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## (8,0) LOCALLY SUPERSYMMETRIC SIGMA MODELS WITH CONFORMAL INVARIANCE IN TWO DIMENSIONS

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The (8,0) conformal supergravity, and an action which describes its coupling to an arbitrary number of (8,0) scalar multiplets are constructed. The 64 + 64 components of the conformal supermultiplet occur as Lagrange multipliers which lead to differential and algebraic constraints on the fields of the scalar multiplets. Solving the algebraic constraints yields an (8,0) locally supersymmetric sigma model based on the manifold  $SO(8+n,m)/SO(8) \times SO(n,m)$ , where  $n, m \geq 0$ .

*1 Introduction* Actions describing the coupling of an arbitrary number of scalar multiplets to conformal supergravity in two dimensions have been constructed with  $(p,q)$  supersymmetry, for  $p, q \leq 4$ <sup>1,2</sup>. Essentially, these models are described by locally supersymmetric sigma models with Wess–Zumino term and heterotic fermions, i.e. fermions without on-shell bosonic partners.

Having possible string applications in mind, it is clearly of interest to inquire as to whether one can construct  $(p,q)$  models with  $p, q > 4$ .

From the work of Alvarez-Gaumé and Freedman [14] it is known that *globally* supersymmetric sigma models exist only for  $p, q \leq 4$ . On the other hand, the dimensional reduction of ten-dimensional Yang–Mills coupled conformal supergravity [15] to two

dimensions is expected to give an (8,8) or (8,0) conformal supergravity coupled to scalar multiplets. This suggests that, in order to construct  $(p,q)$  models with  $p, q > 4$ , one should consider the *local* supersymmetry as an essential ingredient of the theory. This is despite the fact that the fields of the supergravity multiplet do not have dynamics in two dimensions. In fact, a similar situation arises in three dimensions, where the construction of  $N=8$  or  $N=16$  sigma models requires the coupling of a non-dynamical supergravity multiplet [16].

In this letter, we indeed construct an action for an (8,0) locally supersymmetric sigma model. Our strategy in this construction is as follows. We first extract the field content of the (8,0) superconformal multiplet, and the matter scalar multiplet by dimensional reduction of ten-dimensional conformal supergravity [15], and Yang–Mills multiplet [17], respectively. We then construct the transformation rules and the action of the two-dimensional model by the Noether procedure.

We next observe that the fields of the conformal supermultiplet occur as Lagrange multipliers which lead to differential and algebraic constraints on the

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<sup>2</sup> For the (1,1) model see refs [1–3], for the (1,0) and (2,0) models see refs [4–7], for the (2,2) model see ref [8], the (4,4) model was considered in ref [9], the (4,0) model with Wess–Zumino term is given in ref [10].

<sup>3</sup> By  $(p,q)$  supersymmetry we mean a supersymmetry algebra generated by  $p$  left-handed and  $q$  right-handed supersymmetry generators [11–13].

fields of the scalar multiplets Solving the algebraic constraints, we then find that the  $8n$  independent scalars of the theory are described by an  $(8,0)$  locally supersymmetric sigma model based on the grassmannian coset space  $SO(8+n,m)/SO(8) \times SO(n,m)$  where  $n, m \geq 0$  Note that both compact and noncompact spaces are allowed It is amusing that for  $m=1$ , these theories have a global Lorentz symmetry  $SO(8+n,1)$

Several features of our construction are similar to those which arise in the coupling of an arbitrary number of vector multiplets to  $N=4$  conformal supergravity in four dimensions [18]

In this letter, we focus our attention to the construction of the action and the transformation rules of the  $(8,0)$  model, and establishing its geometry We shall neither try to establish an infinite dimensional super Virasoro symmetry, nor compute the anomalies associated with the model presented here In the conclusions we shall comment on these issues, and discuss further properties of the model

A more detailed description of the model presented in this letter will be given elsewhere [19]

