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THE SUPERCURRENT IN TEN DIMENSIONS

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We construct the supercurrent of the $N = 1$ supersymmetric Maxwell theory in ten dimensions. It contains 5760 + 5760 components. There exists a submultiplet of 5632 + 5632 components, which is a constrained $d = 10$ scalar superfield. The remaining 128 + 128 components form an off-shell multiplet with non-local transformation laws. The corresponding multiplet of fields has gauge transformations which suggest an underlying superconformal symmetry.

We analyse in detail the supercurrent [1] corresponding to the $N = 1$ supersymmetric Maxwell theory in ten dimensions ($d = 10$) [2]. Our aim is to obtain information about the off-shell structure of $N = 4$ Poincaré supergravity in $d = 4$, which is related to $N = 1$ $d = 10$ supergravity by dimensional reduction. The supercurrent has been shown to be a useful tool in the construction of off-shell representations of supergravity theories (for a review, see ref. [3]). In this letter we shall restrict ourselves to the analysis of the off-shell multiplets related to the supercurrent in $d = 10$. Partial results about the supercurrent were presented in a recent paper [4].

Somewhat surprisingly, the complete results are rather simple. The supercurrent in $d = 10$ contains 5760 bosonic and 5760 fermionic components. However, it is reducible, and can be decomposed into two submultiplets. One of these is generated by the traceless part of the energy–momentum tensor. It contains 128 + 128 components, and is clearly the smallest off-shell representation of the $d = 10$ supersymmetry algebra. The $N = 4$ conformal supercurrent in $d = 4$ also has 128 + 128 components [5], and it can be obtained from our $d = 10$ multiplet by dimensional reduction. The second submultiplet is generated by the trace of the energy–momentum tensor. This trace-multiplet contains 5632 + 5632 components and forms a constrained scalar superfield in $d = 10$.

Clearly, we find ourselves in a position somewhat different from Howe and Lindström [6], who obtained the supercurrent in $d = 5$. They found that the supercurrent in $d = 5$ has 128 + 128 components, which on reduction and after a field redefinition yield the $N = 4$ $d = 4$ conformal supercurrent. However, in $d = 5$ the supercurrent is not reducible and contains both the trace and the traceless part of the energy–momentum tensor. We have in $d = 10$ two irreducible multiplets and it will be interesting to see what role they play on reduction to $d = 4$.

The relationship with the scalar superfield is interesting in itself, Nilsson [7] showed that the on-shell $d = 10$ $N = 1$ supergravity theory can be completely described by a constrained scalar superfield. This constraint, which is sufficiently strong to put the scalar superfield Φ on-shell, is algebraic. We show that there is a weaker, differential constraint on Φ which reduces it to the off-shell trace-multiplet.

The underlying theory which is used to construct the supercurrent is the $d = 10$ supersymmetric Maxwell theory. It contains a vector field A_μ, and a Majorana–Weyl spinor ‡1. The lagrangian density reads

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{2}\bar{\chi}\partial\!\!\!/\chi, \quad (1)$$

and is invariant under the supersymmetry transformations

$$\delta A_\mu = \tfrac{1}{2}\bar{\epsilon}\Gamma_\mu\chi, \quad \delta\chi = -\tfrac{1}{4}\Gamma^{\mu\nu}F_{\mu\nu}\epsilon. \quad (2)$$

The construction of the supercurrent starts from the energy–momentum tensor and the supersymmetry current. They have the same form as in $d = 4$ and read

$$\theta_{\mu\nu} = 4F_{\mu\alpha}F_{\nu\alpha} - \delta_{\mu\nu}F^2 + \bar{\chi}(\Gamma_\mu\partial_\nu + \Gamma_\nu\partial_\mu)\chi, \quad (3)$$

$$J_\mu = \tfrac{1}{4}\Gamma\cdot F\Gamma_\mu\chi. \quad (4)$$

Using the equations of motion one easily sees that they are conserved. However, contrary to $d = 4$, $\theta_{\mu\nu}$ is not traceless, nor does $\Gamma\cdot J$ vanish. Instead one finds

$$\theta_{\mu\mu} = -6F^2, \quad \Gamma\cdot J = \tfrac{3}{2}\Gamma\cdot F\chi. \quad (5)$$

‡1 We use the same conventions and notations as ref. [4]. See also the appendix of ref. [4] for useful identities concerning Dirac matrices in $d = 10$.

