so, if a bushel of millet-seed makes a sound, also a single millet-seed and a ten thousandth part of a millet-seed will make a sound.' In this way then Zeno presented his argument in questions.[47]

The issue now is how much to believe of Simplicius' account. Though it is perfectly possible for Zeno and Protagoras, being contempories, to have met, it seems unlikely that they had this conversation.[48] As a first reason, I think that Aristotle's silence about Protagoras is telling. But what is most suspicious, is that both the form of the dialogue and the name of Zeno's interlocutor can be explained by reference to Aristotle's theory of dialectic. The conversation between Zeno and Protagoras follows the canonical form of a dialectical discussion as described and prescribed by Aristotle in his *Topica* and *Sophistici Elenchi*: the answerer takes up a thesis in response to a problematic issue presented to him

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46 250a19-22
47 *In Physica* 1108.18-25
48 Cf. McKirihan, *Philosophy* 193; Barnes *Presocratic Philosophers* 258; and Lee, *Zeno* 110, also for further references.

in the form of a question, upon which the questioner tries to get him to admit the contradictory thesis on the basis of his answer to certain propositions. Moreover, Aristotle also recommends drawing up lists of opinions for separate schools in preparation for such dialectical discussions, out of which grew the practice of doxography. In the context of such a practice it is natural that the name of Protagoras should have been assigned to the opponent of Zeno, who in this argument points to a gap between sounds heard and sounds made, as Protagoras was a philosopher who took sense-perception so seriously. Also the dialectical form is easy to understand as a product of such a practice, since it would give historical substance to Aristotle's praise of Zeno as the 'inventor of dialectic'[49] (cf. especially the last line of Simplicius' report). Finally, as Simplicius presents it, Zeno does not wait for an answer to the crucial question, whether the ratio between the sounds is not the same as that between the corresponding amounts of millet-seed. Surely Protagoras would have denied that in any real discussion, just as in fact Aristotle does.

It seems to me that because of this last point, the doubts about Simplicius' testimony as a reliable source as far as the presentation is concerned, should spill over to its contents. For if Zeno were to rely on this crucial premiss without argument, he would in fact be begging the question, and in a rather obvious way. Moreover, the concept of proportion invoked by Simplicius' Zeno seems too mathematical – whereas all the other arguments we know are not couched in mathematical language at all. This is not to say that invoking the concept of proportion is doing a complete injustice to Zeno; it is to say that it is more precise than it needs to be for Zeno's purpose. For the underlying idea of setting up such a proportional relation between sounds made and parts falling, is that to each part there is a corresponding sound. Thus the proportional relation we meet in Simplicius' report is the version of (WP) appropriate to the case, expressed in mathematical language.

A consequence of such scepticism is that it becomes possible to see the argument of the millet-seed in a new light. For unless we are prepared to let Zeno commit the fallacy of *petitio principii* in a rather obvious way, we should take the argument as meaning to establish (WP) for *making a sound*. And from the earlier part of this section we are familiar with a way of grounding (WP): by arguing that parts are prior to the whole made up of them, and that therefore the property the whole has must be due to the parts having that property as well.

As appears from a remark in his response to Zeno's argument, already Aristotle thought that this is the ultimate ground for Zeno's conclusion that every part, however small, of a millet-seed makes a sound. For in the end Aristotle gives only one reason why a part of a millet-seed does not make a sound: because 'nothing ever exists in the whole except potentially.'[50] This is the reason why a part of a millet-seed does not move any air, not even that fraction of the whole amount of air moved by the whole bushel which would correspond to the part of the whole bushel under consideration.[51] Presumably this reason is in general Aristotle's explanation as to why in some cases the proportionalities between the moving power, the thing moved, the distance moved over, and the period need for the movement (the last of course in inverse proportion to the

49 See Diogenes Laertius 8.57 (= DK 29 A10): 'Aristotle in the *Sophist* says that [Zeno] was the first to discover .. dialectic.'

50 250a24-25

51 250a22-24

distance and the moving power) fail to hold. By contrast, the proportionalities do hold if there is no whole consisting of parts, but merely a plurality of parts:

> But if there are two <movers>, and each of these moves each [part] over such a distance in such a period, the powers will also when composed move the composite of the weights over an equal length and in an equal period. (250a25-27)

What, according to Aristotle, distinguishes the cases in which the proportionality does hold from those in which it does not, is that in the former cases the parts are prior to the whole and the whole is nothing more than a plurality of parts, whereas in the latter the whole is more than the parts.

I do not think that it is accidental that Aristotle is prompted to make this point when considering Zeno's argument of the millet-seed; it seems an immediate response to the idea behind Zeno's reasoning there. If I am right, we have in the argument of the millet-seed an argument for the principle (WP), presented, as it behoves Zeno, in a paradoxical way. It is the same argument as we encountered in the Arrow, but now explicitly expressed in terms of parts and wholes.

§ 3.3. No size, no existence

In the argument of the millet-seed we saw that Zeno argued that every part of a whole must have the property possessed by the whole. In one of his two paradoxes of plurality, Zeno argues, as an intermediate point, for the converse of this conclusion, that whatever does not have the property possessed by a whole, is not a part of this whole – at least, so I am going to argue. Let us therefore have a close look at the argument.

Simplicius reports that in his paradox of plurality which has for its conclusion that 'if there are many things, they are large and small, large so as to be unlimited in magnitude, small so as to possess no magnitude'[52], in the half of the argument meant to derive the unlimited size of each entity, Zeno needs the premiss that 'magnitude is possessed by each [of the many things]'[53] and that he established this point by arguing for its contraposition: what does not have size, does not exist. As Simplicius presents it, Zeno's argument for that conclusion is the following:

> In [this paradox of plurality] he proves that (1) whatever has no magnitude nor mass nor any bulk, cannot be. 'For (2) if it were to be attached to something else that is,' he says, 'it would not make that any larger. For (3) as it is no magnitude, it is, when attached, not capable of contributing anything to a magnitude. And in this way, then, (4) that which is attached, would be nothing. And (5) if the other thing will not be anything smaller with it being detached, nor again will increase with it being attached, it is clear that that which is attached was nothing, no more than that which is detached.'[54]

The structure of this argument for what Aristotle calls 'Zeno's principle'[55] is as follows: (3) implies (2), which on its turn, together with (5), implies (4), which is then restated as (1). The argument is valid and, given the truth of (3), depends for its soundness completely on

52 *In Physica* 139.8-9 = DK 29 B2
53 *In Physica* 141.2
54 *In Physica* 139.9-15 = DK 29 B2
55 *Metaphysica* B.4; 1001b7-10

(5).[56] Now why would one accept (5)? According to Aristotle, it is ultimately based on the presupposition that 'what is, is a magnitude'[57], but if that were the case, Zeno would be arguing in a circle, as this presupposition is logically equivalent to (1).[58] At the other extreme, the attempt to exonerate Zeno from such a charge, based on the claim that his opponents themselves would already have adopted (5), so that Zeno would have secured it dialectically,[59] does not seem satisfactory either. Even if it were true that it was Zeno's purpose merely to establish (5) dialectically, it remains unclear why his opponents would put it forward. For (5) is surely not the kind of proposition which people, even philosophers, express regularly; I find it difficult to think of a context other than this very argument, in which (5) could function as a useful contribution. That means that the deviser of this argument, Zeno, must have deliberately come up with (5) to ground his principle. He must have thought that there is something which (5) brings out more clearly than (1), even though accepting (5) would imply accepting (1), and that, analogously, there is something about (5) which makes it more acceptable to his opponents than (1).

So we should take seriously that Zeno tries to establish the very general principle that what does not have magnitude and bulk does not exist, by way of an argument based on attachment and detachment. The notion thus introduced is that of parthood: Zeno seems to imagine a situation in which a point-like entity is added to an entity having size; the whole of these two parts has not become anything larger than the part which does have size. This suggests the following argument for (5), here in the form of (d):

(a) What is, is a part of the whole of reality.
(b) The whole of reality has size.[60]
(c) A part makes a difference in size to the whole of which it is a part.
(d) Therefore, an entity which does not make a difference in size to the whole is not a part of reality and thus does not exist.

The crucial premiss, of course, is (c) – and why should one accept (c)? It is, I think, a combination of two ideas. First, that the whole is only the whole it is due to the parts which compose it; being the sum of the parts, the whole is ontologically secondary to them. Second, that basic entities cannot fail to have magnitude, mass and bulk. It is important to understand how these two ideas interact with each other so as to yield (c). The second idea itself might seem pretty close to the supposition Aristotle ascribes to Zeno, that what is, has magnitude, but in fact there is a crucial difference: it does not say that everything which is, has magnitude, but merely that every *basic* entity has magnitude, mass and bulk. Now this is only contestable by those who posit separate abstract entities outside space and time as basic, but I do not think there was anyone before Plato who was fully aware of this possibility. Granting that, however, one might wonder why one could not hold that these basic entities with magnitude themselves have parts without size, e.g. their properties like

56 See G. Vlastos, 'A Zenonian Argument Against Plurality', in: *Studies* 219-240, at 223 [originally in: J.P. Anton and G.L. Kustas (eds.), *Essays in Ancient Greek Philosophy* (New York, 1971) 119-144, at 121] and 'Zeno' 243 [369], including remarks about the translation.

57 *Metaphysica* B.4; 1001b10

58 Of which Makin, 'Zeno' 846, in fact accuses him.

59 As Vlastos does, 'Zenonian Argument' 222 [120].

60 Moreover, it has a finite size, for even a part with size added to a part with infinite size does not contribute anything to the whole. But Zeno does not seem to have taken into account this possibility.

warmth, their shapes, their limits – as someone like Aristotle would claim. Here the first idea comes in: on this conception of the relation between part and whole, basic entities cannot be wholes having parts, since they would thus lose their status as basic entities; being wholes of parts they are ontologically dependent on their parts. So as soon as one accepts the first idea one must also accept that parts must make a difference to the whole;[61] adding the second idea forces one then to accept (c).

Thus we have again an argument for (WP) on the basis of the priority of the part in relation to the whole.

§ 3.4. Unity and plurality

We have seen that Zeno on several occasions ascribes a property to a part on the basis that the whole of which it is a part has that property. The principle (WP) he employs in doing so, was based on the idea that parts are ontologically prior to the whole. We encountered this idea of priority in the Arrow in the premiss that the flying arrow is always in the now and in the argument of the millet-seed in the position Aristotle argues against in order to restrict the application of proportionalities; it was also to be found in the argument for the non-existence of sizeless entities according the justification I suggested. We have not yet encountered any case in which this ontological priority was stated in so many words. On every occasion it has been merely hinted at, either because we do not have Zeno's words any more, as in the argument of the millet-seed, or because the priority is stated in an oblique way, as in the Arrow, or because Zeno is trading on its intuitive appeal, as I believe to be the case in the quoted sub-argument from the paradox of plurality. Moreover, it always functioned as a ground for something else, namely for (WP), and was never considered in itself.

Now this seems hardly surprising, for how could someone at that stage in the history of philosophy have formulated the idea of ontological priority of parts over wholes? Does it not require a somewhat more developed conceptual vocabulary, e.g. including a concept like that of Aristotle's *separation*? In order to set out the idea in its details, that may indeed be the case, as will become clear at the end of this sub-section. Yet there is a way of expressing the idea which is not beyond Zeno's means – in fact, it goes to the core of Zeno's enterprise –, and that is in terms of the contrast between unity and plurality, the one and the many. The idea is that a whole which is ontologically secondary to its parts is a mere plurality, without any unity of its own; there is nothing but the many. On the other hand, a whole which is more than its parts, has a unity of its own and is not just a plurality any more.[62]

We encounter this distinction between unity and plurality in several ancient reports about Zeno. Philoponus, for example, tells us that he argued that

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[61] On the contrary conception of the relation between part and whole, that the parts are the entities they are because they are parts of the whole, and are thus ontologically secondary to the whole, a part perhaps does not have to make some difference to the whole – because it is possible then that the whole remains the entity it is after having lost one part, as for example with Theseus' ship – and surely does not have to make a difference in size to the whole, as sizeless things like limits or properties could then be parts as well.

[62] Cf. Aristotle's reply to the argument of the millet-seed, above p. 21.

> if nothing is properly one, there are not many either, if at least the many are composed of many units. [And since nothing is properly one], it is impossible for what is to be divided into many.[63]

In the same vein we should understand a quotation from Eudemus appearing in Simplicius:

> And they say that Zeno claimed that if someone could tell him what the one is, he would be able to give an account of the things which are.[64]

The supposition is of course that it is not possible to say what the one is.[65] (There are two reasons given in the ancient reports about Zeno as to why this is not possible, the first being that there will be an infinity of ultimate units[66] and the second that the ultimate units will not have size.[67] Both may go back to arguments we have in Zeno's own words, the first to the second paradox of plurality, to be discussed in the next section, and the second to the argument for the sizelessness of each of the many, from the first paradox of plurality, to be quoted below.)

So here we have one half of the thesis that a whole of parts is nothing more than those parts, expressed in terms of unity and plurality: to say anything about a plurality of entities, we should first find out about the ones from which this plurality is composed. Clearly the whole of parts which is the plurality has no independent status. That, however, is not a very surprising statement; it follows immediately from the concept of plurality. Moreover, it leaves open the possibility that there is a whole of parts which is more than a mere plurality. In order to rule out that possibility on Zeno's behalf, we need evidence that according to Zeno something with parts is not one, but a plurality.

One passage which provides such evidence comes from the pseudo-Aristotelian work *De Melisso Xenophane Gorgia*. In the wider context of a discussion about Xenophanes' unique god, pseudo-Aristotle considers the counterfactual case that Xenophanes would have declared his unique god to be the one:

> [If that were the case] what [is it that] prevents the god from rotating, caused by the parts of the god moving to each other? For indeed he will not declare such a one to be many, as Zeno does.[68]

The author supposes that Zeno would have said that Xenophanes' god, who is one, is turned into many by the fact that he has parts.

Further evidence we get again from Philoponus. He ascribes to Zeno the following argument:

> [S]uppose what is continuous to be one. Then since what is continuous is ever divisible, it is always possible to divide what is divided into several parts. If that, then, is the case, what is continous is therefore many. Hence what is continuous will not be one.[69]

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63 *In Physica* 80.27-29

64 *In Physica* 97.12-13 = 138.32-33 = DK 29 A16

65 Cf. what Alexander says, as recorded by Simplicius: 'As Eudemus reports, Zeno the friend of Parmenides tried to prove that it is not possible for there to be many things which are, on the ground that among the things which are nothing is one, and the many are a multitude of units.' (*In Physica* 99.13-16).

