Chapter 8
Fluid flow caused by collective non-reciprocal motion

Abstract

Using a magneto-mechanical solid-fluid numerical model for permanently magnetic artificial cilia, we show that the metachronal motion of symmetrically-beating cilia establishes a net pressure gradient in the direction of the metachronal wave, which creates a unidirectional flow. The flow generated is characterised as a function of the cilia spacing, the length of the metachronal wave and a dimensionless parameter that characterises the relative importance of the viscous forces over the elastic forces in the cilia.

8.1 Introduction

In lab-on-a-chip devices, working fluids have to be pumped between micro-reaction chambers through micron-sized channels. At these small length scales the viscous forces dominate over the inertial forces. Under this condition, a mechanical actuator has to move in a non-reciprocal manner to cause a net fluid transport (Purcell, 1977). In nature, micron-scale fluid manipulation is often performed using periodically beating hair-like structures called cilia. An example of natural fluid manipulation systems is the expulsion of mucus from the lungs caused by the beating of the cilia attached to the inner layer of mammalian trachea. The ciliary beat consists of distinct effective and recovery strokes, which leads to a non-reciprocal motion. In addition to the non-reciprocal motion of individual cilia, adjacent cilia beat with a constant phase difference leading to a coordinated wave-like motion, which is referred to as metachronal motion. Another example of fluid manipulation is the swimming of Cyanobacteria. Points on the surface of Cyanobacteria oscillate symmetrically and generate waves of lateral displacement along their surface. This wave-like motion causes fluid transport in one direction and the bacteria swim in the opposite direction (Ehlers et al., 1996).

Many examples have appeared in the recent literature of artificial cilia that mimic the natural ciliary motion using different physical actuation forces, imposed by electric fields, magnetic fields or through base excitation (Shields et al., 2010; Evans et al., 2007; den Toonder et al., 2008; Oh et al., 2009; van Oosten et al., 2009; Fahrni et al., 2009; Kim & Netz, 2006; Gauger et al., 2009; Khaderi et al., 2009). In most of the cases the actuation field is uniform (Kim & Netz, 2006; Khaderi et al., 2009; Fahrni et al., 2009; Vilfan et al., 2010), so that all artificial cilia beat in-phase, thus only focusing on the

non-reciprocal motion of individual cilia. The flow generated by synchronously-beating cilia has been analysed in terms of the dimensionless parameters that govern the cilia behaviour (Alexeev et al., 2008b; Ghosh et al., 2010; Alexeev et al., 2008a; Khaderi et al., 2009; Gauger et al., 2009). The formation of the metachronal waves has been investigated using computational models, which suggest that the coordinated motion is due to the hydrodynamic interaction between adjacent cilia, and that the energy spent per cilium decreases in the presence of metachronal waves (Gueron & Levit-Gurevich, 1999; Gueron et al., 1997; Kim & Netz, 2006; Vilfan & Jülicher, 2006). Recently, it has been shown that the flow generated by magnetically-driven non-reciprocally-beating artificial cilia is substantially enhanced and becomes unidirectional when the cilia beat out-of-phase compared to synchronously beating cilia (Khaderi et al., 2011b; Gauger et al., 2009). By modelling cilia that beat out-of-phase and possess only orientational asymmetry (Khaderi et al., 2010) as a porous sublayer, it has been shown that flow can be created in the direction of the metachronal wave (Hussong et al., 2011a). However, from a manufacturing and implementation point-of-view, it is not straightforward to generate non-reciprocal motion of the individual cilia. Therefore, it is of interest to investigate whether cilia can create a flow in the absence of any asymmetry but in the sole presence of waves of lateral displacement, similar to that of Cyanobacteria. This is the subject of the present article.