Supersymmetry variations of (3) and (4), using (2), show the existence of many other currents [4]. From the variation of J_μ one concludes that the supercurrent contains a tensor

$$X_{\alpha\beta\gamma} = \bar{\chi}\Gamma_{\alpha\beta\gamma}\chi. \quad (6)$$

This is the lowest-dimensional bilinear one can form from $F_{\mu\nu}$ and χ, and it is therefore tempting to identify the supercurrent with a superfield $\Phi_{\alpha\beta\gamma}$ ‡2, which satisfies certain constraints [4]. To analyse the superfield $\Phi_{\alpha\beta\gamma}$, it is clearly useful to know which representations are contained in the scalar superfield Φ. Therefore we proceed by first examining the content of Φ.

The scalar superfield $\Phi(x,\theta)$ can be expanded in θ:

$$\Phi(x,\theta) = A + \bar{\theta}\psi + \bar{\theta}\Gamma_{\alpha\beta\gamma}\theta B_{\alpha\beta\gamma} + ..., \quad (7)$$

where θ is a Majorana–Weyl spinor. It contains 2^{16} components. The fermionic representations all form multiplets of 16, since the basic spinor representation of SO(10) has dimension 16. In table 1 we present the SO(10) representations contained in Φ, up to and including the 8-θ sector (higher sectors can be obtained

‡2 This was also noted by P. Howe.

Table 1
The scalar superfield in 10 dimensions.

Sector ($n\theta$)	SO(10)	Dimension	Reduction to SO(9)
0	[0, 0, 0, 0, 0]	1	1
1	$[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}]$	16	16
2	[1, 1, 1, 0, 0]	120	84 + 36
3	$[\frac{3}{2}, \frac{3}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}]$	35 × 16	(27 + 8) × 16
4	[2, 2, 0, 0, 0]	770	495 + 231 + 44
	[2, 1, 1, 1, 1]	1050	924 + 126
5	$[\frac{3}{2}, \frac{3}{2}, \frac{3}{2}, \frac{3}{2}, \frac{3}{2}]$	42 × 16	42 × 16
	$[\frac{5}{2}, \frac{3}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}]$	231 × 16	(160 + 36 + 27 + 8) × 16
6	[3, 1, 1, 0, 0]	4312	2457 + 910 + 594 + 231 + 84 + 36
	[2, 2, 1, 1, 1]	3696	2772 + 924
7	$[\frac{5}{2}, \frac{3}{2}, \frac{3}{2}, \frac{1}{2}, \frac{1}{2}]$	550 × 16	(315 + 160 + 48 + 27) × 16
	$[\frac{7}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}]$	165 × 16	(120 + 36 + 8 + 1) × 16
8	[4, 0, 0, 0, 0]	660	450 + 156 + 44 + 9 + 1
	[2, 2, 2, 0, 0]	4125	1980 + 1650 + 495
	[3, 1, 1, 1, 1]	8085	3900 + 2457 + 924 + 594 + 126 + 84

by reflection around 8). Irreducible representations of SO(N) can be characterized by [N/2] numbers, which represent the form of the corresponding Young tableaux. Their dimensions can be obtained from standard representation theory, useful references are [8,9]. We reduce the SO(10) representations to representations of SO(9) by imposing conditions with derivatives. This reduction is indicated in the fourth column of table 1. For example, the representation 120 of SO(10), an antisymmetric tensor $B_{\alpha\beta\gamma}$, splits into an 84 (a tensor satisfying $\partial^\alpha B_{\alpha\beta\gamma} = 0$) and a 36 (the divergence of B), which are representations of SO(9). In the higher sectors it should be kept in mind that tensors with 5 antisymmetric indices can satisfy a duality condition. This implies in some cases that imposing a condition with a derivative causes the tensor to vanish. This is the case for instance with the 42 × 16 dimensional spinor representation in the 5-θ sector, which therefore does not reduce further.