66 Given by Philoponus, *In Physica* 81.5-7.

67 Given by Eudemus, in Simplicius' report in *In Physica* 99.11 (cf. 97.15-16).

68 *MXG* 4; 979a3-5

69 *In Physica* 43.1-3

Though to me this passage seems more a reconstruction than a report, the core idea, that a unity which can be divided into several parts is many, and therefore not one, confirms the evidence of the previous quotation.[70]

Another reason why I quote this passage is that it actually gives a stronger version of the principle that a whole with parts is not one, but a mere plurality. For by including a modal element in the statement – what is one and *can* be divided into parts, is many –, Philoponus not only attributes to Zeno the proposition that what is one does not actually have parts, but also the proposition that what is one has never had and never will have parts (because it cannot have parts). Though at the present stage of the discussion this may seem a rather small point – is it not a very natural assumption that what is divisible, already has parts? –, we shall see in the next chapter that it is quite important.

Both versions, I want to argue next, can be found in Zeno's own words as well, namely in Zeno's argument for the conclusion that 'each of the many is small so as to be without size':

> [The other conclusion] he proves after having shown that nothing has magnitude because each of the many is the same as itself and one.[71]

The following reconstruction[72] of the argument suggests itself:

(1) Each of the many is self-identical and one.
(2) Everything which is self-identical and one, is partless.
(3) Everything which has size, has parts.
∴ (4) Each of the many is sizeless.

Premiss (2) is only plausible on the conception of a whole of parts being nothing more than a plurality. Thus the whole is a mere sum of its parts, and therefore ontologically secondary to those parts. As a consequence 'each of the many' in (1) cannot range over other entities than the basic units, which themselves are partless.

The remainder of the argument (1)-(4) we will discuss in the next section. But here we should clarify the appearance of the concept of self-identity in the argument. I think the only way this can be done is by understanding self-identity in terms of changelessness – which makes good sense in an Eleatic context. A way of thus making self-identity relevant for the argument would be to take change for an entity which is one as consisting in becoming many, that is, being divided. Then being self-identical for a unit would mean that it will always be and has been always one; conversely it would also mean that a plurality will never turn into a unity and has never been a unity. However, on such an interpretation the fact alone that each of the many is self-identical on its own would not suffice for their partlessness. For a plurality, by always being a plurality,

70 Indirect evidence can also be got from Plato's *Parmenides* 129b1-d6, where Socrates lectures Zeno. The fact that Socrates there explains that being one and being many are not incompatible, and that he takes something to be many if it has parts, indicates that according to Plato Zeno thought that being one and having parts were incompatible.

71 *In Physica* 139.18-19: οὐδὲν ἔχει μέγεθος ἐκ τοῦ ἕκαστον τῶν πολλῶν ἑαυτῷ ταὐτὸν εἶναι καὶ ἕν. I take οὐδέν to be the grammatical subject, as one would otherwise have to supply 'each of the many' as subject, which in the context is rather strained. But nothing substantial hangs on the translation.

72 As Vlastos, 'Zenonian Argument' 220 [118-119], points out, one may compare the following argument set up by Melissus: 'If it is, it must be one; but being one, it must lack body. But if it were to have bulk, it would have parts, and not be one any more.' (Simplicius, *In Physica* 87.6-7 = DK 30 B9)

might seem to be capable of being self-identical as well. The self-identity of each of the many would merely serve to strengthen their unity, that is, to make their unity eternal. Therefore this interpretation gets into trouble with the juxtaposition of self-identity and unity as grounds for the sizelessness of each of the many, for that suggests that each plays an independent part in the argument.

The solution would be to enlarge the scope of self-identity and changelessness by giving up its restriction to being one and being many, and to widen its application to other properties. On such an interpretation, a plurality of entities could gain and lose what one might call structural properties, consisting in the order of the many units. For by the moving around of the units the plurality made up of them instantiates different orderings. The units considered separately, on the other hand, do not change in any way, because the relational properties they have are not internal to them and are thus excluded if they are considerated separately.

It might be objected that on Zeno's conception of a whole as nothing more than its parts a plurality is nothing more than the units making it up, so that if the units are changeless, the plurality is changeless as well, quite apart from a change in relational properties between the units; these relational properties remain external. This objection, however, seems to be based on too strict an application of Zeno's conception of the relation between a whole and its parts, for if it were correct, even the plurality itself would not exist in any way. The whole point of talking about a plurality, also on Zeno's conception of wholes being nothing more than their parts, is to consider units *together* rather than separately. And together they are capable of instantiating the changing structural properties. Of course they do so because the units making up the plurality are related to each other in certain ways, but that does not mean that the externality of the relational properties to each of these units separately carries over to the units taken together, that is, to the plurality. On this understanding of the relation between parts and whole, between unity and plurality, the whole is still nothing more than its parts; but the whole is these parts taken together. Likewise the internal properties of the whole can all be explained by reference to the units and their properties, including the relational properties by which they are related to each other. Thus a changing whole and internally changeless, that is, self-identical, parts are compatible.[73]

If this interpretation is correct, we have found in Zeno's argument for the sizelessness of each of the many not only the principle that a whole of parts is many and not one, but also the stronger principle that what is one, cannot, by being divided, become many either, as it then is not changeless and self-identical.

§ 4. Considerations of homogeneity

At the end of § 3.4 we discussed one premiss of Zeno's argument that each of the many is sizeless, namely proposition (2), which says that everything which is self-identical and one, is partless. But to reach the desired conclusion, Zeno also needs as a premiss proposition (3), which says that everything which has size, has parts. As I will explain, there are problems with (3), or rather, with the combination of (2) and (3), which threaten the validity of the argument. These problems can be solved by invoking another principle. This

[73] Lest worries arise that I am here ascribing a position to Zeno which contradicts (WP), I should point out that (WP) as used by Zeno applies merely to mass-like, additive properties.

principle we know from a probably Zenonian fragment quoted by Simplicius; it also seems to lie at the basis of Zeno's second paradox of plurality. It says that what is, is homogeneous or the same everywhere.

§ 4.1. Two kinds of parts?

At first sight, (3) seems perfectly innocent. For is it not a truism that whatever has size, has parts? On one reading it indeed is a truism: the mere fact that something has size, is enough to allow for a point to be marked on it; and by marking a point we have distinguished two parts, the part on the one side of the point and the part on the other side of it. But what kind of parts are these? They need not be the kind of parts which will ever be separated, or even could be separated; for all we know, they may be inseparably bound together. This is to say that they need not be anything more than parts which can only be distinguished by conceptual means: by giving their boundaries. But the presence of these boundaries does not mean that the parts can come apart along these boundaries: they need not be physically separable parts. And that is all for the good, for (3), and thus the whole argument, would be far less appealing if (3) meant:

(3*) Everything which has size, has parts which are separable by a gap.

Now if we return with this distinction between conceptual and physical parts in mind to proposition (2), we may wonder whether that proposition is true for both kinds of parts. That is, we may wonder whether it may be possible after all on one of these two conceptions of parts for a whole of parts to remain one and self-identical.

As far as the self-identity of each of the many is concerned, there seems indeed to be reason for doubt. For since we interpret self-identity in terms of changelessness, one can question the acceptibility of (2) for conceptual parts. Parts which are merely distinguished by the indication of their boundaries and are not separable, do not move relatively to each other, for such movement brings along separation. Therefore the whole of such parts always remains the same. And with regard to the unity of each of the many there is a similar line of reasoning. For parts which do not come apart remain in a sense unified; thus their whole remains one, even though it has many conceptual parts.

In fact these considerations lead to a partial rejection of the principle of the priority of the parts over the whole for conceptual parts, since there is somehow a unity to the whole of these parts which makes it more than a mere plurality. On the other hand, the priority of the parts over the whole remains intact for physical parts, for indeed these parts are changeless and really one, while the whole of them does change and is a plurality. I say 'partial' and 'somehow', because it is only in the comparison of conceptual parts and wholes with physical parts and wholes that we reject the idea of the priority of parts over wholes to conceptual parts and ascribe a unity to them; with reference to physical parts and wholes, but just taken for themselves, conceptual parts may still be prior to their wholes. Conceptual parts, *in so far as they are no physical parts*, exist merely as a matter of viewing an object as divided and consisting of several parts. Within this world of make-believe, the whole may be nothing more than the sum of its parts. If we compare, however, this fantasy world with the real world of physical parts and wholes, we see that the conceptual parts which seemed to form a plurality actually are a unity, which moreover because of its reality is prior to these conceptual parts.[74]

Thus we can characterize the kinds of parts not only in terms of the pair ‘conceptual’ and ‘physical’, but also by way of the appropriate part-whole principle. Moreover, along with these distinctions comes yet a third one, between two kinds of division. The conceptual parts are divided because there is a boundary between them; they are marked off solely by geometrical means. Physical parts, on the other hand, are divided when they are separated by a gap.[75]

As a result of these distinctions, then, Zeno’s argument for (4) is in a shambles, for (2) is only acceptable in so far as it concerns physical parts, while (3) applies exclusively in so far as it concerns conceptual parts. It seems as if Zeno is trading on an ambiguity in his argument for the sizelessness of each of the many.

It is my belief that Zeno did indeed not distinguish between conceptual and physical parts and that he did not restrict in any way his principle that a whole of parts is not one but many. (We will see more evidence later, in § 4.2.) On the other hand, we should also acknowledge that there is something counter-intuitive to the distinction. For suppose we start breaking up a piece of matter into ever smaller separated parts. According to the distinctions drawn above, the breaking up will stop at the basic physical units. But is there not something arbitrary about that? For *why* are these, the entities of this size, the basic physical units? Why can we not go on as we did? For surely there are still parts in there. Why can we not separate them? What holds them together?

The intuition behind these questions, which seek to undermine the distinction between physical and conceptual parts and the concomitant distinctions set forth above, is expressed in explicit terms in the following argument:

> [S]ince it is everywhere alike, if divisibility belongs to it, it will be divisible everywhere alike, and not divisible here, but not there. Let it then be divided everywhere. It is then clear that again there will remain nothing and it will have vanished, and if it is to be composed, again it is must be composed out of nothing. For if something will remain, it will not turn out to be divided everywhere. Hence it is also clear from these points that what is will be indivisible, partless and one.[76]

Simplicius quotes these lines from Porphyry, who ascribes them to Parmenides. Simplicius himself, however, and before him Alexander, both think that they come from Zeno. And in modern times Makin has argued the case for Zeno’s authorship very convincingly, among other things by pointing to two passages in the commentaries on Aristotle’s *Physica* by Themistius and Philoponus, where the same argument is referred to as being genuinely Zenonian.[77]

The core of the argument consists of the following chain of propositions. Suppose for reduction:

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74 The representation of a physical unity as consisting of several parts can even be said to presuppose this unity, for how can we create these parts if the whole which is imagined to be many, is in reality one?

75 From the previous paragraph it might appear that it is impossible that conceptual parts are also physical parts. Strictly speaking this is true, for parts which are *only* distinguishable by marking off their boundaries, cannot be separable from each other. However, we may widen the criterion, by focussing on the way of division: parts which are separable by a gap are also distinguishable by geometrical means.

76 Simplicius, *In Physica* 140.1-6, with the deletion of one ‘he says’ phrase.

77 These two passages are: Themistius, *In Physica* 12.1-3, and Philoponus, *In Physica* 80.26-27; see S. Makin, ‘Zeno on Plurality’, *Phronesis* 27 (1982) 223-238, at 227-229 and 232-233.

(a) The object of discussion *O* is divisible everywhere.

That means that:

(b) It is possible that *O* is divided everywhere.

However, this possible situation, that *O* is divided everywhere, is absurd, for then *O* would be dissolved into nothing and would have to be recomposed out of nothing. So:

(c) It is impossible that *O* is divided everywhere.

Therefore:[78]

(d) *O* is not divisible everywhere.

In itself, conclusion (d) does not threaten the set of distinctions drawn above. For if one replaces 'divisible' with 'separable into physical parts', then (d) seems soundly established and quite unproblematic, since the whole point of the distinction between physical and conceptual parts is that objects need not be divisible, that is, separable everywhere. On the other hand, if for 'divisible' one were to read 'divisible into conceptual parts', then (d) seems paradoxical: how could something with size not have conceptual parts? – cf. Zeno's premiss (3). However, (d) is reached by illicit means. For the reasoning which should show the absurdity of the possibility referred to in (b) is not sound: since the plurality of conceptual parts is a matter of make-believe, not of reality, the physical whole can have properties which even the sum of all conceptual parts does not have. Having size could be one of them. Or to put it another way: because the conceptual parts are not separated in a conceptual division, the whole of them remains intact – including its size. This is the case whether it is divided at one place, at ten places or indeed everywhere.

With the addition of one premiss, however, the whole picture changes:

(e) *O* is similar everywhere.

For (e) is used to support:

(f) If *O* is divisible, *O* is divisible everywhere alike.

And (f) is used to argue from (d) to:

(g) *O* is indivisible.

And as the whole of reality is in some sense similar everywhere – everywhere it is, forming the whole of what is – one gets:

(h) The whole of reality is indivisible.

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78 This step is, as we shall see in Chapter Two § 1.1, only valid on one reading. As it does not interfere immediately with the point I want to make, I will leave that complication here out of account.

And (h) is not the kind of proposition which is acceptable to a defender of common-sense, no matter what kind of division or part is presupposed.

So by the addition of (e) conclusion (d) – to which no one should object if it is understood to refer to physical parts – leads to conclusion (h), which is paradoxical on any reading. Premiss (e) has this effect because it forces anyone who tries to say that the whole of reality consists of many entities which are separable from each other, to accept that every conceptual part is in fact a part which can be separated from the other parts, as separability somewhere because of (e) implies separability everywhere. Thus there is no room for parts which are merely conceptual and to which the principle that a whole of parts is not one, but a mere plurality, does not apply.

Therefore, if one adds (e) to Zeno's (1)-(3), Zeno's conclusion (4) will follow without any problem. Of course one might think that (e) need not be accepted or that there is otherwise something wrong with Zeno's argument – but these matters I will leave for Chapter Two.[79]

§ 4.2. The unlimited multitude of being

We have now considered an objection against Zeno's argument for the conclusion that each of the many is sizeless. In itself this objection, which is that one should distinguish between conceptual and physical parts, makes a valid point against Zeno, in that he does seem to ignore this distinction. I have suggested that Zeno's failure to acknowledge it can be explained by ascribing to him another principle. This principle, or rather argument, which one might call the argument from homogeneity, turns every conceptual part into a physical part, as soon as one allows for one point where something can be divided physically, that is, separated. In this sub-section I am going to argue that similar considerations are at work behind the scenes of Zeno's second paradox of plurality. Again it will appear that parts for Zeno are primarily physical parts.