**2 Action and transformation rules** The field content of the two-dimensional  $(8,0)$  conformal supergravity and scalar multiplet can be easily obtained by a dimensional reduction and subsequent chiral truncation of the ten-dimensional conformal supergravity and vector multiplet A more detailed discussion of this dimensional reduction will be given in ref [19] Here we only give the result (See table 1) Note that all fermionic fields carry both a spacetime spinor index  $\alpha$  ( $\alpha=1, 2$ ) and an  $SO(8)$  spinor index  $A$  ( $A=1, \dots, 16$ ) which are suppressed for brevity In the table we have given the  $SO(8)$  content of the different fields, their scale weight  $w$ , and the spacetime and  $SO(8)$  chirality of the fermionic fields

Knowing the field content, it is not difficult to make an ansatz for the linearized transformation rules of the different fields Once the linearized transformation rules are known one can find their nonlinear extension by requiring closure of the commutator algebra order by order in the fields Given these nonlinear transformation rules one can then construct a supersymmetric action for one scalar multiplet coupled to  $(8,0)$  conformal supergravity by applying the standard Noether procedure Here we will only give

the action up to terms quadratic in the fermionic fields and the transformation rules up to terms bilinear in the fermionic fields The full result will be given in ref [19]

The action and the transformation rules are given by<sup>23</sup>

$$\begin{aligned}
 e^{-1} \mathcal{L} = & \frac{1}{2} g^{\mu\nu} \mathcal{D}_\mu \phi' \mathcal{D}_\nu \phi' + \frac{1}{2} \bar{\psi} \gamma^\mu \mathcal{D}_\mu \psi \\
 & + \frac{1}{8} g^{\mu\nu} \partial_\mu (\phi' \phi') \partial_\nu \phi + \frac{1}{4} \bar{\psi} \gamma^\mu \phi' \gamma^\nu \mathcal{D}_\mu \lambda \\
 & - \bar{\psi} \gamma^\mu \gamma^\nu \gamma^\rho \psi_\mu \mathcal{D}_\nu \phi' - \frac{1}{4} \bar{\psi} \gamma^\mu \phi' \gamma^\nu \gamma^\rho \psi_\mu \partial_\nu \phi \\
 & - \frac{1}{8} \bar{\lambda} \gamma^\mu \gamma^\nu \psi_\mu \partial_\nu (\phi' \phi') - \frac{1}{16} \phi' \phi' R \\
 & + \frac{1}{2} e^{-1} \bar{\psi} \gamma^\mu \phi' \epsilon \rho^\sigma \mathcal{D}_\rho \psi_\sigma + \bar{\psi} \chi' \phi' \\
 & - \frac{1}{2} \phi' \phi' D_{ij} + \frac{1}{2} \phi' \phi' \bar{\psi}_\mu \gamma^\mu \gamma^j \chi' \\
 & + \text{quartic fermions,}
 \end{aligned} \tag{1}$$

$$\delta e_\mu{}^m = -2i \bar{\epsilon} \gamma^m \psi_\mu,$$

$$\delta \psi_\mu = \mathcal{D}_\mu \epsilon + i \gamma_\mu \eta + \text{bilinear fermion terms,}$$

$$\delta V_\mu{}^\nu = -\frac{1}{4} \bar{\epsilon} \gamma_\mu \gamma^\nu \hat{\mathcal{D}} \lambda + i \bar{\epsilon} \gamma_\mu \gamma^{[1} \chi'^{j]}$$

$$+ \text{bilinear fermion terms,}$$

$$\partial \chi^i = -\frac{1}{2} e^{-1} \epsilon^{\mu\nu} V_{\mu\nu}{}^\rho \gamma^\rho \epsilon + D^j \gamma^i \epsilon - \gamma$$

$$- \text{trace} + \text{bilinear fermion terms,}$$

$$\partial D^u = -i \bar{\epsilon} \gamma^{[1} \hat{\mathcal{D}} \chi'^{j]} + 2i \bar{\eta} \gamma^{[1} \chi'^{j]} - \text{trace}$$