One can obtain the content of $\Phi_{\alpha\beta\gamma}$ by multiplying the representations in table 1 with the external indices [$\alpha\beta\gamma$]. One can compare this with the set of all possible representations that can be constructed from the basic tensorial structures ∂_μ, $F_{\mu\nu}$ and χ, bilinear in $F_{\mu\nu}$ and χ, with $F_{\mu\nu}$ and χ satisfying:

$$\partial^\mu F_{\mu\nu} = 0, \quad \partial_{[\mu} F_{\nu\rho]} = 0, \quad \not\partial \chi = 0. \tag{8}$$

The overlap between the two sets of representations indicates, that the number of components of the supercurrent does not exceed the number of components of the scalar superfield Φ itself. If the supercurrent is contained in Φ, the first component of Φ must be the trace of the energy–momentum tensor, this being the only scalar one can construct from ∂_μ, $F_{\mu\nu}$ and χ.

We therefore consider the supersymmetry variation of the trace of the energy–momentum tensor. One finds

$$\delta\theta_{\mu\mu} = -2\bar{\epsilon}\psi,$$

$$\delta\psi = -\tfrac{1}{8}\not\partial\theta_{\mu\mu}\epsilon - \tfrac{3}{2}\Gamma^{\alpha\beta\gamma}\epsilon T_{\alpha\beta\gamma},$$

$$\delta T_{\alpha\beta\gamma} = \tfrac{3}{4}\bar{\epsilon}\Gamma_{[\alpha}\Lambda_{\beta\gamma]} + \tfrac{1}{288}\bar{\epsilon}\not\partial\Gamma_{\alpha\beta\gamma}\psi, \tag{9}$$

where we have defined

$$\psi = \not\partial\Gamma\cdot J,$$

$$T_{\alpha\beta\gamma} = \partial_\lambda(F_{\lambda[\alpha}F_{\beta\gamma]}) - \tfrac{1}{48}\Box X_{\alpha\beta\gamma} - \tfrac{1}{8}\partial_\lambda\partial_{[\alpha}X_{\beta\gamma]\lambda}, \tag{10}$$

$$\Lambda_{\alpha\beta} = \Box(F_{\alpha\beta}\chi) - \tfrac{1}{6}\Gamma_{[\alpha}\Box J_{\beta]} - \tfrac{2}{3}\not\partial\partial_{[\alpha}J_{\beta]} + \tfrac{5}{216}\Gamma_{\alpha\beta}\not\partial\psi + \tfrac{1}{12}\Gamma_{[\alpha}\partial_{\beta]}\chi. \tag{10 con'd}$$

The spinor–tensor $\Lambda_{\alpha\beta}$ satisfies $\Gamma^\alpha\Lambda_{\alpha\beta} = 0$, so that it corresponds to the 35 × 16 dimensional representation of SO(10). In this manner, one indeed generates the beginning of the scalar superfield Φ. As one has only ∂_μ, $F_{\mu\nu}$ and χ available to construct representations, it is clear that it is impossible to obtain all fermionic representations contained in Φ. For instance, it is impossible to construct the 42 × 16 dimensional representation in the 5-θ sector. This was the first indication of the reducibility of the scalar superfield. As it turns out, the condition which is ultimately responsible for this reduction is the differential identity

$$\partial_\mu[\bar{\chi}\Gamma_{\alpha\beta\gamma\delta(\mu}\overleftrightarrow{\partial}_{\nu)}\chi - \text{traces}]\,\epsilon^{\nu\alpha\beta\gamma\delta\lambda_1\ldots\lambda_5} = 0, \tag{11}$$

which tells us that the 126 dimensional representation in the 4-θ sector vanishes. One then has to show which other structures in higher sectors remain or are caused to vanish by this condition. The result is presented in table 2. Note that the last component is a 4-index symmetric tensor, indicating that on reduction to $d = 4$ this multiplet will contain field components with highest spin equal 4. In terms of F and χ, this tensor is given by

$$\Box(F_{\mu(\lambda}\overleftrightarrow{\partial}_\nu\overleftrightarrow{\partial}_\rho F_{\sigma)\mu} - \tfrac{1}{4}\bar{\chi}\overleftrightarrow{\partial}_{(\lambda}\overleftrightarrow{\partial}_\nu\overleftrightarrow{\partial}_\rho\Gamma_{\sigma)}\chi) - \text{traces, divergences}. \tag{12}$$

One can verify that its variation gives only derivatives

Table 2
The reduced scalar superfield in $d = 10$.