Let me quote Simplicius' report of this paradox:

> [P]roving that if there are many things, the same things are limited and unlimited, Zeno writes literally: 'If there are many things, it is necessary that they be as numerous as they are and neither more than themselves nor fewer. But if they are as numerous as they are, they will be limited. If there are many things, the things that are are unlimited. For there are always other things between the things that are, and again other things between them (ἀεὶ γὰρ ἕτερα μεταξὺ τῶν ὄντων ἐστί, καὶ πάλιν ἐκείνων ἕτερα μεταξύ). And thus the things that are are unlimited.' And thus he proved the unlimited in number from the dichotomy.[80]

We do not need to deal with the first half of the paradox here (for a discussion, see § 5.3). The issue at the core of the argument of the second half concerns the spatial relations between parts. But how does Zeno envisage these relations? In the translation I have opted for one possibility, by rendering μεταξύ as 'between'. It has, however, been argued that a

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[79] A point of detail to which Makin, 'Plurality', has called attention, is that with the help of (e), Zeno's supposition (3), which we saw he also tacitly relied on in his argument for (4), need not be affirmed uncon-ditionally. And that may be quite a relief to Eleatics who want to attack pluralists without accepting something inconsistent with their own views. For because of (e) one can replace (3) with:

(0) There is something with size, *viz.* the whole of reality, which is divisible,

since (0) together with (e) applied to the whole of reality yields (3). Rejecting (0) the Eleatics are not committed to (3), but they can use it unproblematically against those who do adopt (0).

[80] *In Physica* 140.28-34 = DK 29 B3

more suitable translation would be ‘in the middle of’, where this phrase may be complemented with a grammatically singular term. Zeno would then show the existence of an infinite number of parts by invoking a division procedure in which an entity would be divided recursively into three parts, two outer parts and one in the middle.[81]

I do not have any definite philosophical objections against this proposal, but I do have doubts about the possibility of this use of μεταξύ with a genitive singular. The only examples LSJ provides of μεταξύ with a genetive singular are μεταξὺ θύρας (in the opening of the door) and μεταξὺ τούτου (meanwhile). Now θύρα has as part of its meaning that there is a surrounding thing, while μεταξύ in the second example has a temporal meaning. What one would wish for are examples like *μεταξὺ θάλαττης (in the middle of the sea), but they are absent. Therefore we should stick to the customary reading of μεταξύ as ‘between’, which is the most natural one with τῶν ὄντων anyway.

So in the second half Zeno argues that between every two things there are, there is a third. That does not seem obviously true, for why can there not be two entities touching each other? Probably for that reason the reference of ‘the things which are’ has been taken to be pointlike entities, since for them it is true that they are ordered in such a way that between every two points there is a third.[82] If that were the case, however, Zeno would have assumed from the outset that the many things, with which in the context the things which are clearly are to be identified, are points. That does not seem likely, as it would be rather self-defeating for any pluralist to assume that his cherished multitude of existing things are nothing but sizeless entities.

So the question still stands: why should there be always something between two entities? It is my guess that considerations of homogeneity are at work here.[83] A homogeneous chunk of matter is one and therefore not a plurality; in order to avoid the pluralistic hypothesis coming out false, there has to be something between any two entities keeping them separated.[84]

It may be that Zeno based this requirement of separation on exactly the same argument as we encountered in the previous sub-section. Thus any two entities need to be separated in order to avoid there being one homogeneous chunk of matter which is at least divisible at one place. In this way it would be *as if* together with the argument for the sizelessness of each of the many, Zeno has set up a dilemma: if there are many, either they are sizeless – since because of the argument from homogeneity no entity with size is really one at least if it is divisible at one place) –, or they do have size, but must be kept apart from each other by infinitely many units in order to safeguard their independent existence against the argument from homogeneity.[85]

81 Barnes, Presocratic Philosophers 253

82 Caveing, *Zénon* 31

83 My account is related to that of Makin, ‘Zeno’ 845, presented more fully in his *Indifference* 22-24.

84 It is a perhaps unfortunate consequence of this interpretation that Simplicius’ concluding remark comes out slightly misleading, at least if one understands by the dichotomy a division procedure. For in the second paradox of plurality Zeno is not so much dividing a whole in order to bring its parts apart, as assuming the divided state of the things which are and postulating ever more entities in order to make that assumption true. However, this cannot be a real objection to the interpretation, since Simplicius’ remark would equally have to be discounted as misleading on the other two interpretations mentioned above. For if Zeno had envisaged a division procedure according to which in the middle of each entity there is another entity, this should not be called a dichotomy, but a trichotomy. On the other hand, if Zeno had assumed the many here to be points, then there is no division procedure at all.

85 Cf. Makin, *Indifference* 23-24

However, I do not think it has to be this version of the argument from homogeneity which is at work in the present argument. For a second way in which considerations of homogeneity can be behind the argument is that everything which forms a homogeneous chunk of matter, is taken to be one. Homogeneity would thus lead immediately to unity, and not through the chain of reasoning from homogeneity *in respect of divisibility* over absurdum to indivisibility.[86] It is this latter version which seems to play a part in Parmenides' argument for the unity of what is. One authoritative translation runs as follows (quoted earlier in § 1):

> Nor is it divided (διαιρετόν), since it all exists alike
> nor is it more here – which would prevent it from holding together (συνέχεσθαι) –
> nor less there, but is all full of being.
> So it is all continuous (ξυνεχὲς): for what is, draws near to what is.[87]

The idea is that what is, is all alike and full of being, and thus, being neither more nor less anywhere, drawing near to what is, does not allow for internal distinctions and divisions. Therefore it is all continuous and undivided, as well as one, we may presume.[88]

One might object, however, that this version is not sufficient to establish the requirement of separation. For just as appears from the translation of διαιρετόν adopted above, it does not give us more than the undividedness of two adjacent entities, which one might gloss as their unseparatedness. It is hardly surprising that adjacent entities are not separated. And, what is so bad about not being separated?

The translation 'divided' for διαιρετόν, however, seems somewhat misleading, just because it could be glossed as 'separated'.[89] As it is primarily the absence of internal distinctions which Parmenides focuses on, and continuity – the property of holding together – seems to serve as a near-synonym for not-διαιρετόν we should rather translate διαιρετόν as 'with distinctions'.[90] In this way we would have a kind of indivisibility, in that there is nothing to go on when one wants to divide it into units, that is, when one wants to distinguish entities within the whole. Apparently the parts of what is, which Parmenides implicitly refers to in '*what is* draws near to *what is*', do not qualify as independent entities, but are like mere sub-chunks of matter, water in a sea of water.

Thus underlying Parmenides' argument seems a criterion for being one entity: real distinguishability. And given that Parmenides does not countenance distinguishing properties like internal structure, but considers entities to be bare objects, chunks of what is, there can only be a plurality of entities if each entity is separated from other entities. In this way we have derived the requirement of separation in a more direct way from considerations of homogeneity: what is homogeneous is one entity, and there are only several entities if there is some kind of discontinuity.

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86 Aristotle seems to distinguish the two versions in *DGC* 1.8; 325a2-12, to be discussed in Chapter Two § 1.2.1.

87 DK 28 B8.22-25; translation, with one small change, taken from KRS 250-251.

88 That this is meant to be an argument for the unity of what is, appears from DK 28 B8.5-6.

89 And then we get into the issue, addressed by Makin, *Indifference* 32, whether Parmenides meant to argue for the undividedness of what is or its indivisibility.

90 Cf. remarks made by Makin, *Indifference* 32-33.

In Chapter Two we shall return to these two ways of applying considerations of homogeneity. Let me finish here by pointing out an important difference between them. This difference concerns the criterion for being *one* entity, that is, one physical part. On the first version a physical part is one if it is separable from other parts, but cannot itself be split up into separate parts. The second version has a laxer criterion for being one physical part, since parts which are separ*able* need not be at the moment distinguishable.[91] But on the other hand, a part which at the moment is not really distinguishable might become distinguishable later, for example when the indistinguishable whole is split up into distinguishable entities. Thus the second version is incompatible with Zeno's stronger priority principle, which says that entities which are one, remain one, and that pluralities cannot become one. The first version, on the other hand, seems to presuppose this stronger principle.

It is quite another question, however, whether Zeno was aware of all these niceties or whether he distinguished between the two versions at all. What is at all events clear is that Zeno should consider this half of the paradox by itself as a *reductio* of the pluralistic hypothesis. He has shown that no number of entities is sufficient to bring about the required separation.

§ 5. Limits

Up to this point we have paid little attention to limit entities. It was noted in § 3.1 that limit entities as parts cause problems with Zeno's principle (WP) governing the relation between the properties of the whole and those of its parts. We have seen such problems in the Arrow, and they returned later, being found implicit in Zeno's argument for the sizelessness of each of the many (dealt with in §§ 3.4 and 4.1), which surely is meant to be a paradoxical conclusion in itself. They were quite explicit in the fragment quoted in § 4.1, where it was taken to be absurd that a whole with size should be said to have ultimate parts without size. On the other hand, we also saw, in § 3.3, how Zeno ruled out limit entities as existent because they do not contribute anything to the whole.

All of that seems pretty clear and straightforward. The only complication we have come across so far, in § 4.1, concerns the issue of *conceptual* parts which are sizeless. For it was argued that for conceptual parts Zeno's priority principle, which is used to underpin (WP), should not apply without restriction, so that one may wonder whether limit entities as conceptual parts should cause problems involving (WP). But in any case we saw that Zeno did not distinguish between conceptual and physical parts, probably on the basis of an intuition captured by the argument from homogeneity. As far as the ontological relation between parts and whole is concerned, Zeno treats all parts as if they are, as we would put it, physical parts, that is to say, as prior to the whole.

It is my claim in this section that Zeno's argument in the second half of his first paradox of plurality, where he argues that each of the many is 'large so as to be unlimited', and again in his Runner paradox, turns precisely on this idea: that all parts, even if they are limit entities, are separate parts prior to the whole. And this time Zeno does not rely on (WP) to make problems for limit entities, but on some other, even more deep-seated, intuitions.

[91] One way of stating the difference is that the modality involved in distinguishability is epistemological, whereas the modality in separability is ontological.

§ 5.1. ‘So large as to be unlimited’

As far as we know, Zeno’s reasoning for the conclusion that ‘if there are many, .. they are .. large so as to be unlimited,’ consists of two parts, the second of which is described by Simplicius in two passages:

> [Z]eno [says] these things (a) [i.e. that whatever has no magnitude nor mass nor any bulk, cannot be] [in order to prove] that magnitude is possessed by each of the many things – unlimited things, because there is always something in front of what is being taken, because of the division *ad infinitum* (ὅτι μέγεθος ἔχει ἕκαστον τῶν πολλῶν καὶ ἀπείρων τῷ πρὸ τοῦ λαμβανομένου ἀεί τι εἶναι διὰ τὴν ἐπ’ ἄπειρον τομήν).[92]

And:

> [The infinity] in magnitude [he proved] .. [by means of the dichotomy]. For after he has proved that (a) ‘if what is, has no magnitude, it cannot be’, he continues: ‘But if it is, each must have some magnitude and bulk and the one part of it must be apart from the other (ἀπέχειν αὐτοῦ τὸ ἕτερον ἀπὸ τοῦ ἑτέρου). And of the part which juts out (περὶ τοῦ προύχοντος) there is the same account, for this as well will have magnitude and some part of it will jut out (προέξει αὐτοῦ τι). Now it is the same to say this once and to be saying this always, for no such part of it will be the last nor will a part not be related to another (οὔτε ἕτερον πρὸς ἕτερον οὐκ ἔσται). Thus if there are many things, it is necessary that they are small and large, small so as to have no magnitude, large so as to be unlimited (μεγάλα δὲ ὥστε ἄπειρα εἶναι).’[93]

In the second passage Zeno seems to describe a procedure of division which generates a series of parts without a final one: every ‘front part’ taken is recursively divided into two. The series can be represented in a diagram as follows:

In this diagram part Bω is the part jutting out of the whole Aω, or, described as in the first passage quoted, the part ‘in front of’ Aω[94]; part Bω is then ‘apart from’ the remaining part of the whole Aω, that is, AB.[95]

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92 *In Physica* 139.16-18 = DK 29 B2

93 *In Physica* 140.34-141.8 = DK 29 B1

94 A more suitable translation of πρό would be here: ‘in *the* front of’. That this is a possible translation appears from the use of πρό by Plato, *Parmenides* 165a8-b3: πρό τε τῆς ἀρχῆς ἄλλη ἀεὶ φαίνεται ἀρχή ... ἔν τε τῷ μέσῳ ἄλλα μεσαίτερα τοῦ μέσου. The two clauses are clearly meant to be completely parallel. As the things in the middle can only be more central than the previously taken middle if they are parts of that previously taken middle, the parallel shows that the beginning πρὸ τῆς ἀρχῆς is a part of the previously taken beginning. Thus it is the beginning *in the front of* the beginning.
This construal of πρό absolves Simplicius from an accusation of misunderstanding levelled against him by G. Vlastos, ‘Fränkel’s *Wege und Formen frühgriechischen Denkens*’, in: *Studies* 164-179, at 171 [originally: *Gnomon* 31 (1959) 193-204, at 196] and H. Fränkel, ‘Zeno of Elea’s Attacks on Plurality’, in: R.E. Allen and D.J. Furley (eds.), *Studies in Presocratic Philosophy* II *The Eleatics and Pluralists* (London, 1975) 102-142 [orinal version: *American Journal of Philology* 63 (1942) 1-25; 193-206], at 118 and note 70. According to them, Simplicius misunderstood the phrase προέξει αὐτοῦ τι as: ‘some [part] will be more in front than it’, as would appear from the fact that he thought that Zeno argued that there would always be someting ‘*in front of* what is being taken’ (πρὸ τοῦ λαμβανομένου). Their accusation falls flat on the translation proposed here.

It appears that Zeno thinks that the existence of such an infinite series of parts is enough reason to conclude that the many things are 'large so as to be unlimited'. The question any interpretation has to answer is how Zeno could have considered himself entitled to this step. First I will consider the two answers available in the literature and show their shortcomings. Then I will come up with my own interpretation, which, I will argue, gives Zeno a real paradox.