$$+ \text{bilinear fermion terms,}$$

$$\delta \lambda = -i \gamma^\mu (\partial_\mu \phi) \epsilon - 2\eta$$

$$+ \text{bilinear fermion terms,} \tag{2}$$

$$\delta \phi = \bar{\epsilon} \lambda,$$

$$\delta \phi' = \bar{\epsilon} \gamma^i \psi,$$

<sup>23</sup> Our conventions are  $\eta_{nm} = (+, -)$ ,  $\gamma_m \gamma_n = \eta_{mn} + \gamma_5 \epsilon_{mn}$ ,  $\epsilon^{mn} \gamma_n = -\gamma^m \gamma_5 \epsilon^{mn} \epsilon_{pq} = -(\delta^m_p \delta^n_q - \delta^m_q \delta^n_p)$ ,  $\gamma_i \gamma_j + \gamma_j \gamma_i = 2\delta_{ij}$  Symmetry  $\bar{\lambda} \chi = \bar{\chi} \lambda$ ,  $\bar{\lambda} \gamma_\mu \chi = -\bar{\chi} \gamma_\mu \lambda$ ,  $\bar{\lambda} \gamma^{i_1 \dots i_n} \chi = \epsilon \bar{\chi} \gamma^{i_1 \dots i_n} \lambda$  with  $\epsilon = +1$  for  $n=1, 4, 5, 8$  and  $\epsilon = -1$  for  $n=2, 3, 6, 7$

Table 1

Fields of (8,0) superconformal multiplets in two dimensions The +suffix in the fourth column denotes the SO(8) chirality of the fermionic fields

	Field	Type	Restrictions	SO(8)	w
conformal supergravity	$e_\mu^m$	boson		1	-1
	$\psi_\mu$	fermion	$\gamma_5 \psi_\mu = +\psi_\mu$	$8_s^+$	-1/2
	$V_\mu^u$	boson	$V_\mu^u = -V_\mu^u$	28	0
	$\chi^i$	fermion	$\gamma_5 \chi^i = +\chi^i, \gamma^i \chi^i = 0$	$56^-$	+3/2
	$D^u$	boson	$D^u = D^u, D^u = 0$	35	+2
	$\lambda$	fermion	$\gamma_5 \lambda = -\lambda$	$8_s^+$	+1/2
	$\phi$	boson		1	+1
scalar multiplet	$\phi^i$	boson		$8_v$	0
	$\psi$	fermion	$\gamma_5 \psi = -\psi$	$8_s^-$	+1/2

$$\delta\psi = -1\gamma^\mu (\mathcal{D}_\mu \phi^i) \gamma^i \epsilon$$

+ bilinear fermion terms (3)

In (1), (2) and (3) we have used the following definitions The derivatives  $\mathcal{D}_\mu$  are covariant with respect to SO(8) and Lorentz rotations, e g

$$\mathcal{D}_\mu \phi^i \equiv \partial_\mu \phi^i - V_\mu^u \gamma_u \psi,$$

$$\mathcal{D}_\mu \psi \equiv \partial_\mu \psi - \frac{1}{4} V_\mu^u \gamma_u \psi - \frac{1}{2} \omega_\mu \psi$$
 (4)

The spin connection  $\omega_\mu$ , the curvature scalar  $R$  and the SO(8) curvature tensor  $V_\mu^u$  are defined by

$$\omega_\mu = -e^{-1} \epsilon^{\rho\sigma} (e_\mu^m \partial_\rho e_\sigma^m + 1\bar{\psi}_\rho \gamma_\mu \psi_\sigma),$$

$$R = 2e^{-1} \epsilon^{\mu\nu} \partial_\mu \omega_\nu,$$

$$V_{\mu\nu}^u = \partial_\mu V_\nu^u - \partial_\nu V_\mu^u - V_\mu^{ik} V_\nu^{kj} + V_\nu^{ik} V_\mu^{kj}$$
 (5)

The spinor parameters  $\epsilon$  and  $\eta$  characterize a supersymmetry and a super Weyl transformation, respectively They are spinors of SO(8) and satisfy the following chirality conditions

$$\gamma_5 \epsilon = +\epsilon, \quad \gamma_5 \eta = -\eta,$$

$$\gamma_9 \epsilon = +\epsilon, \quad \gamma_9 \eta = +\eta$$
 (6)

