

**THE METAPHYSICS OF CONTINUITY:  
ZENO, DEMOCRITUS AND ARISTOTLE**

Ta oantinken oan ús Beppe

‘Gij gist in mij met ongestorvenheid.  
Wie gij zonder uw dood had kunnen zijn  
wil met zijn voortzetting tesamen zijn,  
volgens de wet der continuïteit.’

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Zeno, Democritus and Aristotle

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# VOORWOORD

Het schrijven van een proefschrift heeft soms wel iets van Zeno's loperparadox, volgens welke een loper om bij het eindpunt aan te komen eerst halverwege aan moet komen, en dan halverwege de resterende weg naar het eindpunt, enzovoorts. De eerste belangrijke ideeën van een proefschrift zijn wellicht redelijk snel geformuleerd, maar daarna moeten ze worden uitgewerkt en vooral in harmonie met elkaar worden gebracht. In omvang mogen de problemen waar men tegenaan loopt steeds kleiner worden, maar hun aantal lijkt niet af te nemen; integendeel, door hun geringe omvang volgen zij elkaar steeds sneller op. In het werk dat op dit moment voor u ligt, zult u dan ook vele niet-gezette stappen kunnen ontdekken.

Desondanks hoop ik een aantal belangrijke stappen te hebben gezet naar het gecombineerde einddoel van, aan de ene kant, een filosofisch beter begrip van de doorgaande lijn in het antieke debat over kwesties van continuïteit en oneindige deelbaarheid, en, aan de andere kant, betere interpretaties van de afzonderlijke teksten en argumenten daarover die uit de Oudheid tot ons gekomen zijn. Deze had ik niet kunnen zetten zonder de ondersteuning van een aantal mensen en instanties. Het onderzoek waarvan dit proefschrift het resultaat is werd financieel mogelijk gemaakt door de Stichting Geesteswetenschappen van de Nederlandse Organisatie voor Wetenschappelijk Onderzoek. De Faculteit Wijsbegeerte van de Rijksuniversiteit Groningen heeft altijd een plezierige werkomgeving geboden. De leden van de beoordelingscommissie wil ik bedanken voor hun soms uitvoerige commentaar, naar aanleiding waarvan ik nog een aardig aantal verbeteringen kon aanbrengen. Mijn promotor John North is mij bijzonder terwille geweest, vooral op enkele belangrijke momenten, zonder overigens na te laten te herhalen dat ik toegankelijker moest schrijven opdat ik meer dan 'those five people in the world' zou bereiken. De grootste dank ben ik echter verschuldigd aan Job van Eck, voor zijn commentaar en onze besprekingen bij alle stappen, maar ook voor de ruimte die hij mij liet in het formuleren van mijn eigen ideeën. Met zijn precisie, nuchterheid en vasthoudendheid bij het zoeken naar filosofisch zinnige interpretaties van moeilijke teksten is hij mij altijd tot voorbeeld geweest. Het verheugt mij zeer dat hij optreedt als co-promotor.

Indirect heeft ook mijn samenwerking met Erik Krabbe bij het vertalen van Aristoteles' geschrift *Over sofistieke redeneringen* een belangrijke bijdrage geleverd aan het tot standkomen van dit proefschrift. Onze gezamenlijke pogingen Aristoteles' argumentaties tot in de details te begrijpen hebben mede mij de vaardigheid en de durf gegeven die nodig waren om het derde hoofdstuk te schrijven.

## *Voorwoord*

Voor de omslag van het boekwerk alsmede de begeleiding bij het drukken ben ik Sabine Verschoor erkentelijk. Bij het opmaken van de tekst heb ik onschatbare hulp gekregen van Marlies de Jonge.

Van Zeno's loperparadox bestaat ook een omgekeerde versie, volgens welke de loper om bij het eindpunt aan te komen eerst halverwege aan moet komen, maar om halverwege aan te komen hij eerst halverwege de weg daarheen aan moet komen, enzovoorts. Zonder mijn familie en naaste vrienden zou het schrijven van dit proefschrift meer op deze versie hebben geleken.

Groningen, 15 november 2002

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# INTRODUCTION

Despite their antiquity, Zeno's paradoxes still offer the most accessible introduction to the subject of continuity and infinite divisibility. Suppose – to paraphrase one of Zeno's division paradoxes – we have a line-segment which is divisible and has parts. In their turn these parts are also divisible and have further parts, *ad infinitum*. What will then be the parts of which the line-segment is really composed? The ultimate parts can only be sizeless, for if not they will still be further divisible, so that the original line-segment, which was of a certain length, is paradoxically composed of parts without length. In order to avoid this paradox, we must therefore assume that the line-segment is one and indivisible. As the same argument can be applied to everything, even the whole of reality must be one and indivisible. Or suppose, to take another of Zeno's paradoxes, that there is a runner who wants to go from A to B. In order to get to B, however, he first has to get to a point C, half-way to B. Suppose he has arrived at C; in order to get to B, again he first has to get to a point D, half-way between C and B; and so forth, *ad infinitum*. Therefore the runner will never arrive at B. Suppose, to take a third paradox devised by Zeno, there is a flying arrow. At every moment – that unlimitedly small time – this arrow occupies a place equal to its own length. Therefore it is at every moment without motion, so that during the whole flight it rests.