§ 5.1.1. Fallacy or failing interpretation?

The most influential account of Zeno's reasoning in this paradox is provided by Vlastos. His reconstruction is as follows:

(1) There is a series of parts without a last term – cf. the diagram.
(2) Therefore there is an infinite number of parts.
(3) Each of these parts has some size – cf. statement (a) in the first passage.
(4) Therefore the whole of these parts has an infinite size.

On this interpretation Zeno must have made an arithmetical mistake: somehow he thought that the sum of such a series would exceed any stipulated limit. Yet this would be quite surprising, for not only would we have to assume that Zeno failed to do some apparently simple arithmetic (e.g. summing the series consisting of the terms 1/2, 1/4, 1/8, 1/16 etc.), but also that he did not grasp by way of a diagram like the one above what his division procedure amounted to.[96] As a mitigating explanation Vlastos claims that knowing that the partial sums of such a series never exceed a certain limit – stated for the first time by Aristotle in *Physica* 3.6; 206b3-11 – was perfectly compatible with committing Zeno's fallacy; according to Vlastos the cases of Epicurus and Simplicius show this.[97] And just as they cannot have made the simple mistake of ignoring that piece of knowledge, but must have made a more subtle one, it is possible that Zeno fell into the same hidden trap rather than committing a blunder. That subtle error, Vlastos proposes, is the assumption that there is a smallest term.[98]

Against this account I have two initial objections. A first small point is that none of the passages from Epicurus or Simplicius employ this particular division procedure yielding an infinite series of unlimitedly decreasing terms. All of them state that if one has an infinite number of parts, all of some size, they will together form a whole of infinite size.[99] And in general that is true – it may only be false if for *every* size the number of parts larger or equal to that size is finite, while the number of parts smaller than that

95 I take it that Vlastos, 'Zenonian Argument' 225-226 [122-123] has convincingly refuted the interpretation according to which ἀπέχειν indicates separation by a gap.

96 Cf. Vlastos, 'Zenonian Argument' 234 [130]

97 Epicurus, *Ad Herodotum* 56-57; Simplicius, *In Physica* 142.14; 168.34-169.1; 459.23-24; 460.23-24; 462.3-5 and *In De Caelo* 608.16; cf. Eudemus apud Simplicium, *In Physica* 459.25-26.

98 Vlastos, 'Zenonian Argument' 234-235 [130-131]; cf. D.J. Furley, *Two Studies in the Greek Atomists* (Princeton, 1967) 69

99 I do not think this can be contested as far as the passages from Simplicius are concerned, but it is controversial with regard to Epicurus.

size is infinite.[100] But then Zeno stands out as being alone in having committed this fallacy.

Secondly, and more importantly, how could Zeno ever have thought that there is a smallest part without there being a last term of the series of decreasing terms? This is just as absurd as supposing that Zeno did not grasp what his division procedure amounted to as represented in the diagram.

Dissatisfied with an account imputing such a fallacy to Zeno, a number of scholars have attempted to argue that Zeno indeed reasoned along the lines of (2)-(4), but did not base (2) on (1) – rather he assumed a division procedure yielding an infinite number of *equal* parts, for if they all have size, as they by (3) are supposed to have, the size of the whole made up by the parts is infinite.

Now the only way of getting such an infinity is to argue that Zeno envisaged a division through and through, that is, one according to which *both* parts produced by a division are divided recursively.[101] The problem, however, with such an interpretation is that it is difficult to square with the evidence. Perhaps for that reason scholars often just shift the discussion to what we may call the Porphyry fragment, which does feature exactly such a division through and through.[102] The only scholar to have argued the case for the argumentatitive identity of the Porphyry fragment and our paradox is Abraham. He makes two related points: the use of τὸ προύχον should not be taken to imply a kind of order of parts, but rather to indicate a symmetrical relation between parts, in the same sense as indicated by ἀπέχειν; and τὸ προύχον should not be taken to refer to one part in particular, but to have a general meaning: 'any part which juts out'.[103] Thus Zeno would have argued that any of the two parts which are apart from each other in the whole, must itself have two parts apart from each other, and so on. The total number of parts then would be infinite, yet the parts being of equal size.[104]

Also against Abraham's interpretation I have two initial objections. First, it gives the impression of being very contrived. Why would Zeno then not have written: 'And of *both these parts* there is the same account', instead of the rather clumsy sentence as it stands? Moreover, even if one were to accept Abraham's account of προύχειν, πρό in πρὸ τοῦ λαμβανομένου remains very awkward if it does not imply an asymmetrical relation. Second, Abraham, as well as the other scholars, taking their clue from the Porphyry fragment, suppose that Zeno first allowed a division through and through (which is infinite) to be completed, and then attributed some length to the *ultimate* parts resulting from such a division. I cannot see how this fits with Zeno's contention that 'no

100 One reason against such a presupposition is that it would be rather coincidental: if one has an infinite variety in size, why then are the different sizes not distributed equally?

101 I cannot take seriously the proposal of Caveing, *Zénon* 37-38, that Zeno thought that by the division as represented in the diagram he had established the existence of an infinite number of points, *in casu* all the pairs of boundary points of adjacent parts, which all must have size.

102 For the Porphyry fragment see Simplicius, *In Physica* 139.27-140.6 (ascribed to Zeno and partly quoted above, in § 4.1.). W.J. Prior, 'Zeno's First Argument Concerning Plurality', *Archiv für Geschichte der Philosophie* 60 (1978) 247-256, at 254-255, seems to presuppose the identity of the two arguments. Makin, 'Zeno' 847, claims that 'it is easy enough to reformulate' our argument into the argument of the Porphyry fragment. Only Barnes, *Presocratic Philosophers* 246, mentions in passing the possibility of a difference between the two arguments.

103 W.E. Abraham, 'The Nature of Zeno's Argument Against Plurality in DK 29 B1', *Phronesis* 17 (1972) 40-52, at 42-43

104 Or at least not forming an infinite series of parts of regularly decreasing size in such a way that the partial sums never exceed a certain limit. For the sake of convenience I assume that in this division through and through each part taken is divided exactly in the middle.

such part of it will be the last': how can ultimate parts – and they alone can be infinite in number if they are all to be equal – fail to be last parts? And surely also the phrase 'nor will a part not be related to another' refers to the relation of a part of stage *n* to a part of stage *n + 1*.[105] But for ultimate parts this is not true.

§ 5.1.2. Saving a paradox for Zeno

So we seem to have reached deadlock. Either Zeno did commit a blatant fallacy or we must assume that he was rather careless in his choice of words. How to escape from this dilemma? A first clue can be found in what both accounts have in common: disagreeing about the justification for (2), they both take Zeno to argue along the lines of (2)-(4). But if one looks closely at the text, (3) is nowhere to be found. We do find (a), but (a) is not used in order to ascribe size to all of the infinite number of parts. Rather, in his actual words Zeno applies (a) separately to each front part taken, in order to argue that it itself still has size and therefore a part jutting out. So (a) is used to ground the ever-divisibility of magnitudes: no division can yield a part which is sizeless and therefore partless. (Zeno thus relies on the unstated, but already familiar, premiss that everything with size has parts.)

Also given that an interpretation which somehow commits Zeno to there being ultimate parts, does not fit his statement that there is no final part, we must conclude that the only reconstruction of the division procedure compatible with this use of (a) is the one represented in the diagram, which yields an infinite series of ever-decreasing parts. So instead of (2) we have the more specific:

(2*) There is an infinite series of unlimitedly decreasing parts

Proposition (a) then serves as a premiss in the derivation of (2*), whereas (2*) alone grounds Zeno's conclusion that each of the many things is so large as to be unlimited.

Immediately, however, two problems present themselves. Firstly, does Simplicius not suggest the argument (2)-(4) in his summary? And secondly, how can (2*) alone be enough to conclude (4)?

As far as the first objection is concerned, it all depends on how one takes μέγεθος ἔχει ἕκαστον τῶν πολλῶν καὶ ἀπείρων: does it mean that the many lack a limit in a numerical sense or in a spatial one? Usually the former is assumed,[106] so that the reasoning (2)-(4) becomes the only viable reading of the summary. There is, however, a problem with it. For thus the number of the many, first given as just 'many', is subsequently specified as 'unlimited'. Then the many can only be the parts generated by the division procedure described in Zeno's actual words. But that fails to fit those words at two places. First, Zeno starts with: 'If it is, *each* must have some magnitude ..', that is, each of the many, and in this place certainly not each of the (infinitely many) parts generated in the division procedure. And second, in the conclusion it is stated that the many things are large so as to be unlimited. But that is a conclusion Zeno is not entitled to if his many things are the parts generated in the division procedure, for it is the wholes of those parts, not the parts themselves, which allegedly are so large as to be unlimited.

105 Contra Abraham, 'The Nature' 43

106 Makin, 'Zeno ' 847

We are not, however, obliged to read ἀπείρων as referring to the infinite number of the many. I think it can be taken as just short for Zeno's actual conclusion that the many things are *so large as to be* unlimited (μεγάλα .. ὥστε ἄπειρα), with the italicized words left out.[107] And if we read Simplicius' summary in this way we are not forced to adopt the reading that according to him Zeno argued from (2) and (3) together to (4).

But then the question remains, how can (2*) lead us to conclude (4)? The answer must be that it cannot. Rather than charging Zeno with a fallacy, though, we should take this as an indication that (4) is not the right interpretation of Zeno's conclusion that the many are 'large so as to be unlimited.' And it is not difficult to come up with a different reading of this phrase: 'the many are so large as *not to have a limit*.'

This proposal is not new; more than half a century ago Fränkel already made the same suggestion.[108] However, the way in which he subsequently reconstructs Zeno's reasoning from (2*) to his conclusion does not seem correct. For Fränkel takes what one might call the constructivist approach: according to him the point of Zeno's introduction of a division procedure yielding an infinite series of unlimitedly decreasing parts is that such a division cannot be completed, so that in fact the series as a whole will never be there, for lack of a last step to finish its construction.[109] The problem with this approach is again that it does not fit Zeno's actual words. For they suggest the following line of reasoning: each part has size, so that it *has* parts itself, of which the front part has size, so that it again *has* parts etc.; therefore there *is* no last part, but each part taken *has* a sub-part; therefore the whole of all these parts is large so as to be unlimited. Thus Zeno assumes he shows by argument all the parts of the infinite series to be actually there: although a construction which is never completed, is used in the proof to describe the series, the series is already there in reality.

Yet Fränkel was definitely on the right track. If one severs his suggestion from the constructivist interpretation, and understands it in the context of an interpretation according to which all the parts of the infinite sequence exist, it provides one with a simple and clear account of Zeno's reasoning. The key idea here is again that a whole is nothing more than its parts, so that everything which can be said about the whole, must be reducible to what can be said of the parts, either collectively, or even to some part

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107 It is with this interpretation in mind that one might be tempted to adopt the emendation of ἀπείρων into ἄπειρον, so that it corresponds to ἕκσστον. (This is proposed by Fränkel, 'Attacks', at 111 and 134 n.45.)

108 Fränkel, 'Attacks' 119-120. He tries to strengthen his case by pointing to passages in Philolaus (DK 44 B11) and Plato (*Philebus* 23cff.), where ἄπειρος means something like 'indeterminate', and to Aristotle's conception of τὸ ἄπειρον. But 'indeterminate' does not mean the same as 'without limit', and Aristotle presented his conception self-consciously as an innovation. One should instead point to a passage in Plato's *Parmenides* 165a5-c5 (quoted in full in note 151), where Parmenides concludes of something which he has argued 'not to have a limit', that it is unlimited, even contrasting 'having a limit' and 'unlimited'.

109 Fränkel, 'Attacks' 119-120, conceives of the argument in terms of 'measuring the thickness of an object', 'reaching the ultimate surface' and 'covering the whole distance'. He explicitly uses Aristotelian vocabulary when he says that the object constructed thus does not have 'an actual and existing plane surface,' yet 'still has a potential limit for its extension.' The same constructivist approach is taken by McKirahan, 'Zeno', 139. Quite interestingly, he does not adopt Fränkel's translation of the conclusion, but reads it as saying that each of the many is so large as to have an unlimited number of parts. I suspect that he thus avoids inconsistency, for saying that an object may have a potential limit, but not an actual one, is not only vague, but also seems to rule out that this object is unlimited in the sense of not having a limit. The trouble for McKirahan is that he is forced to take the predicate ἄπειρον, applied in the conclusion to each of the many, in a numerical sense. This seems in the context doubtful Greek and, more importantly, would destroy the parallel between the conclusion of this half of the paradox and that of the other half, which says that each of the many is small so as not to possess any *magnitude*.

separately. Now Zeno has shown that the whole is divided into parts which form an infinite sequence, as represented in the diagram above. Moreover, these parts exhaust the whole, for because of (a) a part without size does not exist. Therefore the whole must lack something which it, on a normal conception of an entity, does have: a limit. For the front-limit is neither one of the parts, nor does it belong to any of the parts, since the only part to which it could have belonged is the last part – which does not exist.

§ 5.1.3. Objections and answers

This reconstruction of Zeno's reasoning does not only furnish him with a plausible argument, but also seems to me to be the only one which does justice to his use of (a) and the actualistic language suggesting that all the parts of the unlimited series already exist. Moreover, we are already thoroughly familiar with the crucial unstated premiss that a whole is a mere sum of its parts. Nevertheless I do not think that we have as yet fully understood Zeno's reasoning and the ideas behind it. In order to come to a fuller appreciation, I will discuss two objections to the account I have offered, the second of which grows out of my answer to the first.

As a first objection one might protest that if I am right, all the ancient commentators, including Simplicius, our reporter for this paradox, were wrong, since they interpreted Zeno as arguing that each of the many is infinitely large.[110] And if the answer is going to be that Zeno apparently thought that something without a limit is unlimited in the sense of infinitely large, would Zeno not be guilty of trading on a rather obvious ambiguity? For why could Zeno's opponent not just accept the lack of a limit to each of the many, without committing himself to their infinity.[111]

I do indeed think that Zeno argued from *being without limit* to *being infinitely large*. And I want to stress that there is nothing surprising in that. For how else but by having a limit is something going to be limited? Moreover, to good sense I can add authority. Aristotle in a passage from his *Physica* says:

> If the definition of *body* is *that which is bounded by a plane* (τὸ ἐπιπέδῳ ὡρισμένον), there cannot be an unlimited body (σῶμα ἄπειρον).[112]

Though strictly speaking the inference here is from the presence of a limit to the finitude of a body, we are allowed to draw the conclusion that everything which is infinite is so because it is not bounded by a plane, that is, because of lack of a limit.[113]

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[110] For Simplicius, the most explicit pasage is: *In Physica* 141.15-16; for the objection cf. Furley, *Two Studies* 68.