Finally the derivatives  $\mathcal{D}_\mu$  are covariant with respect to Lorentz, SO(8), supersymmetry and super Weyl transformations

$$\hat{\mathcal{D}}_\mu \chi^i = (\partial_\mu \chi^i + \frac{1}{2} \omega_\mu \chi^i - \frac{1}{4} V_\mu^{kl} \gamma_{kl} \chi^i - V_\mu^u \gamma^u \chi^i) + \frac{1}{2} e^{-1} \epsilon^{\rho\sigma} V_{\rho\sigma}^u \gamma^u \psi_\mu - D^u \gamma^u \psi_\mu - \gamma - \text{trace},$$
 (7)

$$\gamma^\mu \hat{\mathcal{D}}_\mu \lambda = \gamma^\mu (\partial_\mu \lambda - \frac{1}{2} \omega_\mu \lambda - \frac{1}{4} V_\mu^u \gamma_u \lambda) + 1\gamma^\mu \gamma^\nu \psi_\mu (\partial_\nu \phi) - 21e^{-1} \epsilon^{\rho\sigma} \mathcal{D}_\rho \psi_\sigma$$

3 The nonlinear theory and the geometry The action (1) is quadratic and the transformation rules (2), (3) are linear in the fields of the scalar multiplet Therefore we can generalize the results of the previous section to an arbitrary number of scalar multiplets

$$\phi_I^i, \psi_I \quad I=1, \dots, p,$$
 (8)

i e the action (1) is generalized to

$$e^{-1} \mathcal{L} = \frac{1}{2} g^{\mu\nu} \mathcal{D}_\mu \phi_I^i \mathcal{D}_\nu \phi_J^j \eta^{IJ} + \dots,$$
 (9)

where all terms are constructed in the same fashion, using a constant real metric  $\eta_{IJ}$  Without loss of generality we can take  $\eta_{IJ}$  diagonal, with eigenvalues  $\pm 1$

The action (9) is linear in the field  $D^u$  of the superconformal multiplet, and its equation of motion imposes restrictions on  $\phi_I^i$  The field equation of  $D^u$  reads

$$\eta^{IJ} \phi_I^i \phi_J^j = \frac{1}{8} \delta^{ij} \eta^{IJ} \phi_I^k \phi_J^k$$
 (10)

We must solve this equation for the fields  $\phi_I^i$  This is possible only if  $p \geq 8$  We solve (10) by the following ansatz

$$\phi_I^i = \frac{1}{2} \sqrt{r} L_I^i, \quad r \equiv \frac{1}{2} \eta^{IJ} \phi_I^k \phi_J^k,$$
 (11)

where

$$L_I^A \mapsto (L_I^I, L_I^a),$$

$$I, A=1, \dots, n+m+8,$$

$$i=1, \dots, 8, \quad a=1, \dots, n+m \quad (12)$$

is an  $(n+8) \times (n+8)$  matrix, which is a representative element of the coset manifold  $SO(8+n, m)/SO(8) \times SO(n, m)$  with  $n, m \geq 0$ . The inverse matrix  $L^{-1} = \eta L^T \eta$  satisfies the following relations

$$L_I^i L^j = \delta^{ij}, \quad L_I^a L^b = 0,$$

$$L_I^a L^b = +h^{ab}, \quad (13)$$

where  $h^{ab} = \text{diag}(n \text{ times } +, m \text{ times } -)$  is a constant tensor which can be used to raise and lower or to contract the  $SO(n, m)$  indices  $a, b=1, \dots, n+m$

The equation of motion of the fields  $\chi^i$  of the superconformal multiplet imposes further restrictions on  $\psi_I$ . The  $\chi^i$  field equation is

$$L_I^i \psi^I = \frac{1}{8} \gamma^i \gamma^j L_I^j \psi^I, \quad (14)$$

where we have used (10) and (11). The solution of (14) is given by

$$\psi_I = \sqrt{r} L_I^a \psi^a + (1/4\sqrt{r}) \gamma^i L_I^i \chi,$$

$$\chi = \frac{1}{2} \sqrt{r} \gamma^i L_I^i \psi^I \quad (15)$$

Finally, the equation of motion of  $V_\mu^u$  is given by

$$\phi^{I(i} \hat{\mathcal{D}}_\mu \phi^{j)} = \frac{1}{8} 1 \bar{\psi} \gamma_\mu \gamma^i \psi + \frac{1}{16} 1 \bar{\psi} \gamma^k \phi^k \gamma_\mu \gamma^j \lambda, \quad (16)$$