Sector ($n\theta$)	Representation of SO(9)
0	1
1	16
2	84 + 36
3	(27 + 8) × 16
4	495 + 231 + 44
	924
5	(160 + 36) × 16
6	2457 + 910
7	120 × 16
8	450

of the 120 × 16 dimensional representation in the 7-θ sector. We conclude that the trace of the energy–momentum tensor generates a multiplet with 5632 + 5632 components.

The on-shell superfield analyzed by Nilsson [7] is obtained by setting the 1050 dimensional representation in the 4-θ sector equal to zero. We have imposed a weaker condition: we set to zero only the 126 dimensional representation in this sector by a differential constraint. This constraint is sufficiently weak to obtain an off-shell multiplet.

However, the trace-multiplet does not contain the traceless part of $\theta_{\mu\nu}$ ($\hat{\theta}_{\mu\nu}$) nor that part of J_μ ($\hat{J}_\mu$) which satisfies $\Gamma\cdot\hat{J} = 0$. Also, we know that the supercurrent contains a 120 of SO(10) in the form of $X_{\alpha\beta\gamma}$ [see eq. (6)] and an 84 represented by $F_{[\alpha\beta}F_{\gamma\delta]}$ (see ref. [4]). In the trace-multiplet we only have one particular combination of these representations, namely $T_{\alpha\beta\gamma}$ in (10), which is a 120 dimensional representation. The number of missing components is therefore 84, 44 for $\hat{\theta}_{\mu\nu}$, and 8 × 16 = 128 for $\hat{J}_\mu$. This adds up to 128 + 128 components.

We now consider the supersymmetry variation of $\hat{\theta}_{\mu\nu}$. We find that indeed the variations close on 128 + 128 components. The transformation rules are:

$$\delta\hat{\theta}_{\mu\nu} = 2\bar{\epsilon}\Gamma_{(\mu\lambda}\partial_\lambda\hat{J}_{\nu)}, \tag{13}$$

$$\delta\hat{J}_\mu = -\tfrac{1}{8}\Gamma_\nu\epsilon\hat{\theta}_{\mu\nu} - \tfrac{1}{36}(\Gamma_\mu{}^{\alpha\beta\gamma\delta} - 6\delta_\mu^\alpha\Gamma^{\beta\gamma\delta})\epsilon \times [\delta_{\alpha\lambda} - (1/\Box)\partial_\alpha\partial_\lambda]G_{\lambda\beta\gamma\delta}, \tag{14}$$

$$\delta G_{\alpha\beta\gamma\delta} = \bar{\epsilon}\Gamma_{[\alpha\beta}\partial_\gamma\hat{J}_{\delta]}. \tag{15}$$

The tensors $\hat{\theta}$, $\hat{J}$ and G are given by

$$\hat{\theta}_{\mu\nu} = \theta_{\mu\nu} - \tfrac{1}{9}[\delta_{\mu\nu} - (1/\Box)\partial_\mu\partial_\nu]\theta_{\lambda\lambda}, \tag{16}$$

$$\hat{J}_\mu = J_\mu - \tfrac{1}{9}[\Gamma_\mu - (1/\Box)\partial_\mu\partial\!\!\!/]\Gamma\cdot J, \tag{17}$$

$$G_{\alpha\beta\gamma\delta} = F_{[\alpha\beta}F_{\gamma\delta]} + \tfrac{1}{6}\partial_{[\alpha}X_{\beta\gamma\delta]}. \tag{18}$$