We study an array of permanently magnetic artificial cilia subjected to a non-uniform magnetic field that travels in space and time, such that the cilia beat symmetrically but out-of-phase. The ciliary motion generates a unidirectional fluid flow in a direction opposite to the metachronal wave. The fundamental mechanism which is responsible for the flow is investigated, and the flow is quantified as a function of the parameters of the metachronal wave, cilia spacing, and a physical dimensionless parameter that quantifies the relative importance of the viscous forces compared to the elastic forces. We find that flow reaches a maximum when the wavelength of the metachronal wave is four times the cilia length.

The chapter is organised as follows. The boundary value problem, solution approach and parameters involved in the system are explained in section 8.2. The collective non-reciprocal motion is analysed in section 8.3.1. The mechanisms that cause the flow are analysed from Eulerian and Lagrangian points-of-view in section 8.3.2. The dependence of the flow on the system parameters is analysed in section 8.3.3. We finally summarise the results of the analysis in section 8.4.

### 8.2 Problem definition

We analyse an infinitely long channel of height \( H \) containing an array of cilia of length \( L \), thickness \( h \) and inter-cilia spacing \( a \). The cilia have an elastic modulus \( E \) and possess a remanent magnetization of magnitude \( M_r \) oriented along their length from the fixed end to the free end. A magnetic field with a constant magnitude in the \( y \)-direction (\( B_{0y} \)) and varying over space and time in the \( x \)-direction is applied according to the following form,

\[
B_x = B_{0x} \sin (\omega t - 2\pi x/\lambda),
\]

where \( \omega = 2\pi/t_{\text{ref}} \) is the angular velocity, \( t_{\text{ref}} \) is the time period of oscillation and \( \lambda \) is the wavelength of the applied magnetic field, see Fig. 8.1. The applied magnetic field has a wave velocity \( \lambda/t_{\text{ref}} \) to the right. The magnetic body couple \( N_z \) acting on the cilia
8.2. Problem definition

Figure 8.1: Schematic picture of the problem analysed. We study an infinitely long channel of height $H$ containing equally-spaced cilia that are arranged perpendicular to the channel. A magnetic field which varies in space and time is used for actuation. The resulting flow is opposite to the direction of the metachronal wave.

at the cilia base can be written as $N_z = M_r B_0 x$. The velocity of the magnetic couple wave, which causes the out-of-phase motion of the cilia, is also equal to $\lambda/t_{ref}$. However, as there are a finite number of cilia per wave length, the metachronal wave velocity also depends on the cilia spacing $a$. The metachronal velocity in cilia per seconds can be written as $\lambda/a t_{ref}$ for $0 < a < \lambda/2$ and $-(\lambda/a t_{ref})/(\lambda/a - 1)$ for $\lambda/2 < a < \lambda$ (see chapter 7 and appendix J). The fluid to be propelled is assumed to be incompressible and Newtonian with a viscosity $\mu$. The cilia are vertically straight when no magnetic field is applied.

The dimensionless geometric parameter $a/\lambda$ scales with the phase difference between adjacent cilia and characterises the response of the system together with $H/L$ and $a/L$. The physical response of the system can be captured through three dimensionless numbers: the fluid number $F_n = 12 \mu L^3/\rho Eh^3 t_{ref}$ - the ratio of the viscous forces to the elastic forces, the inertia number $I_n = 12 \rho L^4/\rho Eh^2 t^2_{beat}$ - the ratio of cilia inertia forces to the elastic forces and the magnetic number $M_n = 12 B_0^2 M_r L^2/\mu_0 Eh^2$ - the ratio of magnetic forces to the elastic forces, where $E$ and $\rho$ are the elastic modulus and density of the cilia (Khaderi et al., 2009). In this chapter we explore the effect of $a/\lambda$, $a/L$ and $F_n$ for a given value of $I_n$, $M_n$ and $H/L$ in the limit of low Reynolds numbers. We model the cilia as Euler-Bernoulli beams and the magnetic field using a boundary element approach (Khaderi et al., 2009). The Stokes equations, which capture the behaviour of the fluid at low Reynolds numbers, are solved using the finite element method. The solid and fluid dynamic equations are coupled implicitly using the fictitious domain method (van Loon et al., 2006). The solution procedure can be summarised as follows. The Maxwell’s equations are solved at every time instant to obtain the magnetic field. From the magnetic field, we calculate the magnetic body couple acting on the cilia$^1$. This body couple is provided as an external load to the implicitly-coupled solid-fluid model, which solves for the velocity of the cilia, and the velocity and pressure of the fluid. Using the velocity of the cilia, its new position is calculated and the procedure is repeated. The reader is referred to Khaderi et al. (2009) for a detailed description of the numerical model.