and can be used to solve for  $V_\mu^u$  algebraically

$$V_\mu^u = Q_\mu^u$$

$$+ \frac{1}{2} 1 (\bar{\psi}^a \gamma_\mu \gamma^u \psi^a + \frac{1}{4} r^{-2} \bar{\chi} \gamma_\mu \gamma^u \chi)$$

$$+ (1/2\sqrt{r}) \bar{\chi} \gamma^u \psi_\mu + \frac{1}{4} 1 r^{-1} \bar{\chi} \gamma^u \gamma_\mu \lambda \quad (17)$$

Here  $Q_\mu^u$  is a composite  $SO(8)$  gauge field defined by

$$Q_\mu^u \equiv -L^{I(i} \partial_\mu L_I^{j)} \quad (18)$$

The equations of motion of the other fields of the superconformal multiplet do not lead to algebrai-

cally solvable equations

Substituting the solutions (11), (15) of  $\phi_I^i, \psi_I$  in terms of  $(L_I^A, r)$  and  $(\psi^a, \chi)$  respectively, and the expression (17) for  $V_\mu^u$  in terms of  $(Q_\mu^u + \text{bilinear fermions})$ , into the action (1) and the transformation rules (2), (3), we obtain the following result for the coupling of  $8(n+m)$  scalar multiplets to  $(8,0)$  conformal supergravity

$$e^{-1} \mathcal{L} = \frac{1}{8} r g^{\mu\nu} \partial_\mu \phi^\alpha \partial_\nu \phi^\beta g_{\alpha\beta} + \frac{1}{2} 1 r \bar{\psi}^a \gamma^\mu \mathcal{D}_\mu \psi^a$$

$$+ \frac{1}{4} g^{\mu\nu} \partial_\mu r \partial_\nu \phi + \frac{1}{4} 1 \bar{\chi} \gamma^\mu \mathcal{D}_\mu \lambda$$

$$- \frac{1}{2} r \bar{\psi}^a \gamma^\mu \gamma^\nu \gamma^i \psi_\mu \partial_\nu \phi^\alpha V_\alpha^{ai} - \frac{1}{4} \bar{\chi} \gamma^\mu \gamma^\nu \psi_\mu \partial_\nu \phi$$

$$- \frac{1}{4} \bar{\chi} \gamma^\mu \gamma^\nu \psi_\mu \partial_\nu r - \frac{1}{8} r R + \frac{1}{2} e^{-1} \bar{\chi} \epsilon^{\rho\sigma} \mathcal{D}_\rho \psi_\sigma$$

$$- \frac{1}{4} 1 \bar{\chi} \gamma^\mu \gamma^i \psi^a \partial_\mu \phi^\alpha V_\alpha^{ai} + \text{quartic fermions}, \quad (19)$$

$$\delta e_\mu^m = -2i \bar{\epsilon} \gamma^m \psi_\mu,$$

$$\delta \psi_\mu = \mathcal{D}_\mu \epsilon + i \gamma_\mu \eta + \text{bilinear fermion terms},$$

$$\delta \phi^\alpha = 2 \bar{\epsilon} \gamma^i \psi^a V_{ai}{}^\alpha,$$

$$\delta \psi^a = -\frac{1}{2} i \gamma^\mu \gamma^i \epsilon \partial_\mu \phi^\alpha V_\alpha^{ai}$$

+ bilinear fermion terms,

$$\delta \chi = -i \gamma^\mu \partial_\mu r \epsilon + \text{bilinear fermion terms},$$

$$\delta \lambda = -i \gamma^\mu \partial_\mu \phi \epsilon - 2\eta + \text{bilinear fermion terms},$$

$$\delta r = \bar{\epsilon} \chi, \quad \delta \phi = \bar{\epsilon} \lambda \quad (20)$$