These results may be strange, but the arguments are simple and still have immediate appeal. If one wants to understand what exactly is going on in them and to account for that lasting appeal, however, matters are quite different. Given that every paradox is a symptom of some kind of conceptual confusion, one may wonder what the concepts and ideas behind these puzzles are. There are two places where one may look for an answer. In the first place, one could turn to the modern philosophical literature that deals with these issues. There are few answers to be found there, however. The discussion of Zeno's paradoxes has been dominated by an approach which is heavily mathematical and focuses primarily on technical details. The main concern seems to be present a model in which these paradoxes are circumvented or simply do not arise.<sup>1</sup> Acceptable

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<sup>1</sup> For example in the influential work by A. Grünbaum, *Modern Science and Zeno's Paradoxes* (Middletown, Conn., 1967) and W.C. Salmon, *Space, Time and Motion. A Philosophical Introduction* (Encino, 1975) 31-67, but also in more recent work, using other mathematical models, as W.I. McLaughlin and S.L. Miller, 'An Epistemological Use of Nonstandard Analysis to Answer Zeno's Objections Against Motion', *Synthese* 92 (1992) 371-384.

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though such mathematical models may be, they do not tell us much about the conceptual background to problems of infinite divisibility and continuity.

A second way to come to grips with these arguments would be to place them back into their historical context and to try to find there clues about the ideas involved in them. In this case we are in the fortunate situation that with Zeno's book of paradoxes, published in the middle of the fifth century BC, a lively debate was sparked off, to which such famous philosophers as Democritus, Plato and Aristotle contributed, and which continued with Epicurus and the Stoics. However, even reading ancient historical literature dealing with continuity proves somewhat disappointing. While over the last twenty years or so there have been a number of truly enlightening contributions to our understanding of the ancient debate,<sup>2</sup> their impact has been limited. Moreover, there is no account available which shows both the continuity and development in the ancient debate, for example by providing an analysis of the concepts used and presupposed in the arguments put forward by the philosophers in Antiquity. Our view thus remains rather patchy.

One of the reasons for this state of affairs may be the continuing normative influence of the mathematical models mentioned above on the history of ancient philosophy. All too often one encounters interpretations and critical evaluations in which the correctness of such a mathematical model is taken for granted. It guides the interpretation, and even serves explicitly as a standard.<sup>3</sup> Another reason for our limited understanding of this part of history, is that many of the arguments put forward in Antiquity are extremely difficult to understand. If they are not short and elliptical, they often seem inconclusive or even fallacious. In this respect philosophers like Plato and Aristotle, even though many of their works are still available to us, do not fare much better than those like Zeno and Democritus, for whose ideas we have to rely on reports and testimonies given by others. One may even feel that these two reasons reinforce each other, for if an argument is obscure, one looks for a model to make things comprehensible, and if the model does not really fit the argument as it stands, one may despair of ever comprehending it.

Being a work in the history of philosophy, the present study follows the second way of trying to understand the conceptual background to issues of continuity and infinite divisibility. My foremost aim is to provide convincing interpretations of many of the obscure and apparently unsound arguments we encounter in Antiquity and to give an account of the continuity and development of the ancient debate on these issues. I have limited my attention to Zeno, Democritus and Aristotle, partly because of reasons of space and time, but also because they represent three different positions in the ancient debate.

At the same time, the issues concern philosophically, as well as cognitively, very basic concepts, as may already appear from the intuitive appeal Zeno's paradoxes enjoy. I think that it is therefore impossible to achieve these historical goals of providing convincing interpretations of arguments and giving an overview of the development of the ancient debate without trying to understand the underlying philosophical

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<sup>2</sup> I want to mention especially the work of Stephen Makin; see the references in Chapters One and Two.

<sup>3</sup> A particularly nice illustration is provided by the title of possibly the best book on the subject, written by M.J. White, *The Continuous and the Discrete. Ancient Physical Theories from a Contemporary Perspective* (Oxford, 1992), in which Aristotle is praised time and again for having been able to come so close to the modern characterization of the continuum. We shall encounter more examples throughout the present study.

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conceptions. One should, for example, be able to explain the fact that these arguments were perceived as carrying conviction. Therefore I shall try to give a philosophically informed characterization of the concepts used and the steps taken in these arguments. If these characterizations help us to discern common patterns among these arguments and to explain more details of other arguments, we may suppose to have discovered some of the ideas behind them. It would be surprising if they did not go some way towards uncovering the conceptual roots of the problem of continuity.

A proper understanding of the arguments often begins with their translation. All translations in this study will be my own, unless otherwise indicated. I shall mark as much as possible phrases and words supplied from the context with square brackets. Other additions are between angular brackets. In my translations, I have tried to keep very close to the original Greek, even to the extent that they suffer with respect to the quality of the English. Too often erroneous interpretations have been partly based on translations which are not literal enough.