To perform the numerical simulations we choose a unit-cell whose width is equal to

$^1$As we have a permanently magnetic film that is magnetized along its length, and because only the magnetic field in the $x$-direction is non-uniform, no magnetic body forces act on the cilia.
one wavelength containing \( \lambda / a \) cilia. The left and right end of the unit-cell are periodic in velocity, while the top and bottom boundaries are no-slip boundaries. We perform simulations for various values of the phase difference \( a / \lambda \), while the inter-cilia spacing \( a / L \) is maintained constant. Therefore, as \( \lambda \) is increased the number of cilia in a unit-cell also increases.

8.3 Results

8.3.1 Collective non-reciprocal motion

We first show that the out-of-phase motion of the cilia leads to a collective non-reciprocal motion. To elucidate this we plot the schematic positions of the cilia at time instances \( t_{\text{ref}}/4 \) after and before the first cilium has reached its extreme position (at time \( t_e \)) in Fig. 8.2. The thin and thick lines represent the extreme and current positions of the cilia, respectively. The arrows represent the direction of motion of the cilia. We would like to remind that for non-reciprocal motion of an actuator, its position after and before an extreme position is reached should not be identical. In the cases of cilia beating in synchrony and anti-phase, the positions of all the cilia before \( (t_e - t_{\text{ref}}/4) \) and after the extreme position \( (t_e + t_{\text{ref}}/4) \) is the same; hence the motion is reciprocal. However, in the case for cilia moving out-of-phase, even though the positions of the cilia 1 and 3 are identical before and after \( t_e \), the positions of cilia 2 and 4 are not. Therefore, even though every cilium performs a reciprocal motion, the cilia collectively perform a non-reciprocal motion and, therefore, will cause a net fluid flow in microchannels. This fluid flow is quantitatively analysed in the subsequent sections.

8.3.2 The fundamental mechanism

Eulerian point-of-view

To illustrate the fundamental mechanism that creates the flow, we take \( a / L = 2 / 7 \), \( a / \lambda = 1 / 7 \), \( H / L = 4 \), \( F_n = 0.15 \), \( I_n = 3 \times 10^{-3} \) and \( M_n = 2.25 \) (based on \( B_{x0} \)). As \( a / \lambda = 1 / 7 \), the metachronal wave travels to the right with a speed \( \lambda / t_{\text{ref}} \). Figure 8.3(a) shows the pressure contours with the streamlines superimposed and Fig. 8.3(b) shows the contours of the absolute value of the horizontal component of the velocity. For clarity, the results are shown for two unit-cells. Because of the travelling magnetic wave and periodicity of the system, the pressure and velocity profiles in the channel remain unchanged in time, but travel with a constant velocity in the direction of the applied magnetic wave (to the right).

At the instant depicted, cilia 2, 3, 4 and 9, 10, 11 move to the right, while cilia 6, 7 and 13, 14 move to the left. The other cilia are nearly stationary (zero velocity). Due to the instantaneous velocity of the cilia, high pressure (hp) and low pressure (lp) regions develop (red and blue regions in Fig. 8.3(a)). Fluid is squeezed out from the hp region and sucked in by the lp regions, as a result of which a series of counter-rotating vortices are formed in the channel. Since the distance between the hp and lp regions opposite to the wave direction is smaller, the pressure gradient is larger, so that the counter-clockwise vortices are stronger (see Fig. 8.3(b)). As a result, the velocity distribution

\(^2\)Note that the cilia will deform by bending – when a magnetic body couple \( N_z \) is applied – and will not remain straight as is assumed in this section for simplicity.
8.3. Results

(a) Synchronously beating cilia.