In (19) and (20) we have used the following definitions. The field  $V_\alpha^{ia}(\phi)$  denotes the  $8(n+m)$ -bein of the coset manifold. It satisfies

$$g^{\alpha\beta} V_\alpha^{ai} V_\beta^{bj} = \delta^{ij} h^{ab},$$

$$h_{ab} V_\alpha^{ai} V_\beta^{bj} + \alpha \leftrightarrow \beta = \frac{1}{4} g_{\alpha\beta} \delta^{ij},$$

$$\delta_{ij} V_\alpha^{ai} V_\beta^{bj} + \alpha \leftrightarrow \beta = [2/(n-m)] g_{\alpha\beta} h^{ab}, \quad (21)$$

where  $g_{\alpha\beta}(\phi)$  is the metric of the manifold. We have also used the relation

$$L^{Ia} \partial_\mu L_I^i = \partial_\mu \phi^\alpha V_\alpha^{ai} \quad (22)$$

The derivatives  $\mathcal{D}_\mu$  are covariant with respect to Lor-

entz, composite SO(8) and composite SO(*n,m*) rotations, e g

$$\begin{aligned} \mathcal{D}_\mu \epsilon &\equiv \partial_\mu \epsilon + \frac{1}{2} \omega_\mu \epsilon - \frac{1}{4} Q_\mu{}^\nu \gamma_{\nu} \epsilon, \\ \mathcal{D}_\mu \psi^a &= \partial_\mu \psi^a - \frac{1}{2} \omega_\mu \psi^a \\ &\quad - \frac{1}{4} Q_\mu{}^\nu \gamma_{\nu} \psi^a + Q_\mu{}^{ab} \psi^b \end{aligned} \quad (23)$$

Here  $Q_\mu{}^{ab}$  is the composite gauge field of SO(*n,m*) rotations defined by

$$Q_\mu{}^{ab} \equiv L^{Ia} \partial_\mu L_I{}^{b1} \quad (24)$$

The 8(*n+m*)-bein fields  $V_\alpha{}^{ia}$  satisfy the following vielbein postulate

$$\begin{aligned} \partial_\alpha V_\beta{}^{ai} - \{ \gamma_{\alpha\beta} \} V_\gamma{}^{ai} - Q_\alpha{}^\nu V_\beta{}^{aj} \\ + Q_\alpha{}^{ab} V_\beta{}^{bi} = 0, \end{aligned} \quad (25)$$

with  $Q_\alpha{}^\nu = Q_\mu{}^\nu (\partial_\mu \phi^\alpha)$  and  $Q_\alpha{}^{ab} = Q_\mu{}^{ab} (\partial_\mu \phi^\alpha)$  In obtaining (19) and (20) we have performed the following field redefinitions

$$\varphi \mapsto \varphi - \ln r, \quad \lambda \mapsto \lambda - (1/\sqrt{r})\chi \quad (26)$$

Note that the fields (*r*, $\chi$ ) and ( $\varphi$ , $\lambda$ ) do not describe physical degrees of freedom The field equations of  $e_\mu{}^m, \psi_\mu$  imply that *r* and  $\chi$  are constant, while  $\varphi$  and  $\lambda$  are gauge degrees of freedom

The action (19) can easily be generalized to include heterotic fermions as well Up to quartic fermions we find

$$\begin{aligned} e^{-1} \mathcal{L}(\text{heterotic}) &= \frac{1}{2} i \bar{\psi}^r \gamma^\mu (\partial_\mu \delta^{rs} + A_\alpha{}^{rs} \partial_\mu \phi^\alpha) \psi^s \\ &\quad + \text{quartics}, \end{aligned} \quad (27)$$

$$\begin{aligned} \delta \psi^r &= -\delta \phi^\alpha A_\alpha{}^{rs} \psi^s \\ &\quad + \text{bilinear fermion terms}, \end{aligned} \quad (28)$$

where  $A_\alpha{}^{rs}$  is a composite gauge field for some Yang–Mills gauge group

#### 4 Comments

(a) From  $[\mathcal{D}_\alpha, \mathcal{D}_\beta] V_\alpha{}^{ia} = 0$  it follows that the curvature tensor of the scalar manifold decomposes as follows

$$R_{\alpha\beta}{}^{a1,b1} = R_{\alpha\beta}{}^{ab} \delta^{11} + R_{\alpha\beta}{}{}^{\nu} h^{ab} \quad (29)$$

