Particle dynamics of branes

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Chapter 8

Conclusions and Future Research

8.1 Summary

In the first chapter we pointed out what we wanted to achieve in this thesis. First we wanted to show that p- and Sp-branes can be linked to lower-dimensional actions whose solutions are respectively given by instantons or S(-1)-branes if we reduce over the worldvolume of the brane. And similarly, if we reduce a p- or Sp-brane over all but one of its transverse directions we find a domain-wall or a cosmology. See also figure 3.5.1. The main goal was to derive the generating solution for the massless geodesics that we obtain in the lower dimension. For the massive theory the focus was on re-writing the second order differential equations as first order equations and explaining why we sometimes still have geodesic motion.

The first goal was explained in section 3.5. Let us summarize the situation for massless theories first. In section 3.5 we showed that the scalar part of the action for massless theories, obtained after a worldvolume reduction, leads to geodesic motion. The gravity part gets decoupled and can be solved independently. This applies both to instantons and S(-1)-branes.

To find the geodesics for both types of (-1)-branes we introduced the concept of a generating solution. By definition, a generating solution is a geodesic with the minimal number of arbitrary integration constants such that the action of the isometry group G generates all other geodesics from the generating solution. The isometry group G is the symmetry group of the lower-dimensional equations of motion. This way we found in chapter 4 the most general Sp-brane with deformed worldvolume via a reduction over a Euclidean torus.

In case we reduce over a Lorentzian torus, the scalar manifold becomes non-

Riemannian. As a result, there are different classes of generating solutions labeled by the sign of the affine velocity. We tackled this problem in chapter 7. We showed how to derive the generating solution for the coset $\mathrm{GL}(p+q,\mathbb{R})/\,\mathrm{SO}(p,q)$.

If we instead do a maximally symmetric compactification, the lower-dimensional theory can contain a potential. This leads to both domain-walls (stationary) and cosmologies (time-dependent). Although we also considered some examples of an instanton with a potential. In general, the presence of the potential upsets the geodesic motion. In chapter 5 we first showed that if we can rewrite the potential in terms of a superpotential in a specific way the equations of motion become first order equations. These cosmologies are called pseudo-BPS. Furthermore, we found examples of scaling solutions that did turn out to be geodesics on the scalar manifold. To explain this we showed in chapter 5 that all pseudo-BPS cosmologies that are scaling solutions must be geodesic. In case the potential is a cosmological constant, the geodesic motion is always preserved.

The resemblance between domain-walls and cosmologies is explained by the domain-wall / cosmology correspondence. A natural question is what pseudo-supersymmetry (pseudo-BPS) means in a real supergravity context. We answered this question in chapter 6. The domain-walls are solutions of an ordinary supergravity, while the cosmologies arise as solutions of the corresponding star supergravity. In this sense the pseudo-supersymmetry of cosmologies corresponds to supersymmetry in the star theory.

8.2 Future Research

Let us give some research problems that are natural to consider next.

- In case of the p-brane we restricted to pure gravity in the higher-dimensional theory. A natural extension would be to extend this with a $(p+1)$-form potential as well. This would allow us to write down the most general p-brane with deformed worldvolume.

- In the non-Riemannian case we restricted the analysis of the generating solution to the coset $\mathrm{GL}(p+q,\mathbb{R})/\,\mathrm{SO}(p,q)$. A natural extension is to look for the generating solutions of the maximally non-compact supergravities given in the right column of table 3.4.1 [69].

- The embedding of the domain-wall / cosmology correspondence was restricted to $\mathcal{N}=2$ supergravities in nine and ten dimensions. Note however that when one continues to lower the dimension, more possibilities could arise since one can then have extended supersymmetry with $\mathcal{N}>2$. This allows for more general reality conditions on the fermions than considered in chapter 6. The matrix ρ appearing in these reality conditions will in general be a $\mathcal{N}\times\mathcal{N}$ matrix. It might be interesting to find out if for $\mathcal{N} > 2$ there can be more than two inequivalent real slices in certain signatures.