The particular combination (18) was already given in ref. [4]. The currents (16)–(18) satisfy the constraints

$$\partial_\mu\hat{\theta}_{\mu\nu} = \hat{\theta}_{\lambda\lambda} = \partial\cdot\hat{J} = \Gamma\cdot\hat{J} = 0, \tag{19}$$

$$\partial_{[\alpha}G_{\beta\gamma\delta\epsilon]} = 0. \tag{20}$$

Notice that the transformation rule (14) contains a nonlocal projection operator. This ensures that the supersymmetry transformations are consistent with the constraints (19), (20). The commutator of two supersymmetry transformations is given by

$$[\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)] = \tfrac{1}{2}\bar{\epsilon}_2\Gamma^\mu\epsilon_1\partial_\mu, \tag{21}$$

on all currents. This follows from the fact that the currents (16)–(18) have been constructed explicitly in terms of bilinears in $F_{\mu\nu}$ and χ, on which the algebra (21) closes on-shell. In general, the product of two on-shell representations is an off-shell representation.

This concludes the construction of the supercurrent in $d = 10$. It contains a total number of 5760 bosonic and 5760 fermionic degrees of freedom.

At this stage we have an interesting check on our algebra. The 128 + 128 component multiplet is the smallest off-shell multiplet in $d = 10$. Therefore, all other multiplets can be obtained from it by adding external indices, or by multiplying the representations in the smallest multiplet by the appropriate spin representation. Note, that the trace-multiplet has 5632 + 5632 = 44 × (128 + 128) components. The 44 corresponds to a traceless symmetric representation of SO(9), i.e. the representation [2, 0, 0, 0]. If we multiply the representations of SO(9) contained in the multiplet (13)–(15), i.e. [2, 0, 0, 0], $[\tfrac{3}{2}, \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}]$ and [1, 1, 1, 0], by [2, 0, 0, 0], we obtain precisely the representations of the trace-multiplet:

$$44 \times 44 = 1 + 36 + 44 + 450 + 495 + 910,$$

$$(44 \times 8) \times 16 = (1 + 8 + 27 + 36 + 120 + 160) \times 16,$$

$$44 \times 84 = 84 + 231 + 924 + 2457.$$

Let us now discuss the reduction of the $d = 10$ current multiplet to four dimensions. The currents (16)–(18) reduce to the $N = 4$ $d = 4$ conformal supercurrent [5]. In order to make the identification with the currents of ref. [5] one has to make redefinitions. These involve derivatives in such a way, that in $d = 4$ there is no need for a projection operator in the transformation rules.

In the reduction to $d = 4$ the trace-multiplet gives rise to 4 irreducible $N = 4$ multiplets. They can be identified by the SU(4) representation of the highest spin contained in them. There is a spin-4 singlet multiplet (640 + 640 components), a spin-3 multiplet in a 6 dimensional representation of SU(4) (2304 + 2304 components) and spin-2 multiplets in a singlet and in a real 20 dimensional representation (128 + 128 and 2560

+ 2560 components). The off-shell spin-3 multiplet has been investigated previously [10,11]. It can be constrained in such a way that it contains only the representations of the on-shell Yang–Mills theory. We now find the spin-3 multiplet in a 6 dimensional representation of SU(4), so that there are in fact six Yang–Mills multiplets. This may be relevant in the construction of off-shell $N = 4$ Poincaré supergravity from the conformal theory, where they should play the role of compensating multiplets. A possible conclusion from our results is, that the spin-3 multiplet itself is not sufficient to achieve this, but that the spin-4 and the two spin-2 multiplets are needed as well.

We remark that the spin-4 and spin-2 singlet multiplets can be generated from bilinears of abelian, on-shell $N = 4$ Yang–Mills theory in $d = 4$, whereas this is impossible for the spin-3 and the spin-2 20 multiplet (however, these multiplets can be generated from non-abelian Yang–Mills bilinears). Therefore, if one first reduces the $d = 10$ abelian Yang–Mills theory and then the $d = 10$ currents in terms of explicit bilinears, only two of the four multiplets contained in the trace-multiplet survive the reduction to $d = 4$. Nevertheless, all four multiplets can and will play a role in $d = 4$ Poincaré theory, since in the reduction to $d = 4$ one should consider the trace-multiplet as an abstract multiplet in $d = 10$, without reference to the explicit form of the bilinear expressions.