[111] Cf. Furley, *Two Studies* 68-69

[112] *Physica* 3.5; 204b5-6

[113] A more immediate parallel for Zeno's inference in antiquity can be found in Plutarch, in his *De communibus notitiis adversus Stoicos*, where he states the following objection against Stoic ideas about limits:

> [I]t is against the common conception that there be no extremity in the nature of bodies, nor anything first or last at which the body terminates, but that the subject, by always making an appearance beyond what has been taken, be reduced to infinity and indefinitude (ἀεί τοῦ ληφθέντος ἐπέκεινα φαινόμενον εἰς ἄπειρον καὶ ἀόριστον ἐμβάλλειν τὸ ὑποκείμενον). For it will not be possible to conceive of one magnitude being larger or smaller than another, if it belongs to both alike to proceed with their parts to infinity (εἰ τὸ προϊέναι τοῖς μέρεσιν ἐπ' ἄπειρον ἀμφοτέροις ὡσ<αὔτως> συμβέβηκεν). (1078e-f)

(I should note that I leave out <τι> added before τοῦ ληφθέντος in several editions, among them H. Cherniss (ed. & transl.), *Plutarch's Moralia* XIII (Cambridge, Mass., and London, 1976) 812.) However one interprets this

What is more, even a consideration of modern set-theoretical representations of lengths of objects leads to the same conclusion. For however one represents the same length, by an open interval on the real number <a, b> or a closed interval [a, b] (or a half-open interval <a, b] or [a, b>), there is no denial that it has limits: on all possible representations the points indicated by the real number a and b.

On the other hand, the set-theoretical representation shows us that there is a distinction to be drawn. For though it does not matter for the presence of a limit whether a length is represented by a closed or open interval, there is still a difference between the two representations: on the one the limit is actually a separate part of the interval, on the other it is not. So one should distinguish between having a limit and having a limit as separate part. Thus if Zeno is to be guilty of anything, it is arguing from not having a limit as separate part to not having a limit at all.

So there we have the second objection: why does Zeno assume that if a limit is going to be possessed by a body, it must be a part of that body? Should it not be an abstract entity rather than a separate part of the limited body? We could even state this objection while staying close to an idea we ascribed to Zeno himself. In § 3.4, it was explained that for Zeno the properties of a whole are reducible to the properties of the parts taken separately or together. Now *having a limit* is thus reducible. For do not all the parts, by having the arrangement they have (see the figure on p. 34) *together* have a limit?

I do believe that ultimately Zeno is unable to defend himself against this second objection. Despite that there is something to be said on his behalf. For the objection presupposes that one already has a conception of a limit alternative to Zeno's conception as a separate part of a body. But how does one arrive at such a conception? In what follows I shall show that both ways of answering this question are unacceptable to Zeno.

According to the first alternative conception of limits, a limit is where something happens, e.g. where two entities touch, or where an entity stops (in case the limit is between an entity and a stretch of void). Thus a limit may be, for example, the plane which has one thing on one side of it and another thing on the other side. In itself, however, the limit is not a part of and does not belong to either of the things between which it is placed; rather it is something independent. This independence also manifests itself in the fact that one can imagine a limit where nothing really happens, e.g. in the middle of a homogeneous entity which has merely conceptual parts. A limit thus conceived is more of a geometrical than of a physical nature, that is, its conceptual home is more the space in which entities are than the entities themselves.[114]

One can indeed ascribe a limit to the whole consisting of the series of parts if one adopts this conception of limits: there is a line to the left of which there are only parts of that whole and to the right of which there are none. However, one can doubt whether Zeno would be willing to adopt this conception, since in his paradox of place he makes trouble for a similar kind of entitity:

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rather obscure passage, it seems reasonably clear that Plutarch holds the mere absence of a limit entity to be responsible for the infinity of a magnitude. Cf. also Plato, *Parmenides* 165a5-c5, quoted in note 151.

[114] For this conception, cf. A. Stroll, *Surfaces* (Minneapolis, 1988) 40-46, and again 47-50. One of the reasons why limits of this kind are not part of bodies is that even a stretch of void does have a limit on this conception – and *prima facie* void is nothing more than space not occupied by a body.

> Zeno raised a puzzle: 'If place is something, in what will it be?'[115]
> [I]f [place] is one of the things which are, where will it be? For Zeno's puzzle requires some account. For if everything which is, is in a place, it is clear there will also be a place of place, and so on *ad infinitum*.[116]

This puzzle depends completely on the stated premiss that whatever is, is in a place. If Zeno takes this proposition seriously – and how else could he be thinking to raise a puzzle? –, he should be reluctant to countenance limits as places where something happens or can happen to entities, places which have independent existence and are not parts of entities.

As Zeno in the paradox of plurality itself assumes that a limit must be a separate, that is, independent part of a body, and in the paradox of place rules out limits as independently existing while not being a part of a body, there is only one conception of limits left to try if we want to escape from Zeno's conclusion that each of the many is so large as to be unlimited: limits as dependent parts of a body. And this seems anyway intuitively a more natural conception. For what else but the limiting surface of a body do we touch, when we touch the body? And does not the limiting surface of a body move around with the body itself?

The way to arrive at this second alternative conception of a limit consists in a kind of limiting procedure: the limit of a whole is conceived through an infinitely converging series of nested parts of the whole – a procedure closely related to the one adopted by Zeno in the paradox of plurality.[117] Now in order to turn the limit thus arrived at into an abstract object belonging to and dependent on the whole, the limit should be identified with the whole converging series, so that it is not an independently existing entity *found* at the end of the series.[118]

This last point, however, Zeno is bound to reject. As he takes each part to be prior to the part coming before it in the series, for him the limit reached by this limiting procedure is the ultimate part. (We are already familiar with this line of reasoning from the argument for the sizelessness of each of the many.) And ultimate parts are by their nature separate and independent parts. So because Zeno takes every kind of part to be governed by his priority principle, he does not have a place for limits which are dependent parts of a whole.

By discussing these two alternative conceptions of limits we have acquired a better understanding as to how Zeno arrived at his conclusion that since there is no final part of the series, there is no limit to the whole made up of the series of parts, and therefore no limit to the whole. But of course, showing that Zeno would not accept any of these alternative conceptions does not mean that we have to agree with him. For does Zeno not, by this very argument with its paradoxical conclusion, show that the conception of limits as separate parts of a whole is to be rejected as well? And then, are the reasons I ascribed to him for the dismissal of the alternative conceptions really convincing?

It may seem that by introducing spatial limits, existing independently from the bodies in space, or by distinguishing between conceptual and physical parts, so that

115 Aristotle, *Physica* 4.3; 210b22-23 = DK 29 A24

116 Aristotle, *Physica* 4.1; 209a23-25 = DK 29 A24

117 Cf. Stroll, *Surfaces* 46-50.

118 This conception may be called Whiteheadian – for an exposition, see D.W. Zimmerman, 'Could Extended Objects Be Made Out of Simple Parts? An Argument for "Atomless Gunk"', *Philosophy and Phenomenological Research* 56 (1996) 1-29, at 17-19, and especially his 'Indivisible Parts and Extended Objects: Some Philosophical Episodes from Topology's Prehistory', *The Monist* 79 (1996) 148-180, at 160-165.

there is room for limits which are dependent parts of a whole, we can escape from Zeno's paradoxical conclusion.[119] In itself this impression is true enough, but things are a bit more complicated than that. For it is possible to recast the basic idea behind Zeno's argument in this part of the paradox of plurality in such a way that we still have a paradoxical conclusion, but now without the escape by way of spatial limits or limits as dependent parts. Consider the following example. Suppose we construct a cube out of layers of clay of two colours, blue and yellow, in such a way that we first have a blue layer with the thickness of 1/2 unit, then a yellow layer of 1/4 unit, then blue of 1/8 unit, and so on. Suppose then that the construction is finished:[120] looking at the limiting surface we were working towards, what colour do we see?[121] Of course this question cannot be answered without assuming there to be a limit which somehow exists as a separate part. Yet it seems possible to pick up the whole cube consisting of these layers of clay, to walk around it and therefore also to look at it from every angle. Should we reject this last intuition in order to safeguard our rejection of limits as separate parts?

This argument suggests, I think, that we need not only ascribe to Zeno negative reasons for his idea that in order to have a limit an entity needs a limit as a separate part. It may be that he was positively guided by the intuitions behind this argument to the idea that limits must be separate parts. In one respect this is rather obvious, for what else is the effect of taking two different colours of clay for the alternating parts making up the cube than that the parts are preserved as the ontologically basic entities, prior to the whole cube made up of them?[122] Thus Zeno's priority principle is enshrined in this argument. But also in another respect this argument may capture an intuition shaping Zeno's ideas. For the image of a finite body as a body one can go around, and on every side of which one can be, and thence see its surface, may have worked in Zeno's mind to produce the idea that if there is no such surface to be seen, the body must be infinite. And there is the merest hint that there was something like that image in his mind. For the use of the preposition 'in front of' in the formulation of the paradox, both in the phrase 'in front of what is being taken' and in the verb 'jut out' (προύχειν), may suggest that we should look at the entity from the side towards which we are working in the division (indicated as 'front' in the figure on p. 34).[123]

Be that as it may, even if the above is mere speculation on my part, it remains the case that this recasting of Zeno's argument should make us think harder about the intuitions at work in our understanding of limits. I for one would as yet not really know

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119 These seem the only two ways of escaping Zeno's conclusion. Particularly, the difference between a closed and an open interval on the reals does not have any bearing on the present issue. The mathematical representation of entities by way of intervals seems to me to combine features of the idea of limits as separate parts (a closed interval contains its limit as a separate part) and limits as existing independently (an open interval is limited by a limit which does not belong to it). In other words, it is a kind of hybrid, and therefore can be ignored for the present enterprise.

120 *We* may not be able to finish such a construction, but why could not reality do the job for us? For why should it be impossible that there are all these parts and that they arrange themselves in this particular way?

121 The example I owe to Nicholas Denyer. It is related to examples like the Thompson-lamp (switching on and off for ever shorter periods of time, in a Zenonian way) which were used to discuss Zeno's Runner paradox. For the difference, see below, note 146.

122 And this priority principle suggests itself anyway in the context of this argument, for surely we do not see the *whole* entity if we watch it from one side? So what do we see then? This part? No, not as a whole. *Etcetera*. In the case of a homogeneously coloured entity, however, one could still try to insist that one can see the *whole* cube without seeing *all* of it.

123 By contrast, the direction suggested by the language in the fragment quoted from Plutarch in note 113: 'beyond' is the opposite one, from 'inside' the entity towards the side we are working towards in the division.

where to start in order to avoid the paradox of the two-coloured cube. That in turn should induce us to have more respect for Zeno's paradox of plurality.

§ 5.2. The Runner paradox

Zeno's argument against motion which is known as the Runner paradox or as the Dichotomy, is structurally very similar to the part of his first paradox of plurality discussed in the preceding section, as it too features a recursive division-procedure generating an endless series of entities. But was his argument the same? In this section I am going to argue that it was. As I go thus against the general tendency of the literature, I shall finish my account by relating it to other interpretations.

§ 5.2.1. What was Zeno's argument?

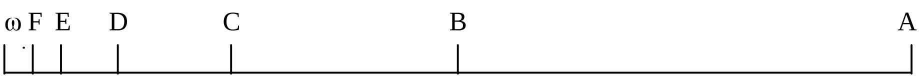
Aristotle offers the following account of Zeno's reasoning:

> There are four arguments by Zeno on motion which give difficulties to those who try to solve them. The first is the one according to which there is no moving, on the grounds that the thing in locomotion has to reach the half before it gets to the end. We have dealt with this one in the discussion above.[124]

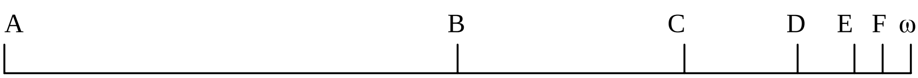
There Aristotle is referring to the following passage, also from the *Physics*:

> That is why Zeno's argument makes the false assumption that it is not possible to traverse unlimited things or to touch unlimited things singly (τῶν ἀπείρων καθ’ ἕκαστον) over a limited time. For both length and time, and in general everything continuous, are called 'unlimited' in two ways: either according to division or by their limits. The things then which are unlimited according to quantity it is not possible to touch over a limited time, but the things which are so according to division it is possible to touch in that way. For time too is itself unlimited in this way. Hence traversing what is unlimited occurs over an unlimited and not over a limited time, and the touching of unlimited things [occurs] with unlimited, not with limited things.[125]

From these two passages it is not immediately clear what construction Aristotle ascribes to Zeno: must the runner, in order to traverse a distance, first traverse half the distance, and of that half first a half of it, and so on, or must he first traverse half the distance and then half of the remaining distance? That is, is the picture Aristotle has in mind this one:



where the runner starts at ω, or this one:



124 *Physica* 6.9; 239b9-14 = DK 29 A25

125 *Physica* 6.2; 233a21-31 = DK 29 A25

where the runner starts at A?

The commentators on Aristotle opted, as far as we can determine, for the former alternative.[126] On that construction one would expect Zeno's conclusion of the impossibility of motion to be somehow derivable from the fact that there is no *first* sub-run for the runner to make, no first sub-distance to traverse. From Aristotle's testimony about the second of Zeno's paradoxes of motion, the Achilles, however, it is clear that Aristotle had the latter construction in mind:

> [The Achilles is the argument] that the slowest one will never be caught up with (καταληφθήσεται) while running by the fastest. For the pursuer has first to go whence the pursued one started, so that the slower one must always be somewhat ahead. This is the same argument as the one by dichotomy, though it differs in that the magnitude taken in addition is not cut into two (ἐν τῷ διαιρεῖν μὴ δίχα τὸ προσλαμβανόμενον μέγεθος). Though the conclusion on the basis of the argument is that the slower one is not caught up with, it depends on the same point as the dichotomy (for in both arguments it follows that there is no arrival at the limit when the magnitude is divided in a certain way; but in this one there is the addition that even the fastest one is represented dramatically as failing in the pursuit of the slowest), so that the solution must be the same as well.[127]

Aristotle's statement that the difference between the two arguments is that 'the magnitude taken in addition is not cut into two' would be quite misleading if there were no 'magnitude taken in addition' in the case of the Runner too. Moreover, in *Physica* 8.8, Aristotle mentions a counting-version of the Runner:

> [S]ome present the same argument by way of different questions, asssuming that together with the moving one should count first the half-motion, with each half coming up, so that with having traversed the whole motion it follows that one has counted an infinite number. But that, as is agreed, is impossible.[128]

If this is just a different version of the same argument – and Aristotle presents it thus –, then the original version cannot have the regressive construction of the first figure, for then the notion of a whole motion gone through does not make any sense.[129]

Another reason for quoting these passages in full is that they constitute the only evidence we have on which to base a reconstruction of Zeno's reasoning accompanying

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126 See Simplicius, *In Physica* 1013.7-10 and Philoponus, *In Physica* 81.8-12; cf. Sextus Empiricus, *PH* 3.76 and *AM* 10.139-141.