(b) Cilia beating in anti-phase.

(c) Cilia beating out-of-phase $a/\lambda = 1/4$.

Figure 8.2: Schematic positions of the cilia at instances $t_{ref}/4$ after $(t_e + t_{ref}/4)$ and before $(t_e - t_{ref}/4)$ the extreme position has been reached by the first cilium at $t_e$. The time $t_e$ corresponds to the extreme position of the first cilium from the left. The arrows represent the direction of motion of the cilia. In the cases of cilia beating synchronously and anti-phase, the positions of all the cilia before and after the extreme position is the same; hence the motion is reciprocal. However, it is not the case for cilia moving out-of-phase; hence the motion is non-reciprocal.
has a dominant horizontal component to the left. Integrating the velocity profile over the channel height results in a net flux to the left. Conservation of mass dictates that the flux at every vertical section through the channel is the same. Since the entire periodic profile, as depicted in Fig. 8.3, travels continuously to the right at a constant pace, the flux remains constant in time. Clearly, the flux magnitude and direction can be directly deduced from the instantaneous pressure distribution profile of Fig. 8.3 as analysed in the following.

Fluid flow occurs in the direction opposite to the net pressure gradient. This pressure gradient is governed by the magnitude of the pressure in the lp and hp regions and the distance between them. The former is governed by the velocity of the individual cilia, whereas the latter is determined by the deflection $\delta$ of the cilia tip. Since the velocity and displacements of the cilia are controlled by the magnetic field and its rate of change, it can be deduced that for $a$ considerably smaller than $\lambda$ the net pressure gradient scales with $\mu \omega \delta^2 / \lambda^3$ (see appendix K). As this pressure gradient is positive, the flow occurs in the negative $x$-direction. Thus, the flow direction is opposite to the metachronal wave, and scales with the square of the amplitude of deflection. When the direction of the applied magnetic wave is reversed, the pressure profile, which is dictated by the cilia velocity, remains alternating. However, the deformed configuration of the cilia changes in such a way that the net pressure gradient is now negative; this creates a flow to the right (again opposite to the metachronal wave). Fluid flow created by oscillating cilia whose motion is kinematically prescribed has been analysed recently using a continuum approach (Hussong et al., 2011a). The formation of the vortices was also observed in this work. A rigorous mathematical analysis of the fluid flow induced by a longitudinally oscillating sheet whose material particles comprise of the tip of the cilia also predicts that the flow will occur in the direction opposite to the metachronal wave (Brennen, 1974).

**Lagrangian point-of-view**

The previous analysis focussed on the channel flow from an Eulerian point-of-view. Instead, we can also adopt a Lagrangian view point and track the motion of fluid particles in time. In the following, all the parameters of the previous section are kept constant, except for $H$, which we now choose to be $2L$. Figure 8.4 shows a portion of the region between the tips of the cilia and the top boundary. We follow the motion of the fluid particles (that initially form a straight vertical line) when they come under the influence of the travelling vortices (see Fig. 8.4). The contours represent the absolute velocity in the $x$-direction, and the direction of velocity is represented by the streamlines. From the velocity field, we can see that the out-of-phase motion of cilia creates a series of counter-acting vortices, and that the velocity field travels to the right, which is also the direction of propagation of the applied magnetic field wave.

We focus our attention on the second particle from the bottom. At $t = 0$ the particle is between two vortices. The velocity of the particle is such that it moves downwards. As time progresses, at $t = 0.2t_{ref}$ the position of the particle is such that it has a low velocity to the right due to the presence of the clockwise vortex. At $t = 0.4t_{ref}$, the particle moves away from the influence of the clockwise vortex, towards the counter-clockwise vortex. Now the particle has a velocity such that it moves upwards. At $t = 0.6t_{ref}$, when the particle is under the influence of the stronger counter-clockwise vortex, it has a higher velocity compared to the instance when the particle was under the influence of less strong
8.3. Results

(a) Pressure contours: red is high pressure and blue is low pressure.