Of course, we can also reduce the currents (16)–(18) from $d = 10$ to $d = 5$. This yields the $d = 5$ supercurrent [6], which contains both the trace and the traceless part of the $d = 5$ energy–momentum tensor. The spin-4 multiplet can be generated in $d = 5$ from the on-shell abelian Yang–Mills theory, if one varies the combination $F_{ab}^2 + 2\Box(W_{ij}W^{ij})$ [where F_{ab} is the Yang–Mills field-strength and W_{ij} the spin-0 5 of Sp(4)]. The trace of the energy–momentum tensor $F_{\mu\nu}^2$ in $d = 10$ reduces to the sum of the trace of the $d = 5$ stress tensor, which is given by $F_{ab}^2 + 3\Box(W_{ij}W^{ij})$, and the combination which generates the spin-4 multiplet. It remans an intruiging problem, to understand why the unimproved supercurrent in $d = 5$ is irreducible.

To the current multiplet (13)–(15) one can associate a multiplet of fields with the same number of components. Because of the constraints (19), (20), these fields will have gauge transformations. Considering the specific form of these constraints, it is not surprising that the gauge transformations are reminiscent of superconformal symmetries. We write the fields and their transformations as:

$$\delta e_\mu{}^a = \tfrac{1}{2}\bar{\epsilon}\Gamma^a\psi_\mu, \tag{22}$$

$$\delta\psi_\mu = D_\mu(\omega(e))\epsilon + F_{\mu\alpha\beta}\Gamma^{\alpha\beta}\epsilon, \tag{23}$$

$$\delta A_{\alpha\beta\gamma\delta} = -\tfrac{1}{36}\bar{\epsilon}[\delta_{\rho[\alpha} - (1/\Box)\partial_\rho\partial_{[\alpha}] \times (\Gamma_{\beta\gamma\delta]\rho\sigma} - 6\Gamma_{\beta\gamma\delta]}\delta_{\rho\sigma})\psi_\sigma. \tag{24}$$

Here $F_{\alpha\beta\gamma}$ is the field-strength of a 4-index antisymmetric gauge field $A_{\mu\nu\lambda\rho}$, which has gauge transformations involving a parameter completely antisymmetric in 5 indices:

$$\delta A_{\alpha\beta\gamma\delta} = \partial^\lambda \Lambda_{\lambda\alpha\beta\gamma\delta}. \tag{25}$$

Then, $F_{\alpha\beta\gamma}$ is given by

$$F_{\alpha\beta\gamma} = \partial^\lambda A_{\lambda\alpha\beta\gamma}. \tag{26}$$

The gauge transformations of ψ_μ are both

$$\delta_Q\psi_\mu = \partial_\mu\epsilon, \quad \delta_S\psi_\mu = \Gamma_\mu\eta. \tag{27}$$

We have verified that the commutator algebra on these fields closes modulo gauge transformations.

In the same way there is a field multiplet corresponding to the complete multiplet of currents. One expects this multiplet to contain the physical fields of Poincaré supergravity in $d = 10$, and indeed it was shown in ref. [4] how this identification should be made. This off-shell multiplet is not contained in a scalar superfield Φ. However, the on-shell Poincaré fields do form a constrained scalar superfield [7]. The precise relation between these on-shell and off-shell formulations is an interesting problem.

There are arguments [12], that in $d = 10$ superconformal algebras do not exist. Nevertheless, the gauge transformations of the fields $(e_\mu^a, \psi_\mu, A_{\alpha\beta\gamma\delta})$ suggest the presence of superconformal symmetry in $d = 10$. Indications for such symmetries were already obtained in ref. [4]. One of the unusual features is the presence of non-local projection operators in the transformation rules. Whether or not the field multiplet forms a gauge field representation of a superconformal algebra is currently being investigated.

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