127 *Physica* 6.9; 239b14-26 = DK 29 A26

128 263a7-11. DK, unlike Lee, *Zeno* 46-49, rightly omit this passage as a fragment, since Aristotle clearly implies that it is not Zeno's. That it is impossible to complete counting an infinite number is stated by Aristotle in *Physica* 3.5; 204b7-10:

> Nor is there number in such a way that it is separated and unlimited. For number or that which has number, is countable; if it is then possible to count the countable, it would also be possible to traverse the unlimited.'

And that Aristotle apparently takes to be self-evidently impossible.

129 By leaving it at this, I am in disagreement with G. Vlastos, 'Zeno's Race Course. With an Appendix on the Achilles', in: *Studies* 189-204, at 190 [originally: *Journal of the History of Philosophy* 4 (1966) 95-108, at 95], whose main reason for adopting the progressive version of the second figure is that Aristotle's reference to the impossibility of traversal over a limited period would be pointless on the regressive version. But who says that the period of time should be unlimited in the forward rather than the backward direction? Vlastos (note 7) thinks that one can only express this situation if the period of time is unlimited in the backward direction, by assuming the run to be completed and describing everything from that perspective, in the perfect tense. I do not see why one needs to do so. Why can one not use aorist-forms, as Aristotle does, to state that in order to move, the runner needs an unlimited period?

the construction of the run. And at first sight it may be difficult enough to arrive at such a reconstruction, for Aristotle has in fact provided us with three different accounts. In the following, however, I shall argue that with some twists we can trust Aristotle's statement that we merely have three different versions of the same argument.

To start with the most extensive report, that from *Physica* 6.2, Aristotle's diagnosis suggests that Zeno thought that a runner going from A to ω passed over an unlimited number of stretches and needed for that an unlimited amount of time. It may be possible, however, to be more precise, for the passage contains three obscure points which, when clarified, may provide clues to Zeno's actual words.

The first of these is that whereas the reference of the phrases 'the things which are unlimited according to quantity' and 'the things which are so according to division' in 233a26-28 is the plurality of things *each* of which is unlimited in one of these two ways, the 'unlimited things' traversed or touched in 233a22-23 must be the unlimited *number* of parts of each of the unlimited things in the previous sense, as is indicated by 'singly'.[130] It is quite remarkable that with the exception of the last line of the quotation (on which I shall have more to say below), this latter sense of 'unlimited things' does not occur elsewhere in *Physica* 6.2. The best explanation I can offer for this discrepancy is Aristotle's single-minded interest in proportions in this passage, which causes him to neglect the point that Zeno is talking about an unlimited *number* (or series) of stretches rather than of unlimited magnitudes – for more on this, see Chapter Three. But then it seems likely that the phrase 'unlimited things' goes back to Zeno himself.

The second difficulty concerns the distinction between 'traversing unlimited things' and 'touching unlimited things'. Is there a real difference? It seems not. As the vocabulary of 'touching' is even more alien to the context of Aristotle's solution, which is completely dominated by the verb 'to traverse', one may conjecture that Zeno himself employed that vocabulary.

This may be reinforced by an elucidation of the third unclarity. What does Aristotle mean when he writes: 'the touching of unlimited things [occurs] with unlimited, not with limited things' (233a30-31)? If one takes a generalizing reading of 'unlimited things', according to which that phrase refers to things each of which is unlimited, the (un)limited things with which these unlimited things are touched must be the (un)limited periods of time needed to complete a run. That reading, however, is made unlikely by the shift from 'traversing the unlimited' (233a29-30) to 'the touching of unlimited things', for then one would expect a similar generalizing plural 'unlimited things' as the internal object of 'traversing' as well. Therefore 'unlimited things' touched refers to the unlimited series of parts of each whole run, and the (un)limited things with which they are touched are probably – what else? – the periods of time together making up the whole period needed for the whole run, that is, the series of those periods.

Thus once again we see a discrepancy between the singular 'the unlimited' in Aristotle's discussion and the plural 'unlimited things', now within one sentence. And this discrepancy coincides with a shift from the vocabulary of traversal to that of touching as well as with the introduction of a turn of phrase, touching 'with (un)limited things', which has something strange about it. The best explanation, I venture, is that these three elements all go back to Zeno himself.

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130 Cf., more explicitly, pseudo-Aristotle, *De lineis insecabilibus* 968a20-21: 'to touch over a limited time unlimited things, touching them singly'.

The conclusion, then, of this close reading of the text is that Zeno argued that in order to finish the run from A to ω, the runner had to touch an unlimited number of things one by one, and that, apparently because it was impossible to touch an unlimited number of things with an unlimited number of periods, the runner should, in order to finish the run, touch these unlimited things over a limited number of periods (presumably all of finite length), but that he actually needed an unlimited number of periods, one corresponding to each sub-run. Strictly speaking this is not the same argument as suggested by Aristotle's diagnosis, for there is at least a difference in formulation between needing an (un)limited amount of time and needing an (un)limited number of periods. One can nevertheless see how to get from the latter to the former: Zeno's requirement that in order to finish the run, the runner should do so over a limited number of periods, can be understood as saying that he should do so over a limited period of time. Similarly his apparent assumption that it is impossible to touch with an unlimited number of things can be interpreted as saying that it is impossible to do so over an unlimited period of time.

If we may again believe Aristotle, Zeno did indeed make this final step of equating the unlimitedness of the number of sub-periods with the unlimitedness of the whole period. For in *Physica* 8.8 he has this to say:

> In the first discussions about motion we tried to solve [the Runner paradox] on the basis of the point that the time has unlimited things in itself. For it is not absurd at all if over an unlimited time someone traverses unlimited things; and the unlimited is similarly present in both length and time. However, though this solution is sufficient with regard to the questioner (for it was asked whether it is possible to traverse or to count unlimited things over a limited time), with regard to the matter and the truth it is not sufficient. For if someone leaves the length aside and refrains from asking whether it is possible to traverse unlimited things in a limited time, but makes these inquiries in the case of the time itself (for the time has unlimited divisions), that solution will not be sufficient any more.[131]

Since Aristotle in the first line clearly refers back to the passage from *Physica* 6.2, we may identify Zeno with the questioner. That means that Aristotle thinks that Zeno confused the two ways in which something can be unlimited and that he assumed that touching with an unlimited series of sub-periods of time implies touching over an unlimited time.[132]

With regard to the counting-version of the Runner paradox as presented by Aristotle in *Physica* 8.8, it is not immediately obvious how to assess Aristotle's claim that it concerns the same argument. The reason is that there might very well be two counting-versions of the Runner. For, in the version given in 263a7-11, Aristotle declares it to be impossible without qualification to count an unlimited number, while in the allusion to the counting-version in 263a16-17 the impossibility seems to concern the counting of unlimited things *over a limited time*. And if one looks at the issue in the context of Aristotle's discussion as a whole in *Physica* 8.8, it becomes even more likely that there are two versions. Firstly, there is the same difference between Aristotle's summary of the original version of the Runner, the one with the runs, where the crucial premiss is

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[131] 263a11-22

[132] It is important to notice that it is this step Aristotle's distinction is meant to invalidate, and not some alleged failure on Zeno's part 'to note the complete parallelism in [respect of the way they are infinite] between the nature of space and that of time' (Ross, Physics 73). He is rather all too aware of this parallelism and uses it to establish the unlimited nature of the time. (As we shall see below, this parallelism is also an indispensable premiss to Zeno's argument in the Achilles.)

merely that 'it is impossible to traverse unlimited things',[133] on the one hand, and the allusion to it in 263a16-17, quoted above, on the other. Secondly, the solution to the Runner paradox which Aristotle does consider sufficient 'with regard to the matter and the truth', obviously does not apply to a puzzling argument with the premiss that it is impossible to traverse unlimited things *over a limited time*, as the puzzle is stated in terms of the periods of time itself. It can, however, apply to a version of the Runner which does not feature such a qualification, because such a version, either in the original Runner-form or in the counting-form, runs exactly parallel to the puzzle stated in terms of the periods of time itself. And that is what Aristotle says, both when he introduces the Runner paradox without the 'over a limited time'-qualification, and when he mentions 'these inquiries in the case of the time itself'. In both cases he refers to the discussion immediately preceding it. In 263a4-5 he does so by saying: 'In the same way one should respond also to those who present Zeno's argument in the form of questions', while in 263a22-23 he states that for the real solution 'one must state the truth which we set forth in the recent discussions.' Because he is completely focused on the deeper problem, requiring a real solution, Aristotle does not bother about the clause added by Zeno in his crucial premiss, and alludes to it only in passing.[134]

So both the counting-version and the original version of the Runner as alluded to in 263a16-17, can be equated with the argument ascribed to Zeno in *Physica* 6.2. In both of them the crucial premiss is, just as in *Physica* 6.2, that it is not possible to go through unlimited things *over a limited time*. And both the counting-version and the original version of the Runner as presented in 263a4-11, leaving out the qualification 'over a limited time', differ from the argument of *Physica* 6.2. They both are based on a premiss which differs from the one explicitly ascribed to Zeno.

The Achilles was the last argument claimed by Aristotle to be identical to Zeno's Runner. Unfortunately we do not hear much about an argument; Aristotle only reports about a construction with two runners. But *prima facie* there is a problem with identifying the Runner with the Achilles, as the conclusion that the fastest will never catch up with the slowest seems immediately derivable from the fact that for every run completed by the fastest there is a run by which the slowest is ahead, without having recourse to a premiss saying that it is impossible to touch unlimited things over a limited time.[135]

It is clear that the constructions differ. In the case of the Runner there is a set distance. First this distance is divided in the Zenonian way, and then the resulting parts are correlated with periods of time, whose number as a consequence is unlimited. In the Achilles, on the other hand, there is no such set distance; rather the runs are generated one by one without taking them from a pre-existing whole. In order to do so, Zeno does not set up a one-way correspondence from sub-distances to periods of time, but employs such a correspondence in both directions. Thus, starting with the distance d_1 as the distance by which the slowest is ahead, the fastest needs time T_1 to traverse d_1. In the same T_1 the slowest traverses distance d_2, by which he is ahead at the end of T_1. To

133 This summary can be found in *Physica* 8.8; 263a5-6:

> .. Zeno's argument, [that] .. one must always traverse the half, and that these [halves] are unlimited, and that it is impossible to traverse the unlimited [things].

134 The above is a somewhat more carefully worked out version of the discussion in Ross, Physics 73.

135 This comes close to Vlastos' statement of the 'distinctive logical structure of the Achilles' in his 'Race Course' 202-203 [104-106].

traverse d_2 the fastest takes T_2, and so forth. The reasoning is: d_1 corresponds to T_1, T_1 (also) to d_2, d_2 to T_2, and so forth.

The effect is that the construction, by assuming a two-way correspondence between distances and periods (embodied by two runners, one for each direction of correspondence), provides a kind of existence-proof for an endless series of runs and periods: for every distance d_x there is a further d_y to be traversed, and for every period T_x there is a further T_y during which to move. There we have in the Achilles the unlimited number of sub-distances and periods we are familiar with from the Runner.

The only possible difference left between the Runner and the Achilles concerns the inference Zeno makes in the Runner from the unlimited number of the periods to the unlimitedness of the whole period. Because of the paucity of evidence one may despair of finding anything similar in the case of the Achilles, but perhaps there is something to go on. For Aristotle formulates Zeno's conclusion as:

(c) The slower one is always something ahead.

This is ambiguous as between:

(c_1) During every period of the endless series of periods the slower is ahead

and:

(c_2) During the whole (period) of time the slower is ahead.

Not only is this ambiguity as between a numerical sense and an extensive sense of 'always' very similar to the ambiguity Aristotle finds in Zeno's use of 'unlimited', but it is also the case that the former ambiguity must be explained in terms of the latter. For the distinction between the two senses of 'always' turns on the distinction between two ways in which stretches of time may be called (un)limited. In the case of (c_1) the number of periods quantified over may be unlimited, but they are taken from a delimited stretch of time. In the case of (c_2), on the other hand, the number of periods quantified over may be limited, but they are taken from an unlimited stretch of time.

If this indeed goes back to Zeno, we have in the Achilles the same argument, though not exactly the same construction, as in the Runner. Thus the evidence provided by Aristotle allows us to conclude that in the Runner as well as in the Achilles Zeno established first through a construction – under the assumption of a correspondence between distances and periods -, that there is an unlimited series of periods during which there is motion, and then concluded that because this series is unlimited, the whole made up of the parts from the series is unlimited.

§ 5.2.2. Time again

It will be clear that, on my construal of Zeno's reasoning in the Runner, the structure of the argument is identical to that of the paradox of plurality discussed in § 5.1. It may therefore seem that in order to explain Zeno's step from the unlimitedness of the series of periods to the unlimitedness of the whole made up of the parts from the series, I can confine myself to referring to that section. Though for the general outline that is true, on some points the explanation will differ. Moreover, no paradox of Zeno has received such close scrutiny from a systematic, as opposed to a historical, perspective, as the Runner has; it will be useful to give at least some indication as to where to situate my account in that debate.

§ 5.2.2.1. Limits in time

To remind ourselves: Zeno's argument in his first paradox of plurality was that (i) because there are no sizeless entities, there is a series of parts (ii) generated by a recursive division procedure and (iii) together exhausting the whole, which (iv) lack a final part; the series is therefore endless, so that (v) the whole made up of the parts of the series is unlimited in size. In the argument of the Runner as reconstructed so far we clearly have (ii), (iv) and (v). We may also assume (iii), because there is little point in drawing a conclusion about the whole if the series on which the conclusion is based does not exhaust the whole. What we do not have is (i), but its absence can be readily explained from the formulation of the Runner. For the function of (i) – (a) in § 5.1 – is to ensure that a division does not result in two parts one of which is a mere point. In the Runner this possibility is defined away as each remaining half is divided into two *equal* parts. (Similarly in the Achilles it is understood that a distance requires a period to traverse it, and *vice versa*.)