From the usual definition of the Riemann tensor, and from (22), one finds

$$\begin{aligned} R_{\alpha\beta}{}^{ab} &= \kappa^2 (V_\alpha{}^{ai} V_\beta{}^{bj} - \alpha \leftrightarrow \beta) \delta_{ij}, \\ R_{\alpha\beta}{}{}^\nu &= \kappa^2 (V_\alpha{}^{ai} V_\beta{}^{bj} - \alpha \leftrightarrow \beta) h_{ab}, \end{aligned} \quad (30)$$

where we have introduced the gravitational coupling constant  $\kappa$  Consequently, in the global supersymmetry limit both curvatures vanish since  $\kappa$  goes to zero In that limit the fields (*r*, $\chi$ ) decouple from the physical scalar multiplet fields ( $\phi^\alpha, \psi^a$ ), and can consistently be set equal to (1,0) The lagrangian then reduces to free kinetic terms for ( $\phi^\alpha, \psi^a$ ) This lagrangian coincides with the one given in ref [20] which describes the field theory of the AdS<sub>3</sub> singleton multiplet

(b) We expect that the (8,0) conformal supergravity theory described in this paper corresponds to the gauge theory of the conformal superalgebra  $\text{OSp}(2,8) \oplus \text{OSp}(2,8)$  [21]

(c) In the conformal gauge  $\varphi=1$  and  $\lambda=0$ , the (8,0) model for *n*=0 and without the heterotic fermions coincides with the model one would obtain by dimensional reduction and subsequent chiral truncation of the *N*=8, *d*=3 Poincaré supergravity model of Marcus and Schwarz [16] This is remarkable because the compensating fields  $\varphi$  and  $\lambda$  belong to the conformal supermultiplet itself, as opposed to being independent matter fields as is usually the case in a Brans–Dicke type theory The only other known conformal supergravities which contain their own compensators are the conformal supergravity in *d*=10 [15] and in *d*=6 [22]

(d) In view of the previous remark, since an *N*=16 Poincaré supergravity theory coupled to  $E_8/\text{SO}(16)$  scalar manifold exists in three dimensions, we expect that an (16,0) model in two dimensions where the matter scalars parametrize the coset  $E_8/\text{SO}(16)$  and action similar to that given in section 3 can be constructed However such a model probably only exists on-shell, since the off-shell (16,0) superconformal multiplet contains a large number of components with undesirable canonical dimensions which makes the construction of an action highly nontrivial

(e) The signature of the metric  $g_{\alpha\beta}$  has 8*m* positive, and 8*n* negative signs which do not correspond to a lorentzian signature An intriguing possibility might be that, say, for *m*=0, the 8*n* scalars of the theory, together with the unphysical scalars *r* and  $\varphi$  form the coordinates of a special (8*n*+2)-dimensional

manifold with lorentzian signature

(f) We expect that a Wess–Zumino term can be added to the (8,0) model of this paper, in much the same way it is added in the (4,0) model [10]

(g) It seems conceivable that one can gauge any subgroup of the global  $SO(8+n,m)$  group by introducing an (8,0) Yang–Mills multiplet  $(A_\mu, \zeta)$ , where  $\zeta$  is an eight-spinor of  $SO(8)$ . This would work in a similar way as in ref [4]

(h) From the (8,0) model, by truncation, one can obtain a (2,0) model with the field content  $(e_\mu^m, \psi_\mu^i, A_\mu, \lambda^i, \varphi)$  where  $i=1, 2$  is an  $SO(2)$  index. Here the vector field  $A_\mu$  has only one gauge transformation. It is interesting to compare this theory with the one based on the superconformal multiplet  $(e_\mu^m, \psi_\mu^i, B_\mu)$  where the vector field  $B_\mu$  has two independent gauge transformations [8]

(i) Concerning the anomalies, the perturbative Lorentz anomaly in two dimensions will be absent provided that the number of heterotic fermions  $\psi^r$  is fixed to be  $8(m+n+22)$ . The sigma model anomaly [23] can be cancelled by the Hull–Witten mechanism [11], provided that one adds a Wess–Zumino term to the action. Whether the trace anomaly, or the global anomalies [24], or anomalies in any one of the other local symmetries of the theory such as supersymmetry are absent remains to be investigated.

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