(b) Contours represent the magnitude of velocity in x-direction and the streamlines represent the direction of velocity.

Figure 8.3: Fundamental mechanism causing fluid flow: (a) Contours of pressure and (b) Contours of absolute velocity in x-direction for \( a/L = 2/7 \) and \( a/\lambda = 1/7 \) (wave moving to the right) at \( t = 0.35r_{\text{ref}} \). Due to the velocity of cilia, regions of positive and negative pressure are established in the channel. The deformed position of the cilia causes a lower pressure gradient in direction of the wave compared to that of in the opposite direction. This leads to a high velocity and a net flow to the left.

clockwise vortex (compare Figs. 8.4(b) and 8.4(d)). Therefore, the particle effectively moves to the left. The displacement perpendicular to the channel is less for the particles near the top boundary. This leads to equal velocities in the channel direction when they come under the influence of the clockwise and counterclockwise vortices, which results in no net displacement of the particles near the top boundary.

It can be seen that the fluid particle near the free end of the cilia moves unidirectionally and that its displacement is much larger compared to the displacement of the rest of the particles. Therefore, the contribution to the flow from the fluid particles near the cilia is much larger compared to other particles. This results in a flow that is nearly unidirectional, even though the cilia motion is oscillatory. An unidirectional flow is also observed when the cilia motion is non-reciprocal in the presence of metachronal waves (chapter 7) and inertia (chapter 5). A similar approach (Lagrangian point-of-view) was used in (Ehlers et al., 1996) to analyse the swimming of microorganisms (based on tangential travelling waves).

8.3.3 Parametric study

We now characterise the flow generated by the cilia as a function of the system parameters such as the cilia spacing \( a \), wavelength \( \lambda \), amplitude of cilia deflection and fluid number \( F_n \). The output of the cilia is quantified by the area flow created per cycle, which is found by integrating the instantaneous velocity flux through the channel over a representative
8. Fluid flow caused by collective non-reciprocal motion

Figure 8.4: Motion of particles with time: The field-of-view is the region between the oscillating end of cilia and top boundary of the unit-cell with the bottom left corner at \((x, y) = (0.5L, 0.95L)\) and the top right corner at \((x, y) = (1.55L, 2L)\). The velocity in the channel direction is larger in the direction opposite to the wave than in the direction of the wave. The white curves represent the trajectory of particles and the black dots represent the particles. Particles near the cilia move unidirectionally and show larger displacement, whereas the particles near the top boundary do not show any displacement.

cycle. The nature of the fluid flow, as seen in section 8.3.2, is unidirectional. We focus our attention on the cases where the direction of the metachronal wave velocity is positive, i.e. \(a < \lambda/2\), and perform the analysis for \(H = 2L\), \(M_n = 1.13\), \(F_n = 0.15\).
8.3. Results

Figure 8.5: Flow as a function of (a) phase difference $a/\lambda$ and (b) wavelength $L/\lambda$ for various cilia densities $a/L$. The fluid flows in a direction opposite to the metachronal velocity (shown using dashed lines in (a)). The flow is maximum when $\lambda = 4L$. The arrow shows the direction of increase of $\lambda$.

and $I_p = 3 \times 10^{-3}$ unless mentioned otherwise. Figure 8.5(a) shows the flow and the metachronal wave velocity as a function of $a/\lambda$. When $a = 0$, the cilia move in-phase. When $a = \lambda/2$, the positive wave velocity and the negative wave velocity are equal and this leads to a standing wave (see chapter 7 and Fig. 8.2(b)). Under these conditions, we do not see metachronal waves and thus no fluid flow is observed. As the magnetic wave travels to the right (see Fig. 8.1), the fluid flows to the left; the opposite occurs for $\lambda/2 > a > \lambda$. To get more insight into the behaviour of the system, we plot the flow as a function of $L/\lambda$ for the cases shown in Fig. 8.5(a) in Fig. 8.5(b). The arrow shows the direction of increasing $\lambda$. The fluid flow increases when the inter-cilia spacing is decreased, and for a given inter-cilia spacing, the flow initially increases with $\lambda$, reaches a maximum at $\lambda = 4L$ and then decreases.