The main difference between the paradox of plurality and the Runner is that for the latter we cannot simply repeat the discussion of § 5.1.3 about the different ways an entity can be limited, in order to understand Zeno's step from (iv) to (v). It is clear that the series of periods needed to complete the run does not have a final part and therefore does not have a limit as an independent part, and it is also clear that the periods together cannot have a limit as a dependent, abstract 'part', since the way to arrive at the conception of such a limit, a limiting procedure, must because of Zeno's priority principle yield an independent part. What is not immediately clear is how to deal with the possibility of a limit as an independent but abstract entity. But on reflection, that does not seem to be so difficult either: with periods of time, one might argue, we do not have the distinction between concrete body and abstract place (time is already abstract enough), so that there is no room for limits which, by being the place where something like ending happens to a period of movement, are independent of that period while not being a part of it.

As a consequence of a small alteration, however, Zeno's argument could become more difficult to explain along these lines. For if we assume that the 'unlimited things' with which the unlimited things are touched are not the unlimited series of periods, but the unlimited series of runs, then there is room for an independent, but abstract limit: a moment in time. (And it should be noted that such an interpretation cannot be ruled out on the basis of Aristotle's reports, even though he says that Zeno assumed that touching unlimited things in a limited time is impossible. For time may serve as a kind of medium for events, *in casu* the sub-runs and the whole run, and thus as their measure, so that the unlimitedness of the whole run made up of the unlimited series of sub-runs can be expressed in temporal terms.[136]) Now the trouble is that on this picture it is not easy to give an argument on behalf of Zeno for the point that the whole cannot be limited by an independent limit serving as a 'place' where something like ending happens to the run. For does Zeno not himself acknowledge moments in time, in the guise of the now? For suppose we have made a run in the Zenonian way: could Zeno then not imagine that the

[136] Similarly Zeno could have said that a body consisting of an unlimited series of parts takes up an unlimited place.

series of sub-runs is completely in the past, though the now is the limit to the whole run made up of the series?

The answer to this question, I want to argue, should be negative. Zeno does not think about the relation between the past and the present in this way. Rather he thinks of them as different *periods* of time, even though on occasion he also holds that the now is without room for movement and therefore without duration. I have two reasons for ascribing this conception to Zeno. The first is that Plato, in one of his puzzling arguments about being in time in the *Parmenides*, clearly thinks of the past, the present and the future in this way. This appears especially from the following passage:

> Then is it not the case that [the one] stops becoming older just then when it hits upon the now and does not become but rather is then already older? For going on it could never be caught by the now. *For what goes on is in such a state that it touches both, the now and the later, letting off the now while taking a hold of the later, becoming between them,[between] the later and the now.*[137]

One can recognize in this passage a concern with issues similar to those raised by Zeno in the Arrow and the Runner (no movement in the now, the relation between a process and its goal or limit): they are part of Plato's recasting of Zeno's arguments. Apparently Plato thinks that the conception of the now as an independent time between the past and the future was involved in those arguments.[138]

If the lead given by Plato is not enough, there is also another consideration pleading for ascribing the conception of past, present and future as periods standing next to each other: such a conception is part and parcel of the idea, endorsed by Zeno in the Arrow, that it is in the now that everything happens. For if the now were seen as something which merely provided a limit to the past and the future, it would be either an independent or an dependent part of them, so that the claim that everything is always in the now, and not in the past or in the future, comes out false.

That the now is not a limit which limits a period of time, however, does not mean that it cannot be an entity to be found at the end of an unlimited division. Behind Zeno's presupposition that in the now everything is over against what is equal, one may suspect the following argument. Starting with the idea that there are three periods next to each other, the past, the present and the future, one recursively applies the idea that part of the present is not really present: it is in the past or in the future. Now for each stage of this thinning out of the now it remains true that there are three periods. Therefore there is, once one has finished this procedure, a durationless now which is still not a part of either the past or the future; rather it is an independent entity standing between and next to the past and the future. Since it not in any way part of either of them, and is not a limit either in the sense of being a place where the past and the future end (because it stands next to them), it does not serve as a limit to them. Thus it is a limit entity which does not limit anything: it is a quasi-period without duration.[139]

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137 152b6-c7: Ἆρ᾽ οὖν οὐκ ἐπίσχει τότε τοῦ γίγνεσθαι πρεσβύτερον, ἐπειδὰν τῷ νῦν ἐντύχῃ, καὶ οὐ γίγνεται, ἀλλ᾽ ἔστι τότ᾽ ἤδη πρεσβύτερον; προϊὸν γὰρ οὐκ ἄν ποτε ληφθείη ὑπὸ τοῦ νῦν. Τὸ γὰρ προϊὸν οὕτως ἔχει ὡς ἀμφοτέρων ἐφάπτεσθαι, τοῦ τε νῦν καὶ τοῦ ἔπειτα, τοῦ μὲν νῦν ἀφιέμενον, τοῦ δ᾽ ἔπειτα ἐπιλαμβανόμενον, μεταξὺ ἀμφοτέρων γιγνόμενον, τοῦ τε ἔπειτα καὶ τοῦ νῦν.

138 A similar conception seems to underly two earlier passages, 151e6-152a2 and 152b3-6.

139 This argument has become known as the argument for the retrenchability of the now (see e.g. G.E.L. Owen, 'Aristotle on Time', in: P. Machamer and R. Turnbull (eds.), Motion and Time, Space and Matter (Columbus, 1976) 3-27, at 11-12 [also in: G.E.L. Owen, *Logic, Science and Dialectic. Collected Papers in Greek Philosophy*

One might object that this discussion of Zeno's conception of the now is not relevant to the issue we are dealing with here. After all, we do not need the now itself, but merely a moment in time, in order to have a limit to the whole run made up of the unlimited series of sub-runs. Here it is important, however, to realize that this is tantamount to denying the original point that Zeno did acknowledge moments in time in the guise of the now. It may be that he did, but as we saw, not in such a way that moments can serve as limits. Therefore we must ask whether there is a way to conceive of moments in time without invoking primarily the *present* moment.

If we are to answer that question, it must be that the only ways are those already discussed, *viz.* as independent or dependent parts of a period – ways which failed to provide Zeno with a limit. There is therefore no moment in time available to serve as a limit to the whole run comprising of the unlimited series of runs made by the Runner. And as it has already been established that there is no limit to the whole, in either of the alternative ways in which there could be a limit, we may conclude that the whole run is unlimited, that is, takes an unlimited time.

§ 5.2.2.2. *To finish*

It may appear that I have given an account of Zeno's Runner paradox which is completely different from anything available in the literature. On the one hand, by ascribing to Zeno the *direct* step from the unlimitedness of the series to the unlimitedness of the whole that is constituted by the series, I am in disagreement with those who suspect that Zeno may have argued arithmetically that an unlimited number of sub-runs, each of some duration or length, must yield an unlimited whole run.[140] On the other hand, by interpreting it as having for its main premiss that it is impossible to make an unlimited series of sub-runs *in a limited time*, I reject all those accounts according to which the core-premiss is that it is impossible to make an unlimited series of sub-runs, without further qualification.[141]

This appearance is partly correct, but substantially incorrect. It is correct in that I do think that Zeno added the qualification 'in a limited time', and that he did not reason arithmetically. It is also correct in that my account of Zeno's crucial step from the unlimitedness of the series to the unlimitedness of the whole, as provided in the previous sub-section, differs from anything one can encounter in the literature. It is not correct, however, on the point that my account would be wholy unrelated to the issue of the impossibility to make an unlimited number of runs, without further qualification. It is not only intimately connected to that issue, but even explanatory of many of the ideas invoked in analyses of that purported impossibility. What is more, it also allows us to pin-point one issue in particular on which Zeno would disagree with the modern 'solution' to his Runner paradox.

We do not need to discuss here all previous attempts at analysing the alleged impossibility of making an unlimited number of runs, but it is helpful to give a broad classification of them. There are two groups to be distinguished. First, there are those

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(M. Nussbaum (ed.)) (London, 1986) 295-314, at 302-303]. Recently it has been argued by J. Westphal, 'The Retrenchability of "the Present"', *Analysis* 62 (2002) 4-10, that this argument commits the fallacy of composition. In its assumption that a period cannot be present because every period contains several sub-periods only one of which could be properly called 'present', there is hidden the idea that every part of a whole which is *F* must be *F*. For Zeno this would just be another case of his part-whole-principle (WP).

140 Cf. remarks by Vlastos, 'Race Course' 192 [96], and Barnes, *Presocratic Philosophers* 265.

141 Vlastos, 'Race Course' 192-193 [97] and further, and Barnes, *Presocratic Philosophers* 267-272.

analyses which focus on the problem of 'getting there', and base the impossibility on the assumption that the whole run, including its finish, must be described in terms of a discrete ordering; dense orderings are not allowed. One version of this is of course the demand that there be a final sub-run.[142] The other group consists of accounts which focus on 'being there'; according to them one must arrive at some inconsistency once one starts describing the state of completion of such an unlimited series of sub-runs. Of these, the most well-known (perhaps for its simple fallaciousness?) is Thompson's Lamp – suppose a lamp is switched on and off in a Zenonian way: in which state is the lamp at the moment of finish?[143]

My account of Zeno's reasoning is basically of the latter type, as it argues that it is impossible to get there (it takes an unlimited time) because it is impossible to be there (the whole run made up of the series does not have a limit to be on when one has finished). But it also bears features of the analyses of the first group. For according to the assumption that the now is a quasi-period next to the past and the future, there is a discrete order at least among these three parts of time. This discreteness will spill over to the whole of time, as the past and the future are also separate parts of time, having a limit on their own. This limit must be a moment in time and therefore again a (possible) now, separated from its past and future. And so forth.[144]

This argument not only shows that my account has something in common with the first group of analyses. It also explains the assumption of discreteness, by deriving it from intuitions about the nature of time.

While my analysis belongs in its basic features to the second group, it differs from them in important respects. This becomes evident as soon as we examine the most famous of them, the so-called Thompson-lamp. Suppose there is a lamp which can be in either one of two conditions: on or off. During a time-interval between t_0 and t_ω this lamp is switched on and off in a Zenonian way: on at t_0, off at t_1 (half-way between t_0 and t_ω), on at t_2 (half-way between t_1 and t_ω), and so on. In which state then is the lamp at t_ω? According to Thompson, it cannot be in either state: 'It cannot be on, because I did not ever turn it on without at once turning it off. It cannot be off, because I did in the first place turn it on, and thereafter I never turned it off without at once turning it on.'[145] Disregarding the obvious fallacy which is being committed here (that the lamp is switched on/off and then always is switched off/on, is only true before t_ω; there is no specification which gives us the state at t_ω), the difference between my account and the Thompson-lamp is that I do not wonder in which state the Runner is at t_ω, but whether it is possible at all that there is a time t_ω serving as a limit to the period of the run.[146]

Now it is just on this point that my account collides with the modern solution to Zeno's Runner paradox. This modern solution is presented in different guises.

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142 Vlastos, 'Race Course' 199-200 [103], and Grünbaum, *Modern Science* 37-39, 74 and 76-77.

143 J.F. Thompson, 'Tasks and Super-Tasks', *Analysis* 15 (1954) 1-13, at 5. For an overview of similar arguments, see Grünbaum, *Modern Science* 78-82.

144 If one is not satisfied with this result, because it does not involve a discrete system of periods rather than quasi-periods, one merely needs to blow up the now to a real period. The assumption that the now is a real period can then be incorporated into the argument by making it one horn of a dilemma, the other horn being that the now is a quasi-period. It follows from this assumption that the whole run must be limited by a final sub-run.

145 Thompson, 'Tasks' 5.

146 Similarly the point of the argument with the two-coloured cube of § 4.1.3 is not to show that the cube can neither have a yellow side or a blue one (unless one thinks that the only way to have a blue or a yellow surface is by having a final part of some size), but to drive home the point that there should be a limit to the cube, which it clearly lacks.

Sometimes it is stressed that summing the length of the unlimited series of sub-runs – something made possible by a limit-definition for the sum of an infinite series – does not yield more or less than the total length of the original run as a whole.[147] On other occasions it is emphasized that there is a dense rather than a discrete ordering of points: there is no last point (terminating a sub-run) before the limiting point, since between any two points, one terminating a sub-run, the other being the limit of the whole run, there is another point terminating a sub-run.[148] The crux of the modern solution, however, is captured by the set-theoretical representation of the series of sub-runs in terms of intervals on the real number-line, with its concomitant ontology of a continuum made up of points (of time or space, for example). The real numbers or points making up the whole continuum are taken as given; the intervals isolate, as it were, continuous stretches of numbers or points from them. The trick now is that the same length or the same period can be represented in two ways: by open or by closed intervals. Thus the run from A to ω can be represented by the interval [A, ω] as well as the interval [A, ω>. It does not matter which we choose, because both have the same length. Similarly point ω serves as a limit to both intervals; it is merely not included in the open interval.

It is precisely the existence of the underlying continuum which is objectionable to Zeno.[149] For him there is no such thing, either in time or in space. If there is to be a plurality, then in time there are *separate* events or, somewhat more abstract, *separate* periods of time called past, present and future, while in space there are *separate* objects.[150] It is these separate entities which have to be limited, by themselves, not by something outside of them, since that would threaten their being separate.[151] But if that is the case, then the limit has to be somehow part of the run as a whole, the period as a whole or the object as a whole, since one should not invoke a limit from an underlying continuum. (And as we saw, Zeno does not countenance such limits either, because he rejects the notion of place or space, or because he sees the now as a quasi-period just as

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[147] E.g. Vlastos, 'Race Course' 197-198 [101]; cf. Grünbaum, *Modern Science* 76.

[148] Grünbaum, *Modern Science* 75, and D. Bostock, 'Aristotle, Zeno, and the Potential Infinite', *Proceedings of the Aristotelian Society* 73 (1972/73) 37-51, at 46

[149] I borrow the term 'underlying continuum' from P. Benacerraf, 'Tasks, Super-tasks, and the Modern Eleatics', *The Journal of Philosophy* 59 (1962) 765-784, at 778.