As shown in appendix K the pressure gradient responsible for the flow increases when the wavelength decreases, as well as when the deflection of the cilia increases. However, there is an additional dependence. When the wavelength of the magnetic field is increased in Fig. 8.5(b) for a particular cilia spacing, the pressure between the cilia decreases, which reduces the hydrodynamic drag and causes an increased deformation of the cilia (see Fig. 8.6). On the other hand, the pressure gradient will decrease with an increase in the wavelength\(^3\), so that the flow created is due to the relative competition between these two effects. The deflection effect dominates when the wavelength is small, creating an increased flow as we increase the wavelength until $\lambda = 4L$. When we increase the wavelength any further, the effect of the increasing deflection is overcome by the decreasing pressure gradient, which causes the flow to decrease. This is in contrast to the situation where the cilia sweep an asymmetric area (see chapter 7), where it is shown that the flow created does not depend on the phase difference between adjacent cilia. This is due to the different mechanisms that drive the flow: The pressure gradient in the current situation and the asymmetric area in (Khaderi et al., 2011b).

\(^3\)Note that the pressure gradient will continue to decrease with an increase in wavelength, whereas its increase because of the increased cilia deflection is limited.
Fluid flow caused by collective non-reciprocal motion

Figure 8.6: Maximum transverse displacement as a function of $L/\lambda$ for different cilia spacings ($a/L$). When $L/\lambda$ is large the phase difference between adjacent cilia is large, causing them to move against each other, which leads to high drag forces on the cilia causing low deformation.

Figure 8.7: Flow as a function of the fluid number ($F_n$) for various cilia spacing at $L/\lambda = 1/4$. The maximum transverse displacement of the free end is shown as an inset.

The flow as a function of the fluid number ($F_n$) for various cilia spacings at $L/\lambda = 1/4$ is shown in Fig. 8.7; the flow decreases with an increase in the fluid number. As the viscous forces increase with the fluid number, the cilia exhibit lower deflections when $F_n$ is increased (see the inset in Fig. 8.7). As the pressure gradient scales with the cilia deflection, the flow created decreases with the deflection.

When $\lambda >> a$, the fluid experiences an oscillating surface whose material particles are made up of the tip of cilia. The velocity of propulsion in such a case is given by the envelope theory under the assumption that the cilia spacing is much smaller than the wavelength. Brennen (1974) has shown that for a continuous distribution of cilia, the fluid velocity scales with the inverse square of the wavelength, the frequency of oscillation $\omega$ and the square of the amplitude of oscillation (Brennen, 1974). Moreover, the flow is in
8.4. Conclusions

In this chapter we analysed the fluid transport created by cilia that beat symmetrically and out-of-phase when actuated by a non-uniform magnetic field, leading to the formation of metachronal waves. Although at the scale of individual cilia the beating is reciprocal, because of the metachronal waves the cilia collectively exhibit a non-reciprocal motion. Using a magneto-mechanical solid-fluid numerical model we analyse the fundamental mechanisms that cause this fluid flow. The out-of-phase motion of cilia creates a net pressure gradient, which results in a unidirectional flow whose direction is opposite to the direction of the wave. The flow increases with the tip deflection of cilia and decreases with the wavelength. Analysis of the motion of fluid particles reveals that the major contribution to the fluid flow comes from the particles located near the free end of the cilia. The flow created reaches a maximum value when $\lambda = 4L$. 

![Figure 8.8: Flow as a function of deflection.](image_url)