[150] As D.N. Sedley, 'Two Conceptions of the Vacuum', *Phronesis* 27 (1982) 175-193 has explained, Zeno and his contemporaries did not conceive of the vacuum as empty space, but as what we might anachronistically call a place filler – a weird kind of object.

[151] Plato, in one of his allusions to Zenonian ways of reasoning in the *Parmenides*, lets Parmenides draw the distinction (as part of the seventh deduction, which is concerned with the consequences for the other things of the hypothesis that *one* is not):

> Then [each mass will] also [appear] to have a limit in relation to another mass, but not to have itself in relation to itself either a beginning or a limit or a middle ... because always when someone takes in thought some [part] of them as being one these [*sc.* beginning, limit and middle], then in front of the beginning there always appears another beginning, after the end another end, and within the middle other things more central than the middle, though smaller, since it is not possible to take each of them as one, because the *one* is not. ... Thus each of the other things must appear both unlimited and having a limit .. if [*one*] is not while the other things than the one are.
>
> Οὐκοῦν καὶ πρὸς ἄλλον ὄγκον πέρας ἔχων, αὐτός γε πρὸς αὑτὸν οὔτε ἀρχὴν οὔτε πέρας οὔτε μέσον ἔχων ... ὅτι ἀεὶ αὐτῶν ὅταν τίς τι λάβῃ τῇ διανοίᾳ ὥς τι τούτων ὄν, πρό τε τῆς ἀρχῆς ἄλλη ἀεὶ φαίνεται ἀρχή, μετά τε τὴν τελευτὴν ἑτέρα ὑπολειπομένη τελευτή, ἔν τε τῷ μέσῳ ἄλλα μεσαίτερα τοὺ μέσου, σμικρότερα δέ, διὰ τὸ μὴ δύνασθαι ἑνὸς αὐτῶν ἑκάστου λαμβάνεσθαι, ἅτε οὐκ ὄντος τοῦ ἑνός. ... Οὕτω δὴ ἄπειρά τε καὶ πέρας ἔχοντα ἕκαστα τἆλλα δεῖ φαίνεσθαι, ἓν εἰ μὴ ἔστιν, τἆλλα δὲ τοῦ ἑνός. (165a5-b3; c3-5, with some omissions)

separate as past and future.) The whole point of the Runner paradox, as well as the first paradox of plurality, is that there is no such part.

§ 5.3. The unlimited

To repeat the previous assertion: the whole point of the Runner paradox, as well as the first paradox of plurality, is that there is no part of a whole, whether a run, a period or an object, which could serve as a limit. Up to now it has been taken for granted that Zeno indeed had shown that there is no such part. But what has Zeno actually said to establish it?

In this section I want to show that the principles we have had reason to ascribe to Zeno in the preceding sections are sufficient to provide him with a convincing answer to this question. The argument is not very difficult, and we have already encountered a version of it in § 5.2.2.2, but it will be instructive to go through it again, for two reasons. First, it shows how coherent Zeno's position is, and second, it tells us something about Zeno's concept of the unlimited.

Let me first delimit the challenge of the first paragraph somewhat. When I say that we should ask what Zeno has said to establish that there is no part of a whole which could serve as its limit, I am only talking about the kind of parts accepted by Zeno, namely separate parts, obeying his priority principle. As I said above, in § 5.1.3, we may escape from Zeno's paradox by rejecting his requirement that a limit be a separate part, and that remains true. But that still seems to leave enough of a challenge, as might appear from the following summary of sections § 5.1 and 5.2. In these sections we discussed the background to the step made by Zeno from the unlimitedness of a series of parts to the unlimitedness in size of the whole made up of the series. On the one hand, the only conception of a limit acceptable to Zeno was as a separate part, while on the other hand it belonged to the formulation of the paradox under discussion that the existence of such a limit is ruled out. In both sections a procedure of recursive division is given to establish the existence of a series without a last part. On the assumption that such a series exhausts the whole, we could explain Zeno's conclusion that that whole lacks a limit in any sense, and is therefore unlimited in size.

This summary might be taken to expose a weak spot in Zeno's reasoning. For could we not attack Zeno's assumption that the unlimited series exhausts the whole? Indeed, the idea that the unlimited series of parts of ever decreasing size exhausts the whole is crucial to Zeno. As already remarked in passing (p. 39), there is little ground for Zeno to make the step from the unlimitedness of the series to the unlimitedness in size of the whole, if this idea is not correct. But what has Zeno given us in terms of reasons for this assumption? In the paradox of plurality he could point to the preliminary result that whatever does not have size, does not exist (despite the fact that this is not used to that purpose, but rather in order to assure the unlimitedness of the series). In the Runner paradox, however, we do not have anything like that. But even though we have this partial underpinning of the exhaustiveness assumption, why could we not reject it? We might, for example, allow for the existence of sizeless entities as basic entities, in order to undermine Zeno's conclusion of 'no size, no existence' (see § 3.3).

What this line of attack presupposes is the existence of a series with an unlimited number of parts, yet with a last part: the limiting part. It would only be this series which exhausts the whole, the objection goes. And why should this whole be unlimited, now that it has a limit? For the unlimitedness of the series is thus not so much a qualification of the series as such (in the sense that it is endless), but merely of its numerical aspect. It contains an unlimited number of parts, but that kind of unlimitedness need not have implications for the size of the whole.

The problem with this attempt at undermining Zeno's conclusion is that the series with an unlimited number of parts, but still with a final part, does have as a part the original series without the final part. How could that have a limit? To repeat an argument from § 5.2.2.2, we cannot take a limit for it from an underlying continuum, because whatever is limited must be limited as a *separate* object. By Zeno's priority principle all parts, even limits, are separate from the remainder, so that the remainder without the limit is a separate object in its own right, and thus should have a limit on its own; it cannot be limited by a limit external to it. So there is still a whole without a limit, which is therefore of unlimited size.

It is, then, again Zeno's priority principle which does the work, in that it separates the remainder from its limit, both in terms of being distinct entities and of needing limits on their own. But the argument above also highlights something else we have more or less been taking for granted: that for Zeno, 'unlimited' denotes not a numerical property of a set of entities or the size of an entity, but primarily a *series* without a last term. By extension, this core notion of the unlimited may indicate the unlimitedness in number or size, by conceiving of the unlimited in number in terms of a series with always another object, or of the unlimited in size in terms of a line going *on and on and on* without a final stretch, but the serial aspect is never lost.

We encountered this serial aspect in Zeno's literal use of ἄπειρον as 'without a limit', which referred first to the series and then to the whole made up of the series. It was also present in the second half of his second paradox of plurality, where the unlimitedness in number of the many things is shown by an iterative argument. We also have it, I want to show next, in the first half of that paradox. To remind ourselves, let me quote Zeno's argument again:

> If there are many things, it is necessary that they be as numerous as they are and neither more than themselves nor fewer. But if they are as numerous as they are, they will be limited.[152]

The notion of order behind this argument is indicated by the addition 'neither more nor fewer'. It focuses our thoughts on a final entity which could have been absent (if there had been fewer) or beyond which other entities could have been present (if there had been more). Thus it introduces the idea of a limit marking off the things inside (including itself) from the things outside. In Zeno's conclusion this idea is merely explicated.

This may be true, one might object, but does this not still amount to a mistake on Zeno's part? For why can he not say that an unlimited series does not contain more or fewer than itself, but just as many? Surely we can imagine the unlimited series to be extensionally determinate and to be just as numerous as itself, neither having a limited number of parts nor having more than an unlimited number of parts.

There are two points to be made on Zeno's behalf in response to this objection. The first is that the ancient Greek concept of number, which I take to be behind Zeno's words τοσαῦτα, πλείονα and ἐλάττονα, rules out that a number is unlimited. A number for the Greeks is a multitude of units, not some abstract 'numerical aspect' of a multi-

[152] See note 80.

tude.[153] As a consequence, two multitudes of which the one includes the other, but contains some units outside the other, are different numbers, the one being more than the other, which is fewer. The existence of an unlimited number, however, would throw this conception into disarray. Moreover, a number, ἀριθμός, can be counted, ἀριθμεῖσθαι – as is explicitly stated by Aristotle in *Physica* 3.5; 204b7-9 (quoted in note 128).[154] Occasionally a number is even defined as being limited, for example by Aristotle: 'Number is a limited multitude.'[155]

The second point is an elaboration of some elements of the first. One might think that the connection between being a number and being countable is a mere play on words. There is, however, a link between the two concepts, which may also explain why a number is primarily taken to be a multitude of units. In § 3 we saw that according to Zeno's priority principle a whole is nothing more than its parts taken together. If we apply this principle to multitudes of units, of the kind envisaged by Zeno in the paradox under discussion, we get the following problem: how can we refer to such a multitude without being acquainted with each of the units separately? To grasp the multitude as a whole is to do neither more nor less than grasp the units together. This is not a real problem in the case of a limited multitude, but in the case of an unlimited multitude it is impossible to refer to the whole. For grasping the whole is done by going through the units making up the whole – in fact by counting the number, that is, the multitude.

The upshot of this idea is that an unlimited multitude cannot be referred to as a whole – there are no unlimited wholes. But it is precisely as a totality, so it seems, that the multitude of the things which are, are presented in the first horn of Zeno's second paradox of plurality: *together the things which are*, are as many as they are. Thus we can understand why the objection against his argument did not occur to Zeno: he could not conceive of an unlimited series as a whole.[156]

Thus the notion that the multitude of things which are, constitutes a whole number – to use a pleonastic phrase –, which is nothing more than a collection of units, immediately gives Zeno his conclusion that they are limited. Conversely, he should hold then that an unlimited multitude is a multitude which does not constitute a whole number, because for every unit there is another, without there being a last. This would then make the second paradox of plurality similar to the first and to the Runner, in that in all of them the requirement for being a limited whole is to have a final part. In the first paradox of plurality and the Runner it is shown that there is no such final part which can serve as a limit, whereas in the second paradox it is assumed that there is a whole, which therefore has a final part. Moreover, this parallel also adds to our understanding of the first paradox of plurality and the Runner. In the discussion of these two paradoxes we

153 For a discusssion of this distinction see P. Pritchard, *Plato's Philosophy of Mathematics* (Sankt Augustin, 1995) chapters 2 and 4, and J. Klein, *Greek Mathematical Thought and the Origin of Algebra* (New York, 1992) [original text: 1934-36; transl.: 1968], especially chapter 6.

154 Cf. Plato on counting: 'We will posit counting as nothing else than investigating how large some number happens to be.' (*Theaetetus* 198c)

155 *Metaphysica* 1020a13; cf. Eudoxus' definition of number as 'a definite multitude' (πλῆθος ὡρισμένον), *apud* Iamblichum, *In Nicomachus* 10.17.

156 A similar idea we encounter in again Plato's *Parmenides*, when Parmenides, after having argued that the one is unlimited in multitude (ἄπειρα τὸ πλῆθος), continues:

> [Because the parts [of the one] are parts of a whole, the one would be limited according to the whole (κατὰ τὸ ὅλον). Or are the parts not contained by the whole? – Necessarily. – But then a limit would be a thing which contains (τὸ περιέχον πέρας ἂν εἴη)? – Of course. – So the one is surely both one and many, both a whole and parts, both limited and unlimited in multitude. (144e8-145a3)

often employed sentences like: ‘The whole of the unlimited series is unlimited.’ As a turn of phrase there is no problem with this, but now we have learned that taken literally there is for Zeno no whole of the unlimited series. Similarly we should not conceive of the ‘whole’ of the series as having a limit at infinity, but as an open-ended line which goes on and on and on.

§ 6. Conclusion

In the interpretations and analyses I offered of Zeno’s arguments, two principles played the leading roles. In the first place, there is the principle that parts are ontologically prior to the whole which is composed of them. Zeno expressed this principle in terms of unity and plurality, saying that he could only understand the plurality of things if he knew what the unit was. There are two versions of it, a plain one and a modalized one. According to the former, an entity which is in fact divided, is a plurality and not a unity, though it is not excluded that a unity can be divided and thus turned into a plurality. According to the latter, by contrast, a unity will always be a unity and cannot become a plurality. Zeno was seen to adopt the stronger version as well.

This priority principle explains why Zeno assumed the whole-part principle that the properties of a whole need to be reducible to the properties of its parts. We saw that he applied this derived principle in the paradox of the millet-seed to argue that every part, no matter how tiny, of something which makes audible sound when it falls down, also makes an audible sound. The absurdity in this case is caused by the fact of nature that we do not hear everything. In the paradox of the Arrow and the argument for ‘Zeno’s principle’ that what does not have size, does not exist, however, the absurdity is inherent in the very nature of the parts involved, because they are limit entities. If there is to be motion over a period of time, there must be motion in the present now, because it is in the present, not in the past or the future, that things happen. However, because the present now is infinitely small, there is no room for motion in it. Therefore there is no motion over the period of time consisting of present moments. If something is to be a part of a whole, it must contribute something to that whole. Something without size, however, cannot contribute to a whole. Therefore it is not part of a whole, not even part of the whole of reality.

These problems with limit entities were also discovered to be the ground for the second half of the first paradox of plurality and the famous Runner paradox. In these structurally identical arguments, Zeno argued that since an object or motion cannot have a limit as a part which is independent of the whole, while there is no other way of being a part than as an independent part, the whole must be without limit and therefore unlimited. Moreover, even if an object or motion were to have a limit as an independent part, it would not help, because in that case the whole without the limit would be equally independent and therefore should have a limit apart from the independent limit, and so forth. So if the whole is not to contain a part which is unlimited, it must consist of consecutively ordered limit entities, that is, entities without size. But how would this whole have size then?

In all of these arguments posing difficulties for limit entities, the unlimited divisibility of objects or motions was presupposed. To escape these paradoxes, then, one might consider setting limits to this divisibility. Against such an escape, Zeno argued that there is no third way between an entity being everywhere (and thus infinitely) divisible and being indivisible. He did so on the basis of his other main principle, the homogeneity of entities. Two objects which are not separated form an entity consisting in one homogeneous stretch of stuff. If this entity is somewhere divisible, it must be divisible everywhere, so that if that is impossible, it cannot be divisible at all.

On the basis of a similar consideration, Zeno also seemed to infer that between every two independent objects there must be a third, since a third object would break the homogeneity of the two objects. This line of reasoning could of course be repeated for any two objects, so that the number of objects must be unlimited. That, however, was unacceptable for Zeno. Conceiving of reality as a determinate whole, he could only think of it as limited in number, on the grounds that an unlimited series of parts cannot form a determinate whole, but goes on and on.